2D/3D beach morphology: the role of the wave potential stirring

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CONTENT

1. 2D vs. 3D morphology. Rhythmic morphology.
2. Sediment transport. Stirring function.
3. Bed evolution equation. Potential stirring.
4. Bed-surf instability. Shore-transverse bars.
5. Development of crescentic bars.
6. Conclusions

1. 2D vs. 3D MORPHOLGY. RHYTHMIC MORPHOLOGY.

‘2D’ morphology ($z_b(x)$)

Sea bed topography (and/or shoreline) approximately uniform along the coast

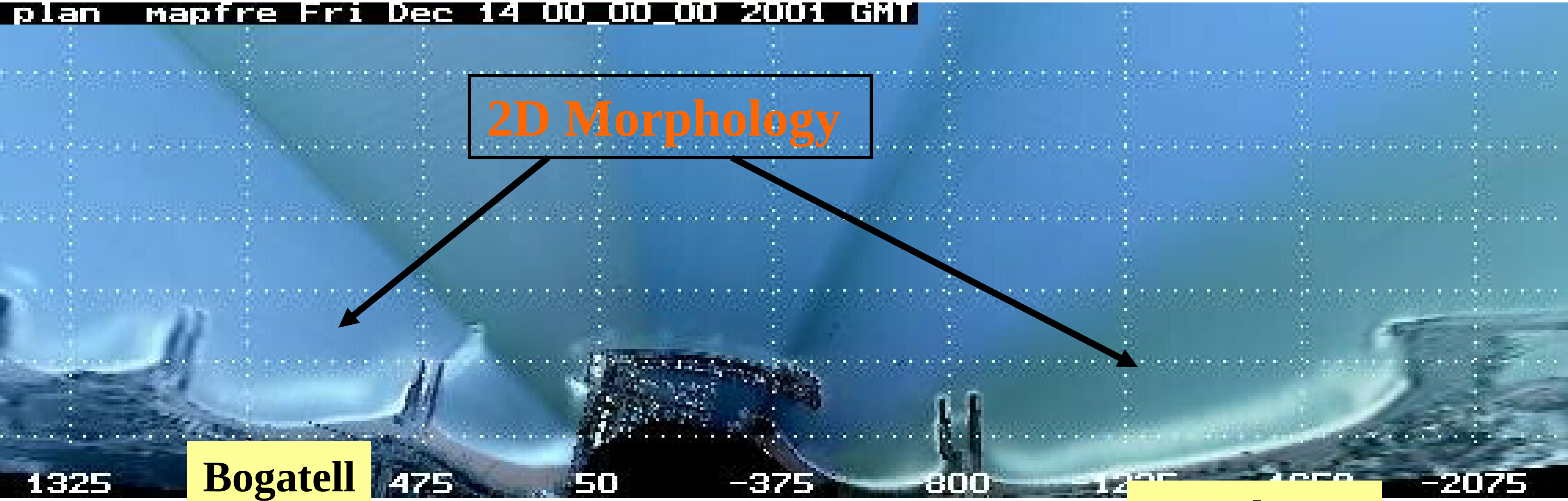
‘3D’ morphology ($z_b(x,y)$)

Sea bed topography (and/or shoreline) with alongshore gradients

Australian beaches

1. 2D vs. 3D MORPHOLOGY. RHYTHMIC MORPHOLOGY.

A beach may change from 2D to 3D and viceversa



Courtesy of ICM, CSIC, Barcelona

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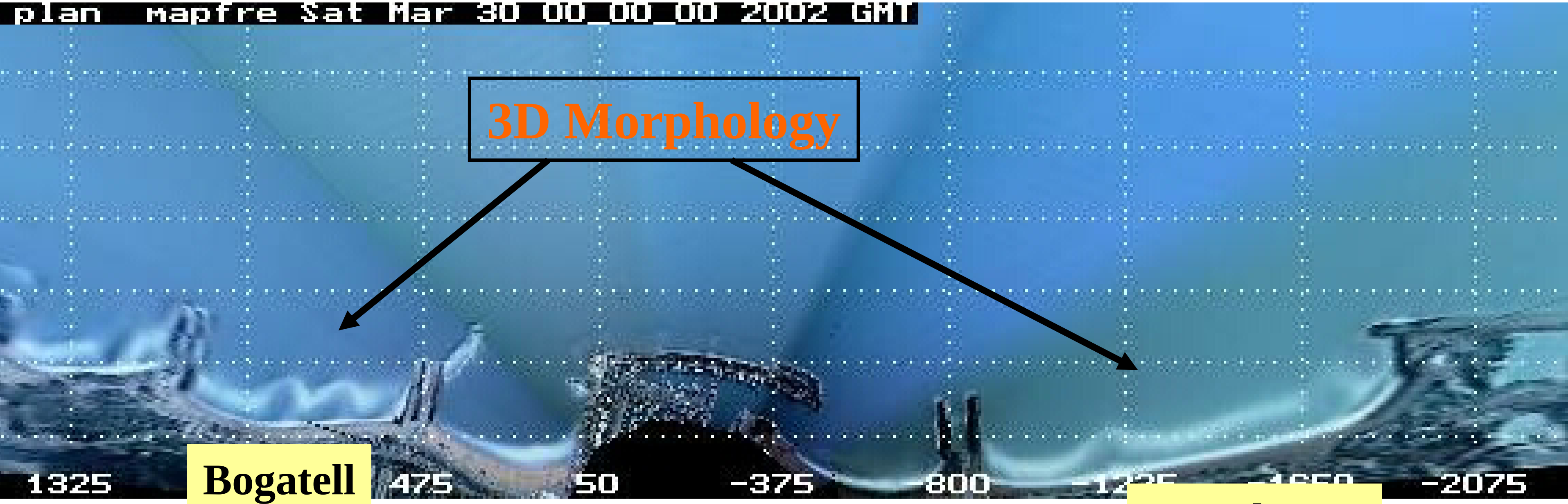

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1. 2D vs. 3D MORPHOLOGY. RHYTHMIC MORPHOLOGY.

Basic field knowledge

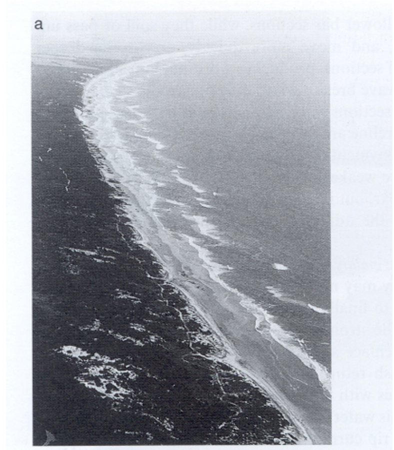
Classification of wave conditions with respect to sediment size
(which if time allows give rise to a particular beach equilibrium state)
(Wright y Short, 1984; Sunamura, 1988; Lippmann y Holman, 1990)

Dissipative conditions

All the incident wave energy is dissipated

Fine sand, large waves
wave period relatively short

Intermediate conditions



3D Morphology (in the surf zone)

