



# *“Effect of Non-Hydrostatic Pressure Distributions on Bedload Transport”*

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## ***Introduction***

$$\tau^* = \frac{\tau_b}{(\rho_s - \rho)gD}$$

The assumption of **hydrostatic pressure** is implicit in the definition of the Shields stress parameter



$$\frac{dp}{dz} = -\rho g$$

**hydrostatic pressure distribution**

This hypothesis is accurate in the case of uniform and rectilinear flow, but fails where local non-hydrostatic pressure distributions can affect the mobility of sediment and hence the bedload transport rate.



Study of the effect of the **non hydrostatic pressure distribution** on sediment transport

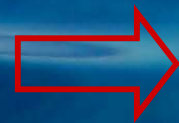
Proposal of general criteria, experimentally based, to estimate sediment transport rate

# Theoretical background

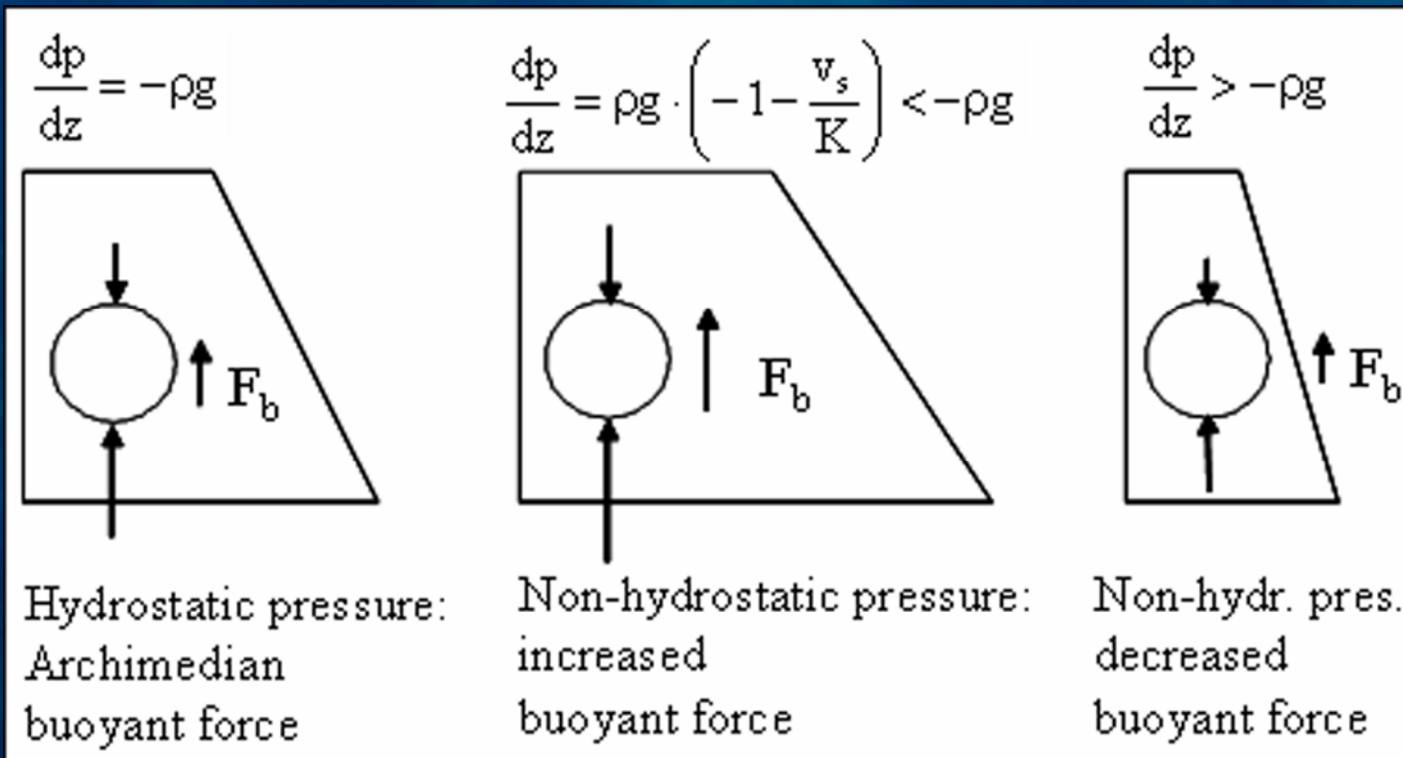
A ground-water flow induces a non hydrostatic pressure distribution along the vertical