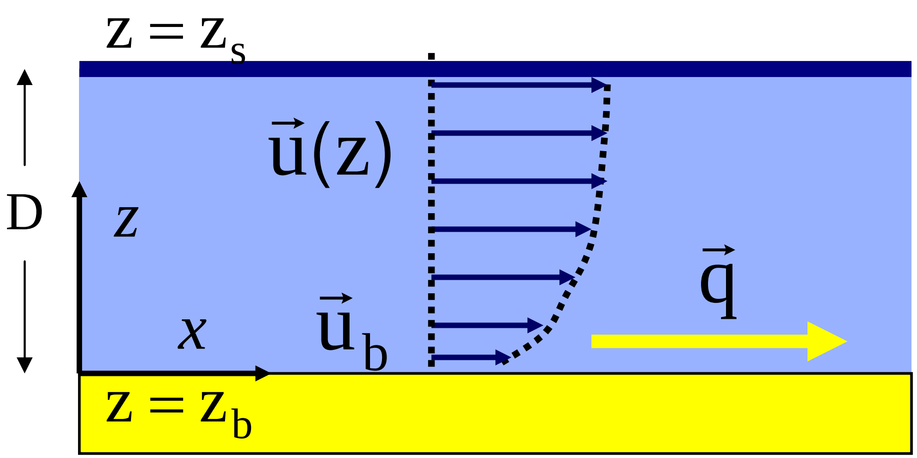
Reflective conditions

A signifiicant fraction of wave energy is reflected (up to ~ 30 %)

Coarse sand, small waves
wave period relatively long

2. SEDIMENT TRANSPORT. THE STIRRING FUNCTION

Bedload transport by a current



Vertically averaged sediment flux
= total volume crossing the horizontal length unit per time unit
= $m^3 / m \times s = m^2\, s^{-1}$

Bagnold, 1963

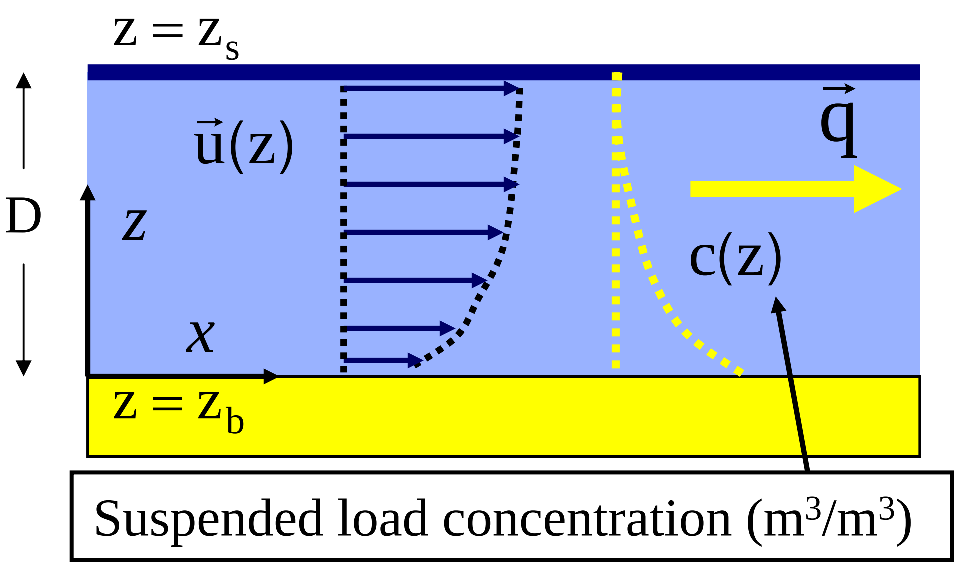
$$\vec{q} = r\left(u_b^2 - u_{cri}^2\right)\vec{u}_b \longrightarrow \vec{q} = \hat{r}\left(v^2 - v_{cri}^2\right)\vec{v}$$

$$\vec{v} = \langle \vec{u} \rangle \frac{1}{D}\int_{z_b}^{z_s} \vec{u}(z)\, dz \longrightarrow \vec{u}_b = \mu \vec{v}$$

$$\vec{q} = \alpha(v)\ \vec{v}$$

2. SEDIMENT TRANSPORT. THE STIRRING FUNCTION

Suspended load transport by a current



$\vec{v} = \langle \vec{u} \rangle$

$$\vec{q} = \int_{z_b}^{z_s} c(z)\,\vec{u}(z)\,dz = \langle c \rangle \langle \vec{u} \rangle \int_{z_b}^{z_s} g(z)\,f(z)\,dz = \mu D \langle c \rangle \langle \vec{u} \rangle = \alpha\,\vec{v}$$

$\vec{u}(z) = f(z)\,\langle \vec{u} \rangle$

$c(z) = g(z) \langle c \rangle$

Stirring function or equivalent depth integrated concentration $\alpha = \mu D < c >$

$$\vec{q} = \alpha(v)\,\vec{v}$$

2. SEDIMENT TRANSPORT. THE STIRRING FUNCTION

However, many complications arise:

- ❑ Vertical profiles vary in space and time:
  - ➢ in particular, c(x,y,z,t) has its own dynamics
- ❑ Current + wave oscillatory flow
  - ➢ very often in different directions

Still, there are relatively simple formulations that work reasonably well in the nearshore with waves + currents.
For example:

- ➢ Bailard
- ➢ Soulsby-Van Rijn

It makes sense to assume:

$$\vec{q} = \alpha(v, u_o, D, \ldots)\ \vec{v}$$

u_o = bed wave orbital velocity

3. BED EVOLUTION EQUATION. POTENTIAL STIRRING

sediment conservation $$\frac{\partial z_b}{\partial t} + \nabla \cdot \left(\alpha \, \vec{v} \right) = 0$$

water conservation $$\frac{\partial D}{\partial t} + \nabla \cdot \left(D \, \vec{v} \right) = 0$$

$$\frac{\partial z_b}{\partial t} + \nabla \cdot \left(\frac{\alpha}{D} D \, \vec{v} \right) = 0 \longrightarrow \frac{\partial z_b}{\partial t} + D \vec{v} \cdot \nabla \left(\frac{\alpha}{D} \right) + \frac{\alpha}{D} \nabla \cdot \left(D \, \vec{v} \right) = 0$$

$$\frac{\partial z_b}{\partial t} + D \vec{v} \cdot \nabla \left(\frac{\alpha}{D} \right) - \frac{\alpha}{D} \frac{\partial D}{\partial t} = 0$$

$$D = z_s - z_b$$

$$\frac{\alpha}{D} << 1 \quad , \frac{\partial z_s}{\partial t} \approx \frac{\partial z_b}{\partial t}$$

$$\left(1 + \frac{\alpha}{D} \right) \frac{\partial z_b}{\partial t} + D \vec{v} \cdot \nabla \left(\frac{\alpha}{D} \right) = \frac{\alpha}{D} \frac{\partial z_s}{\partial t}$$

$$\frac{\partial z_b}{\partial t} = -D \, \vec{v} \cdot \nabla \left(\frac{\alpha}{D} \right)$$

3. BED EVOLUTION EQUATION. POTENTIAL STIRRING

Bed evolution equation BEE

$$\frac{\partial z_b}{\partial t} = -D\,\vec{v}\cdot\nabla\left(\frac{\alpha}{D}\right)$$

$\frac{\alpha}{D} =$ **'potential stirring'** or depth averaged concentration $\sim \langle c \rangle$

CONSEQUENCE:

Acretion **condition** $\left(\frac{\partial z_b}{\partial t} > 0\right)$**:** $\vec{v}\cdot\nabla\left(\frac{\alpha}{D}\right) < 0$ **Current *against* the gradient in potential stirring**

Erosion **condition** $\left(\frac{\partial z_b}{\partial t} < 0\right)$**:** $\vec{v}\cdot\nabla\left(\frac{\alpha}{D}\right) > 0$ **Current *with* the gradient in potential stirring**

4. 'BED-SURF' INSTABILITY. SHORE-TRANSVERSE BARS.

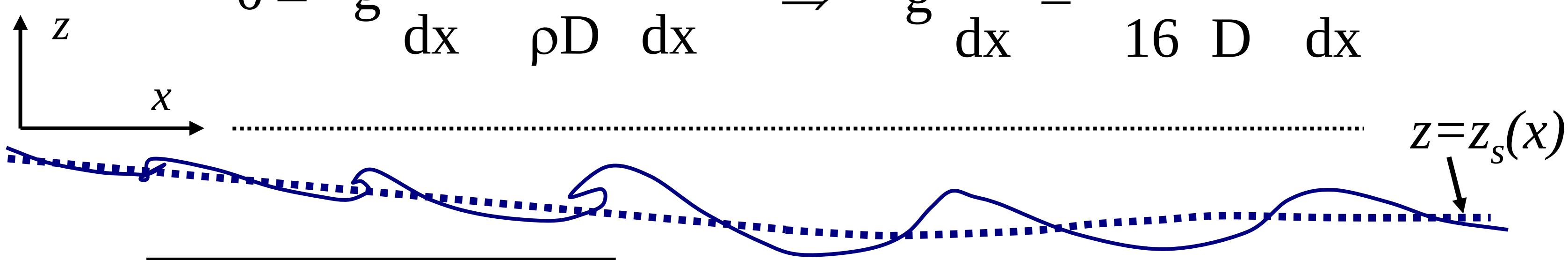
Shore-normal wave incidence

No alongshore gradients

Mean hydrodynamics:

- water conservation $\Rightarrow$ $v_x = 0$
- alongshore momentum balance $\Rightarrow$ $v_y = 0$
- cross-shore momentum balance :

$$0 = -g\frac{dz_s}{dx} - \frac{1}{\rho D}\frac{dS_{xx}}{dx} \quad \Rightarrow \quad g\frac{dz_s}{dx} = -\frac{3}{16}\frac{gH}{D}\frac{dH}{dx}$$



surf zone

$$\frac{dH}{dx} > 0 \quad \Rightarrow \quad \frac{dz_s}{dx} < 0$$

shoaling zone

$$\frac{dH}{dx} < 0 \quad \Rightarrow \quad \frac{dz_s}{dx} > 0$$

4. 'BED-SURF' INSTABILITY. SHORE-TRANSVERSE BARS.

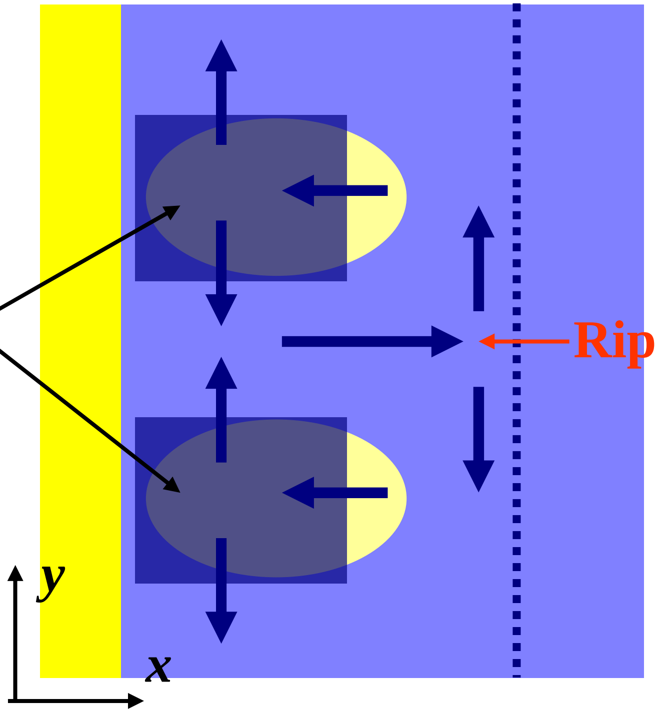
Shore-normal wave incidence
With alongshore gradients

Mean hydrodynamics:

By assuming the same momentum balance in x:

$$g\frac{dz_s}{dx} = -\frac{3}{16}\frac{gH}{D}\frac{dH}{dx}$$

Larger set-up in the mean water level



There is no possible equilibrium without currents.

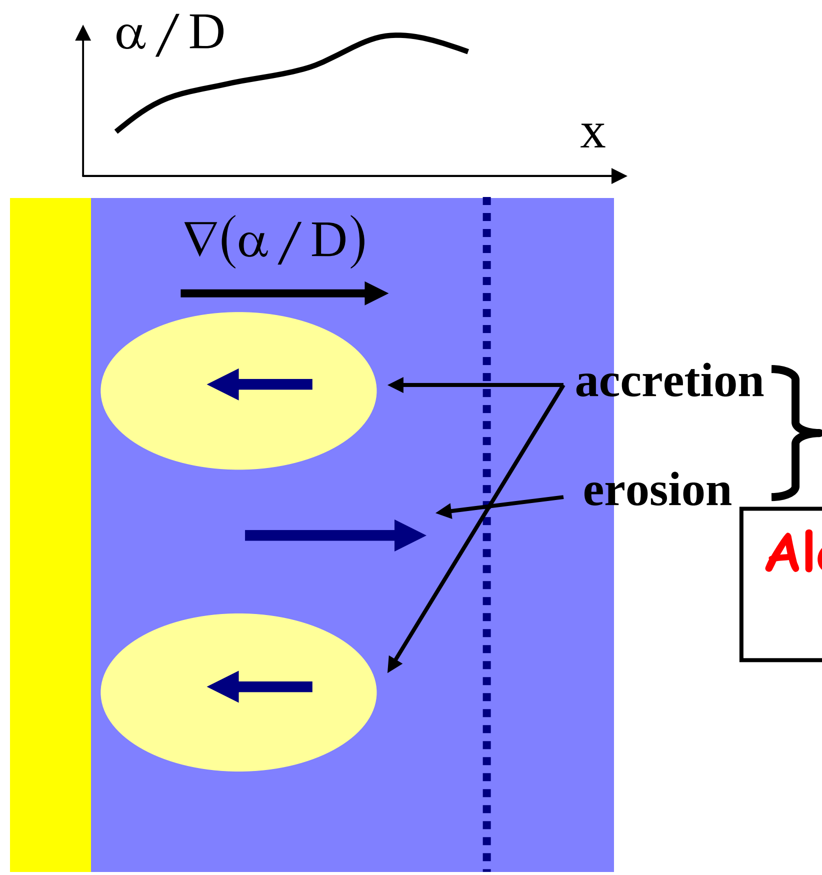
4. 'BED-SURF' INSTABILITY. SHORE-TRANSVERSE BARS.

Shore-normal wave incidence

Morphodynamic instability:

Wich is the direction of the potential stirring?