Darcy's law

$$v_s = -K \frac{\partial h_p}{\partial z}$$



$$\frac{dh_p}{dz} = \frac{d}{dz} \left( \frac{p}{\rho g} + z \right) = -\frac{v_s}{K}$$



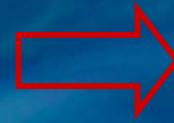
$$\frac{1}{\rho g} \frac{dp}{dz} = -1 - \frac{v_s}{K}$$

general expression for the pressure distribution

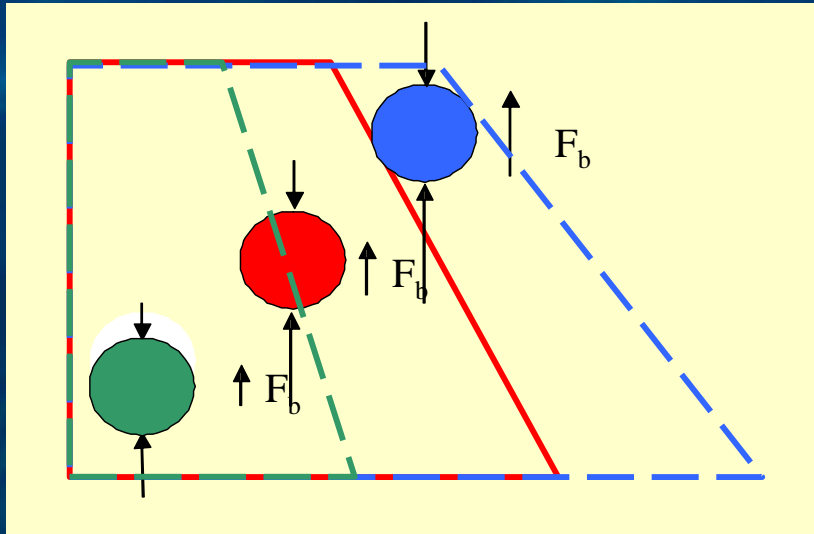
Vertical pressure gradient  $dp/dz$  near the bed and buoyant force  $F_b$  acting on a bed particle.

## Theoretical background

The generalized expression for the Shields parameter



$$\tau^* = \frac{\tau_b}{\left( \rho_s - \rho \left( 1 + \frac{v_s}{K} \right) \right) gD}$$