Seaward



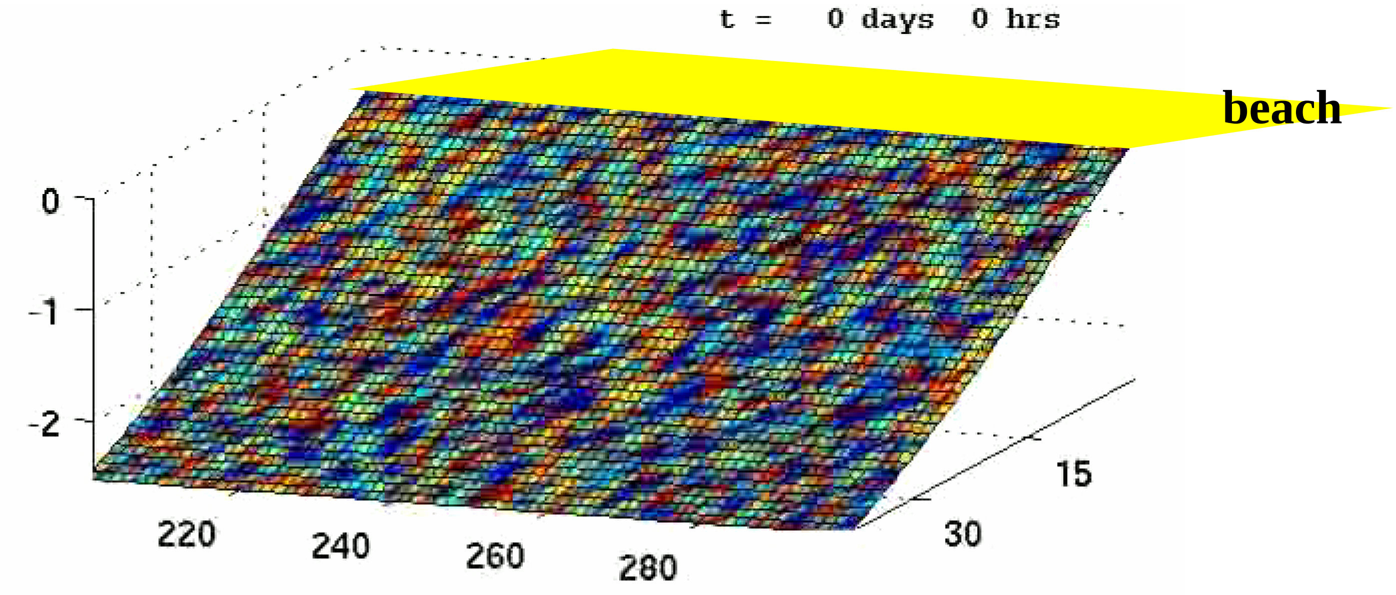
Positive feedback

Alongshore uniform topography is unstable

- **Formation of transverse bars**
- **Associated circulation**

4. 'BED-SURF' INSTABILITY. SHORE-TRANSVERSE BARS.

Numerical simulation of the formation of transverse bars from initial small bed perturbations.
Shore-normal incident waves with H=1 m
morfo55 model.
(Garnier et al., 2005)



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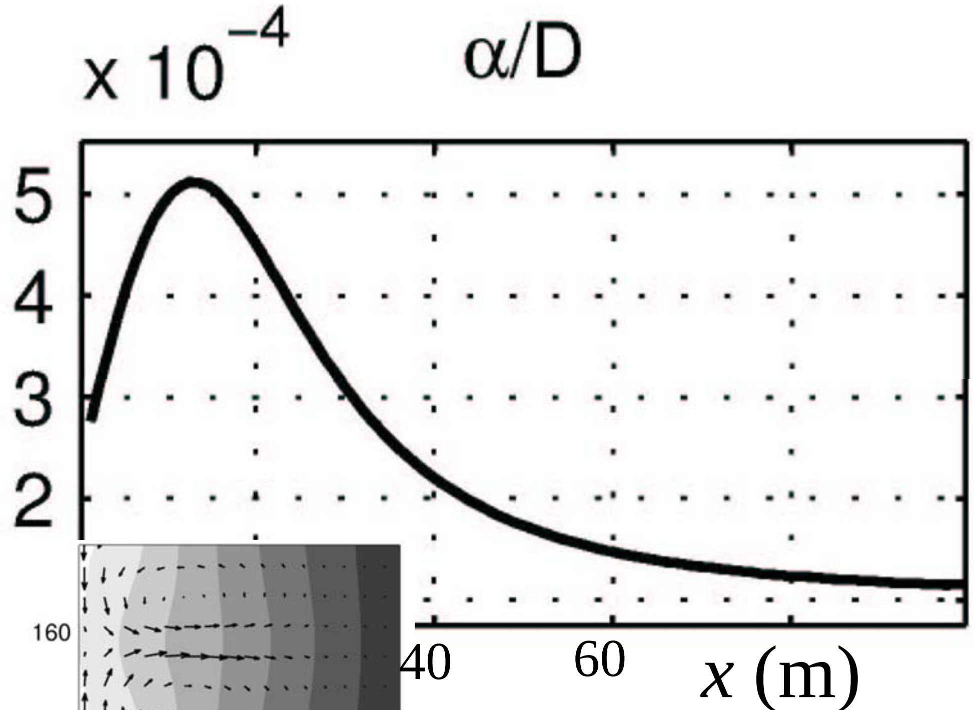
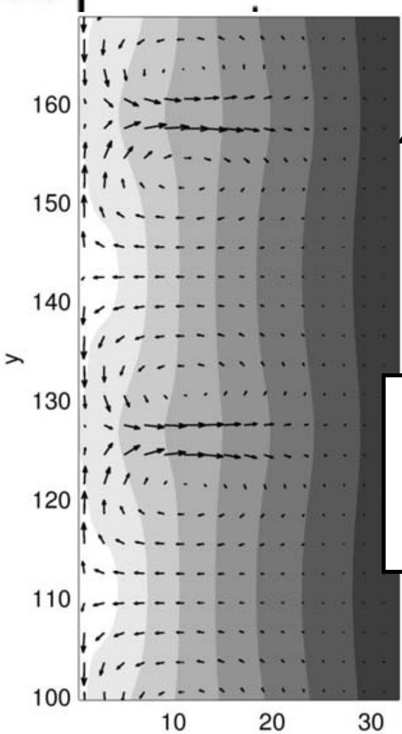
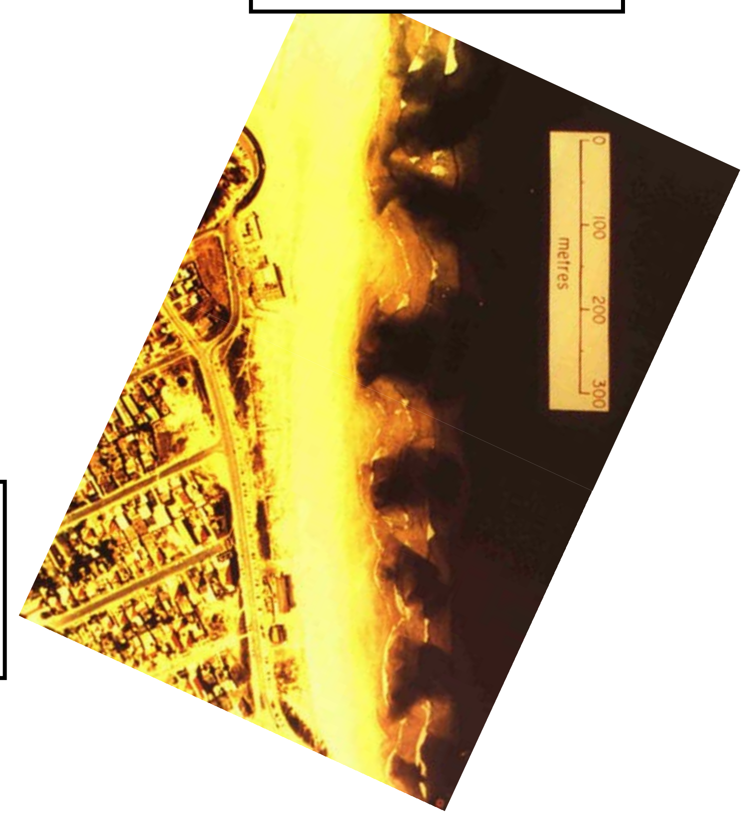




Transverse bars and rip currents.
Nonlinear model morfo55
(Garnier et al., 2005)

Aerial picture of transverse bars



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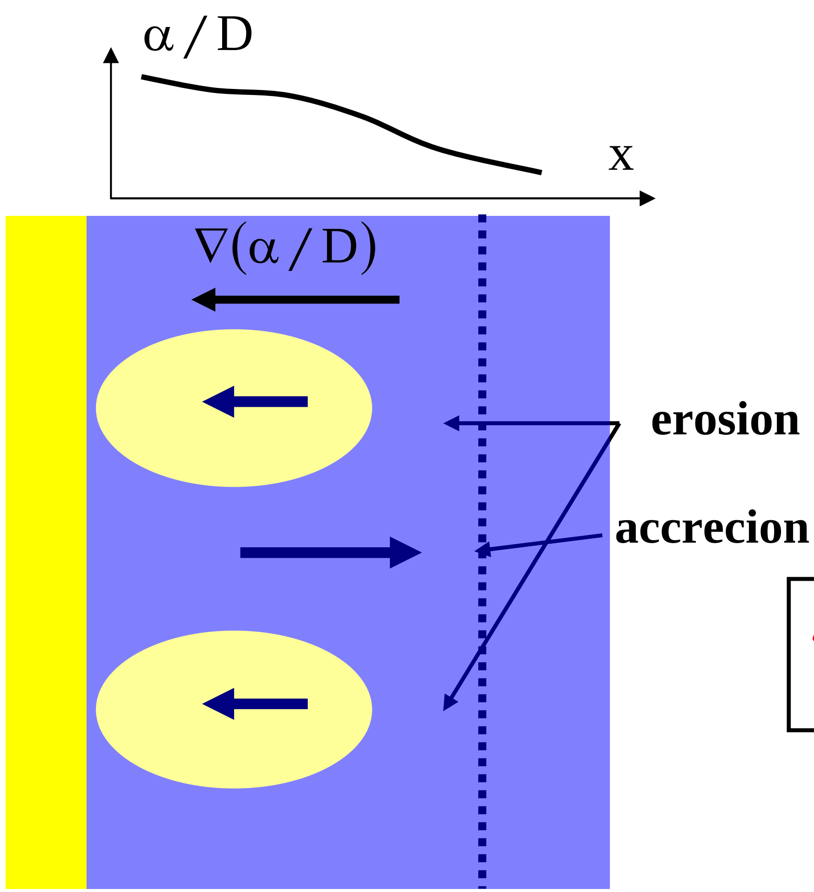


Negative feedback

Alongshore uniform topography is stable

4. 'BED-SURF' INSTABILITY. SHORE-TRANSVERSE BARS.

Therefore:

- ❑ **seaward gradient in potential stirring in the surf zone**
  - ➢ **INSTABILITY**
- ❑ **shoreward gradient**
  - ➢ **STABILITY**

BUT what determines this gradient?

$$\vec{q} = \alpha(v, u_o)\ \vec{v} \approx \alpha(u_o)\ \vec{v} = \mu\, u_o^n\ \vec{v}$$

initially, $v << u_o$

Wave orbital velocity $u_o \approx \frac{1}{2}\gamma_b \sqrt{gD}$

$$\frac{\alpha}{D} \propto D^{(n/2)-1}$$

4. 'BED-SURF' INSTABILITY. SHORE-TRANSVERSE BARS.

$$\frac{\alpha}{D} \propto D^{(n/2)-1}$$

- bedload, $n = 2$ $\Rightarrow$ $\frac{\alpha}{D} \approx$ cte. $\Rightarrow$ **stability**
- suspended load, $n > 2 \Rightarrow$ $\frac{\alpha}{D}$ Seaward increasing $\Rightarrow$ **instability**

bedload: coarse sand, small waves → reflective conditions

Suspended load: fine sand, large waves → dissipative or intermediate conditions

Observations show that 3D morphology occurs in intermediate conditions

BUT what happens for dissipative conditions?

4. 'BED-SURF' INSTABILITY. SHORE-TRANSVERSE BARS.

Infragravity waves
In addition to ordinary wind or swell waves with T ~ 1-20 s
There are low frequency waves with T ~ 20 s - O(1 min.))

Because of their low frequency these waves do not break at the shoreline ⇒
Their amplitude is maximum at the shoreline (shoaling).

Wind/swell wave orbital velocity **'+'** infragravity wave orbital velocity
shoreward decreasing *shoreward increasing*

$\alpha \approx cte. \quad \Rightarrow \quad \dfrac{\alpha}{D}$ seaward decreasing ⇒ STABILITY

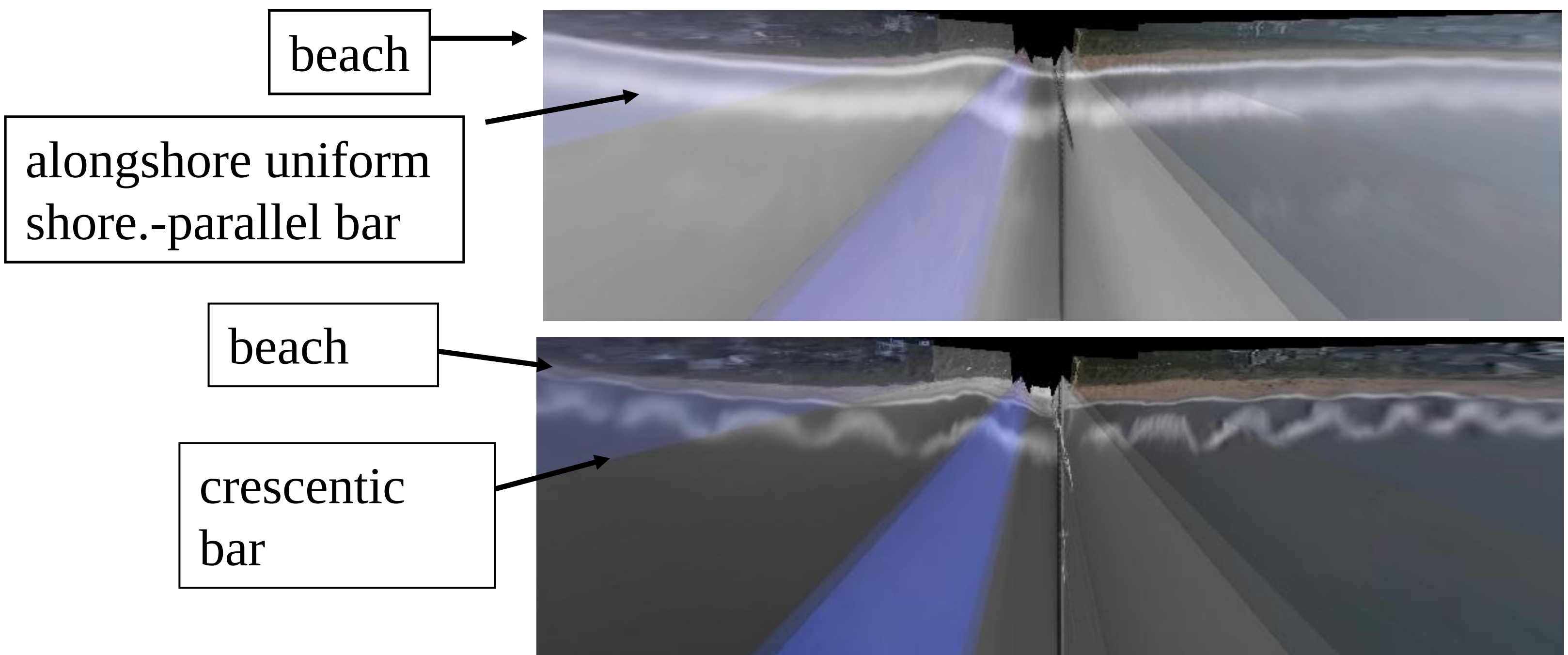
The ratio (energy in the infragravity band / wind/swell wave energy)
is maximum for dissipative conditions ⇒