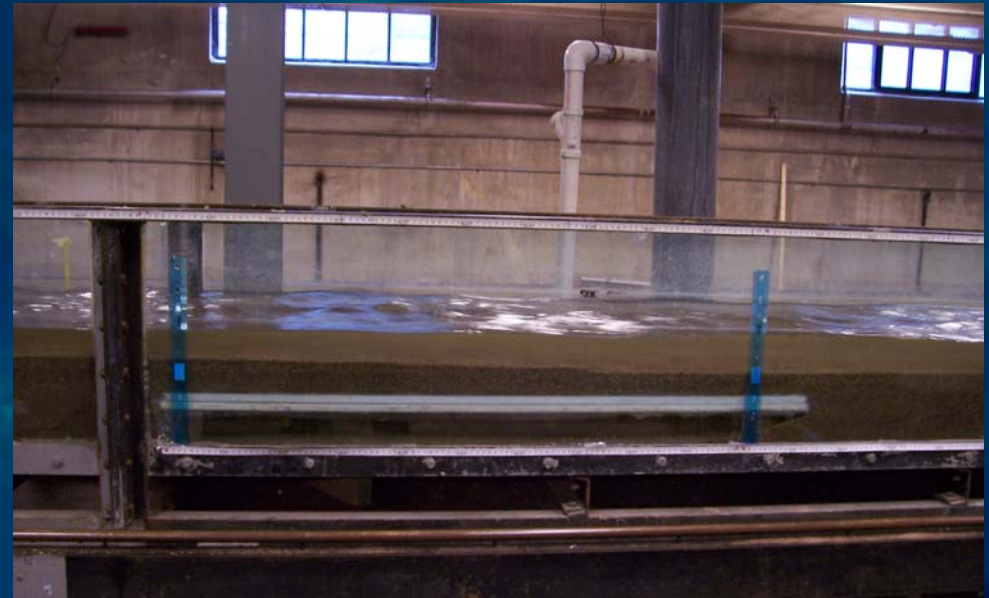
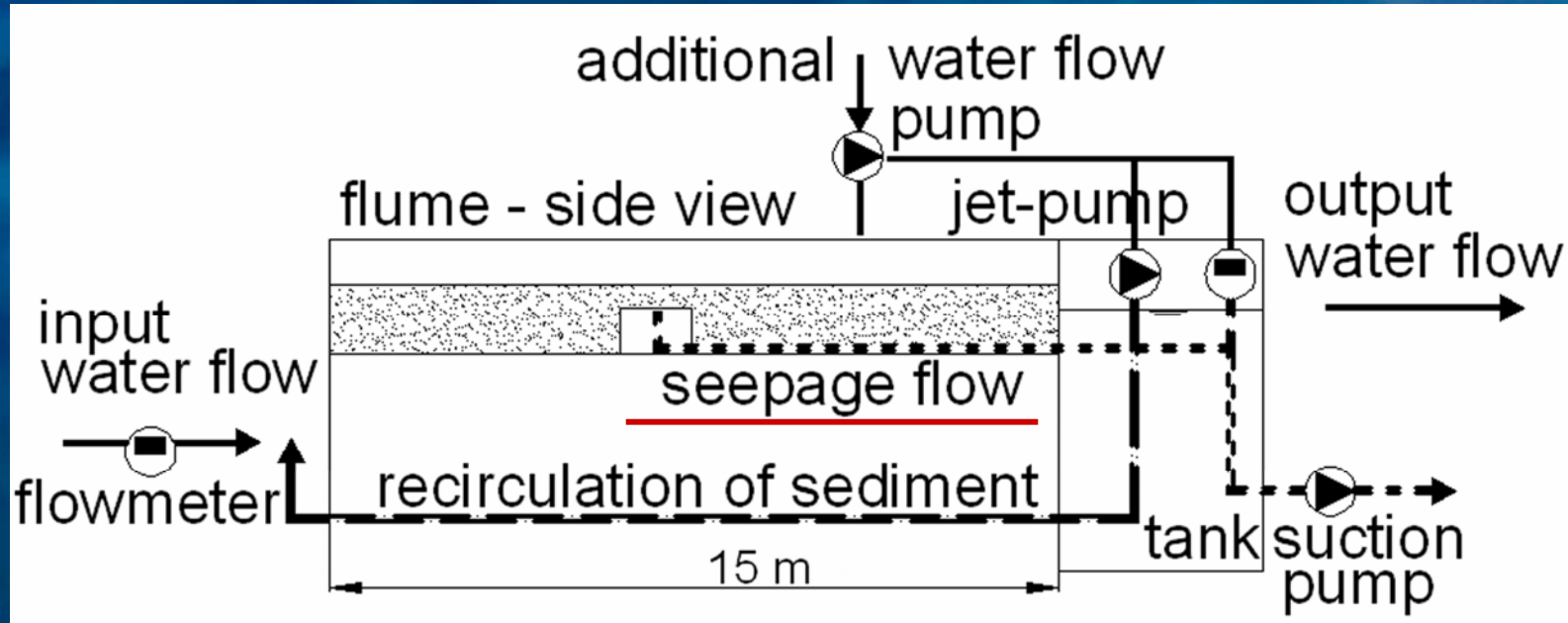


Both upward and downward seepage flows through a sediment bed were used to create a non-hydrostatic component of pressure affecting the particles at the bed surface. The velocity  $v_s$  increases or decreases the Shields parameter

Experimental study of the groundwater flow, and its effect on sediment transport and bed morphology

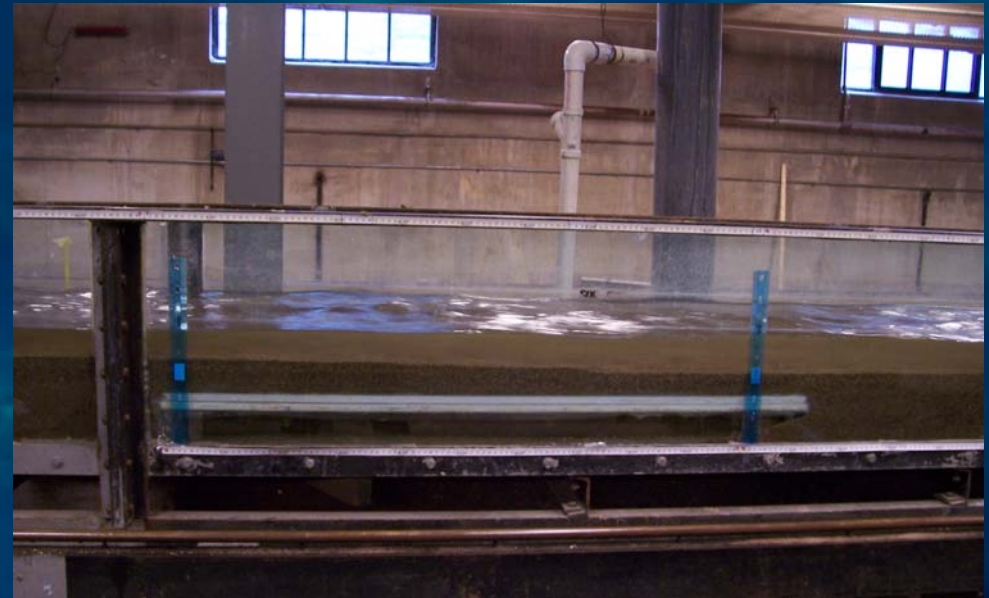
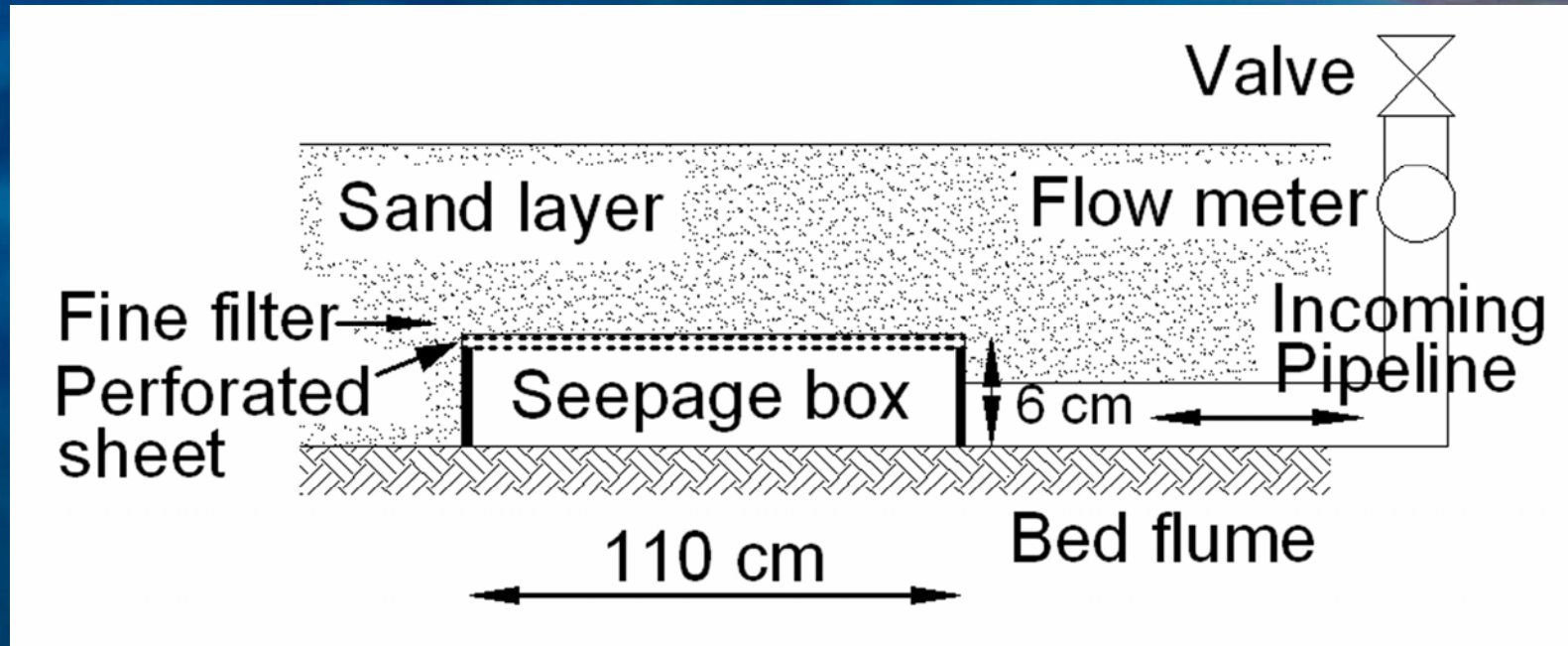
## *Experimental set-up*