Dissipative conditions ⇒
Morphodynamic stability under alongshore non-uniform perturbations

5. DEVELOPMENT OF CRESCENTIC BARS.

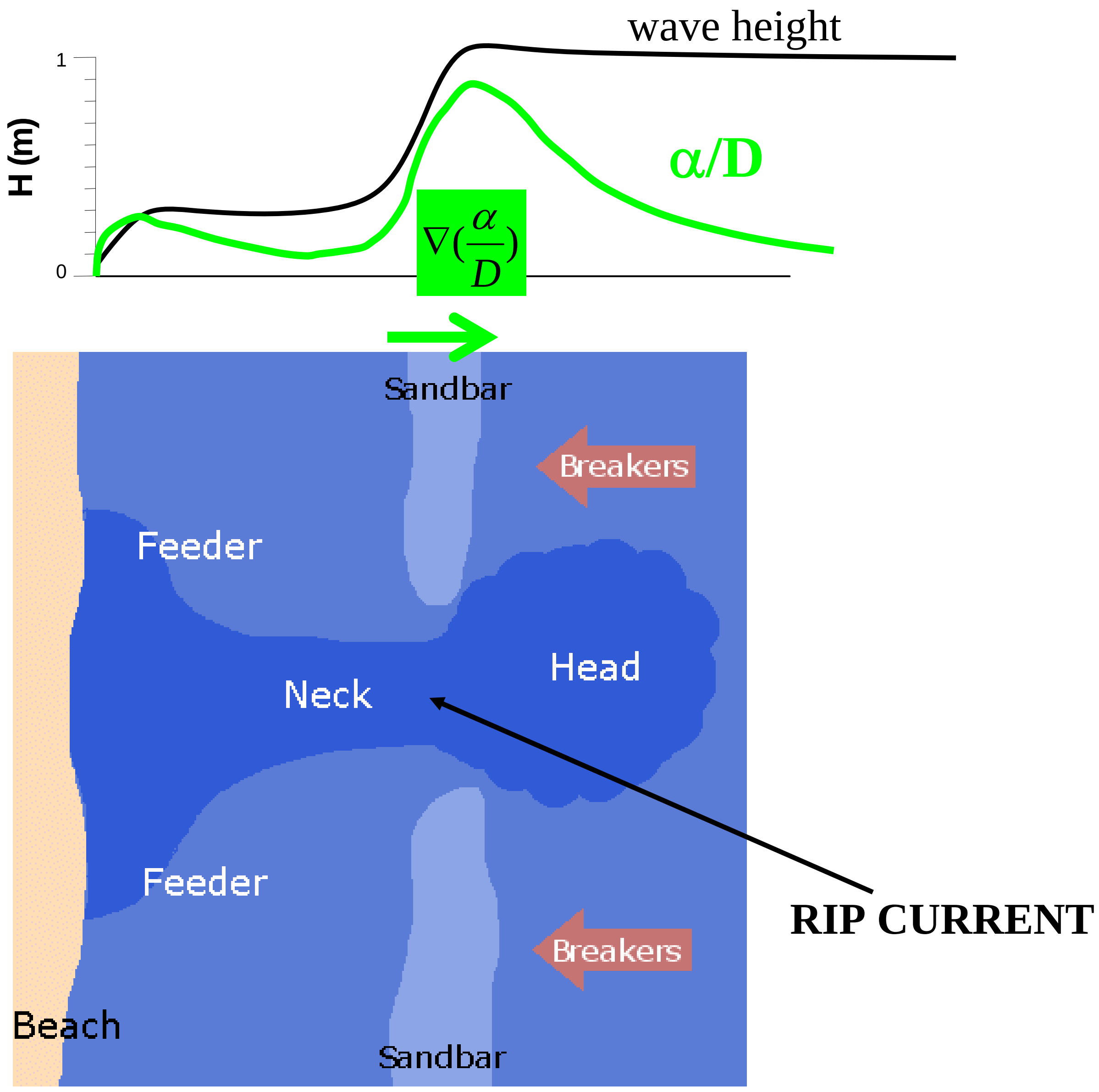
Crescentic bars are the most common (at least best known) type of rhythmic topography.

Can bed-surf instability explain why a shore-parallel bar becomes crescentic?



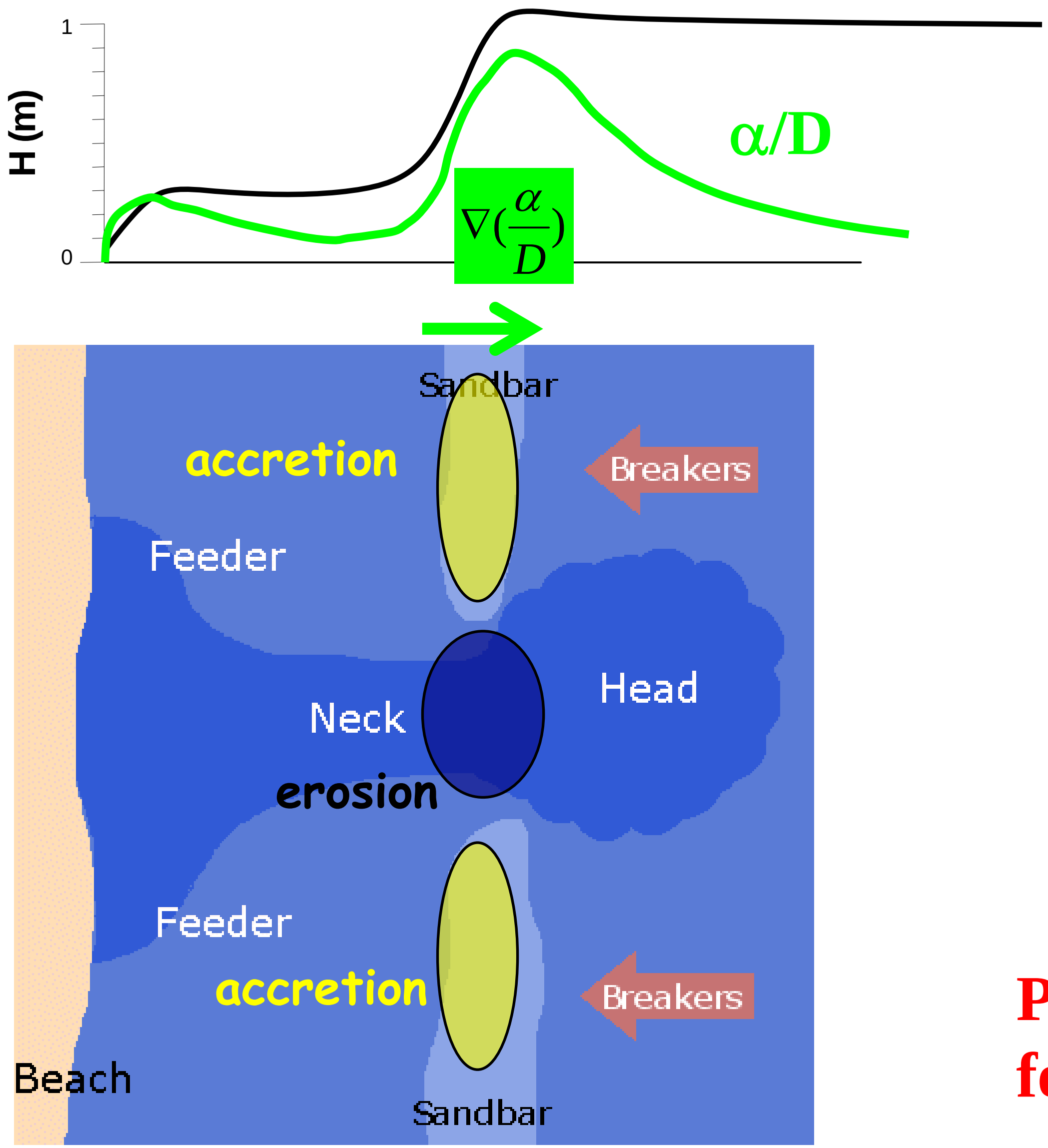
Duck beach, NC, USA

5. DEVELOPMENT OF CRESCENTIC BARS.



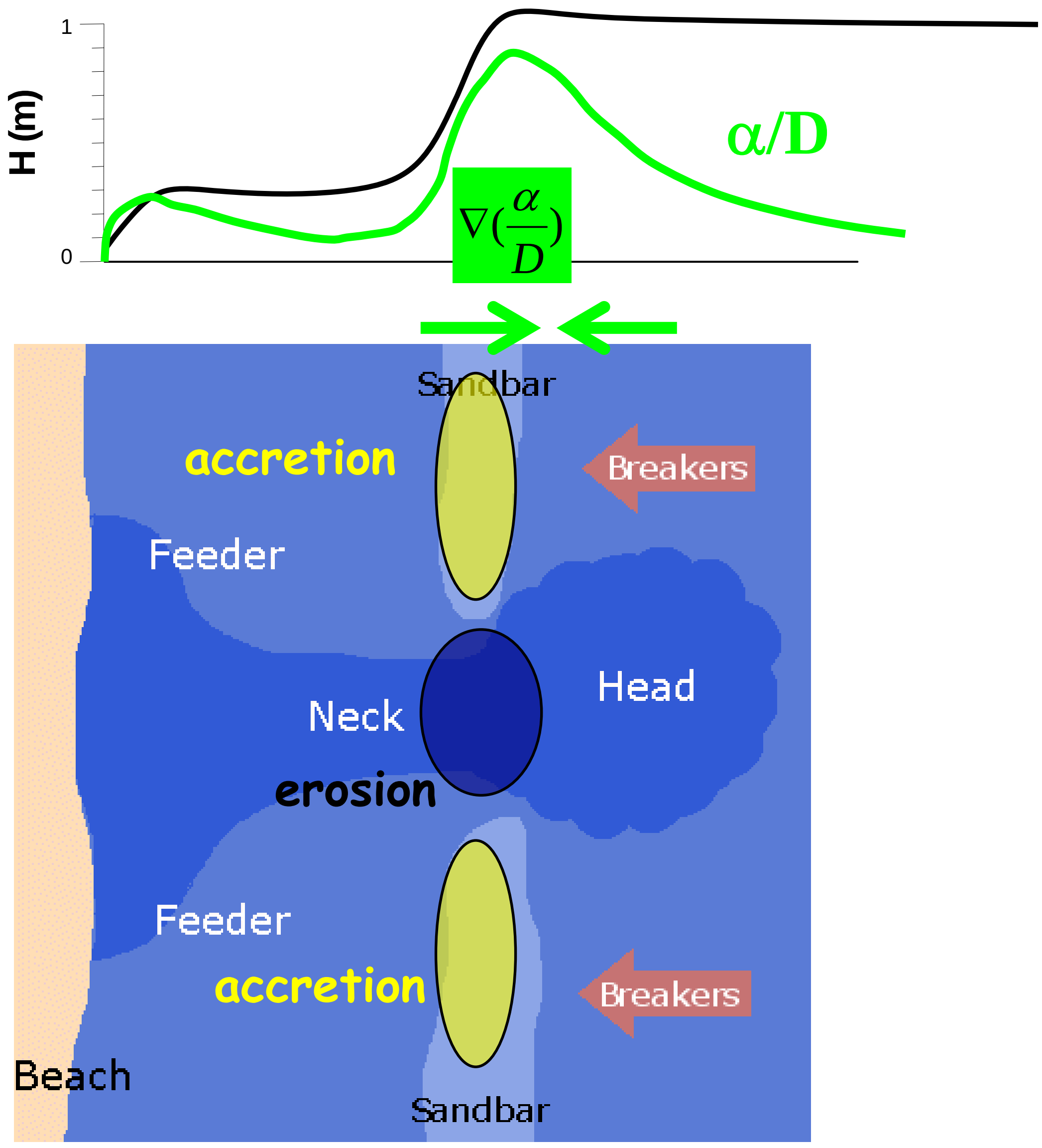
Courtesy of NOAA

5. DEVELOPMENT OF CRESCENTIC BARS.



Positive feedback

5. DEVELOPMENT OF CRESCENTIC BARS.



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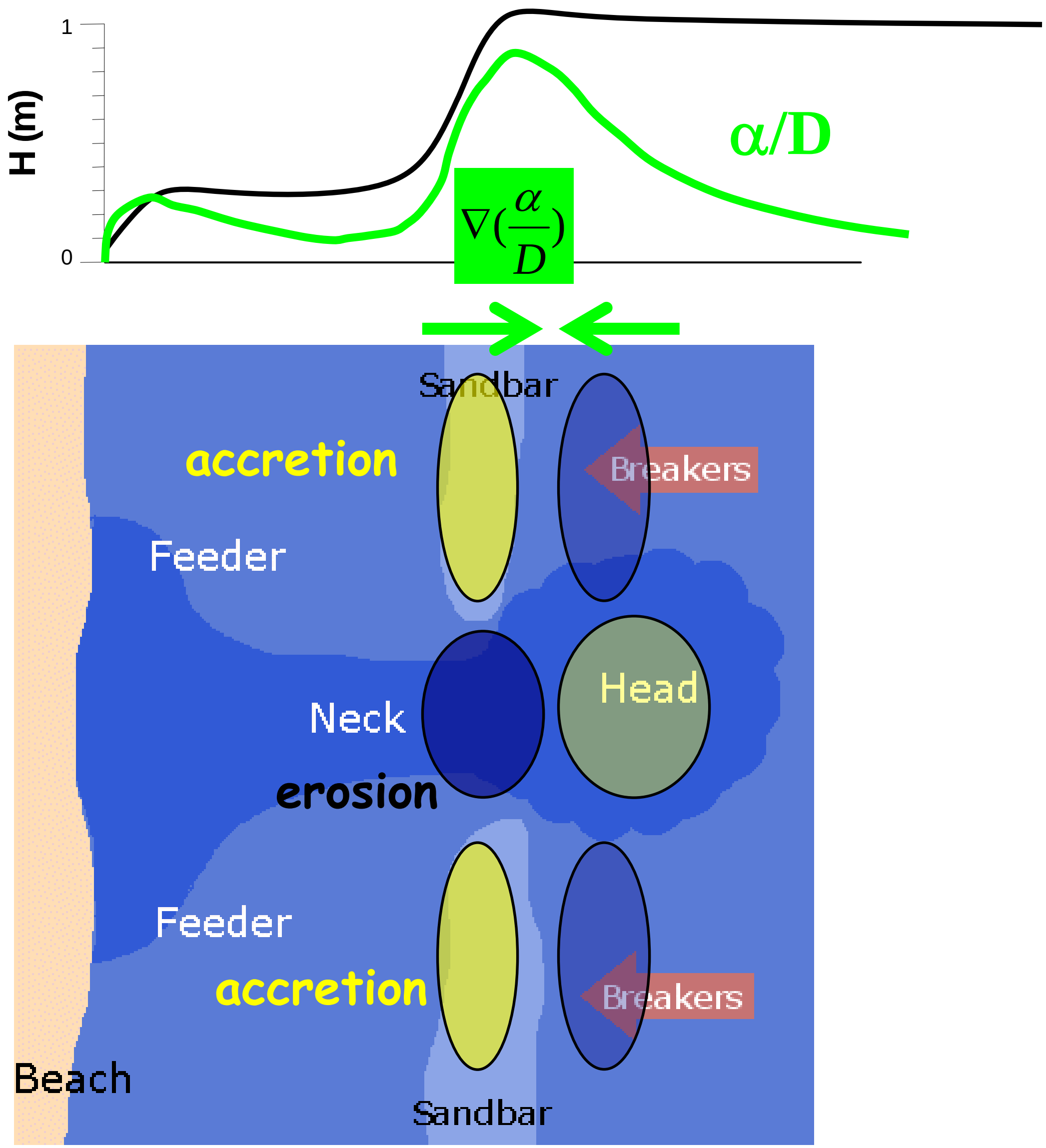

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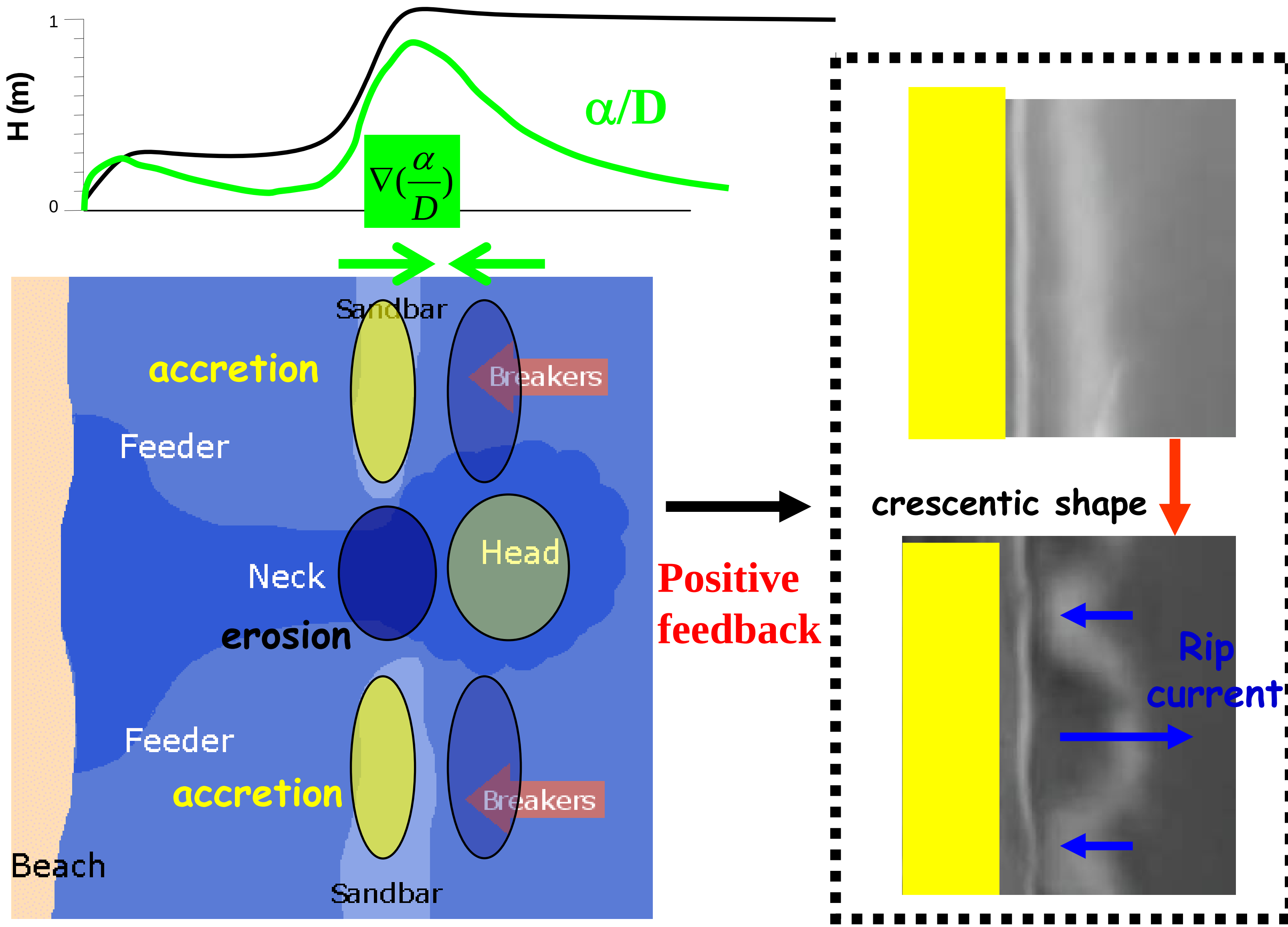

6. CONCLUSIONS.

- The potential stirring or depth averaged equivalent concentration is a promising tool to understand and predict
  - the stability/instability of 2D morphology
  - the emerging morphology when 2D morphology is unstable

- Field experiments are needed to measure depth averaged equivalent concentration profiles and check against observed morphodynamics

- Limitations: more complex sediment transport processes:
  - anisotropy
  - transport driven by waves without mean current
  - dynamics of sediment concentration, space and time lags, …
  - dynamics of the vertical structure (undertow, ….)