The system was a sediment-recirculating, water-feed flume



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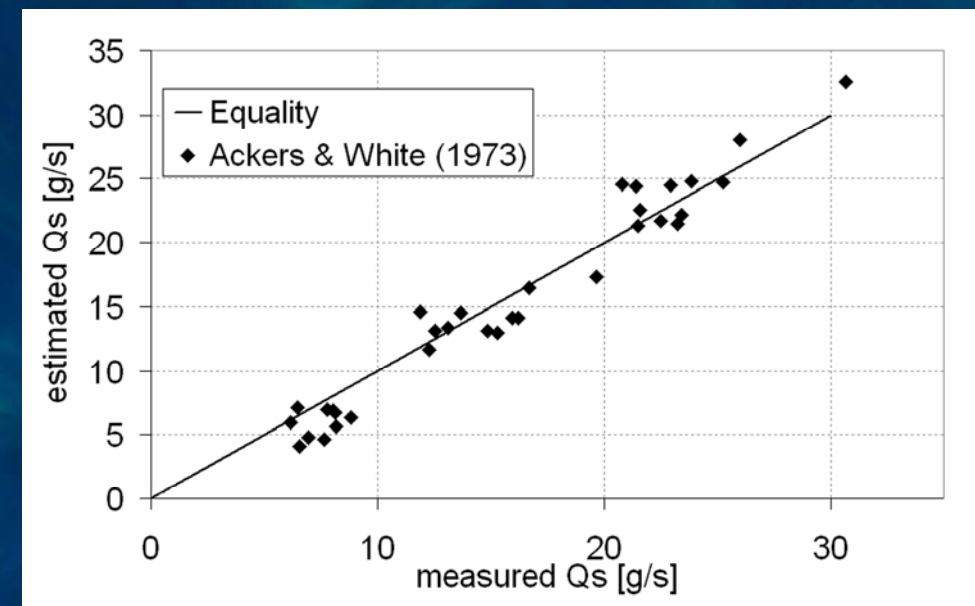
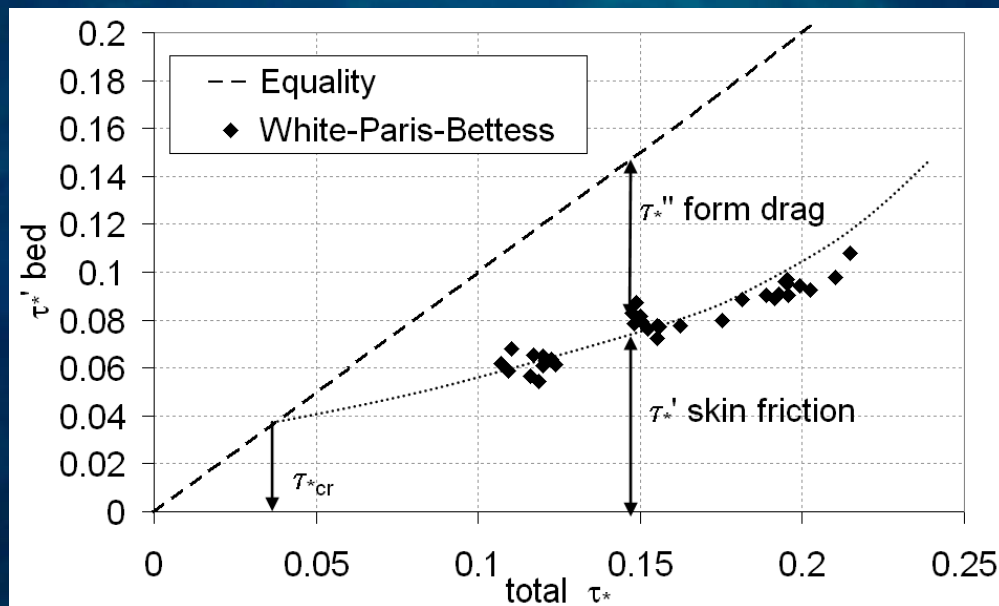
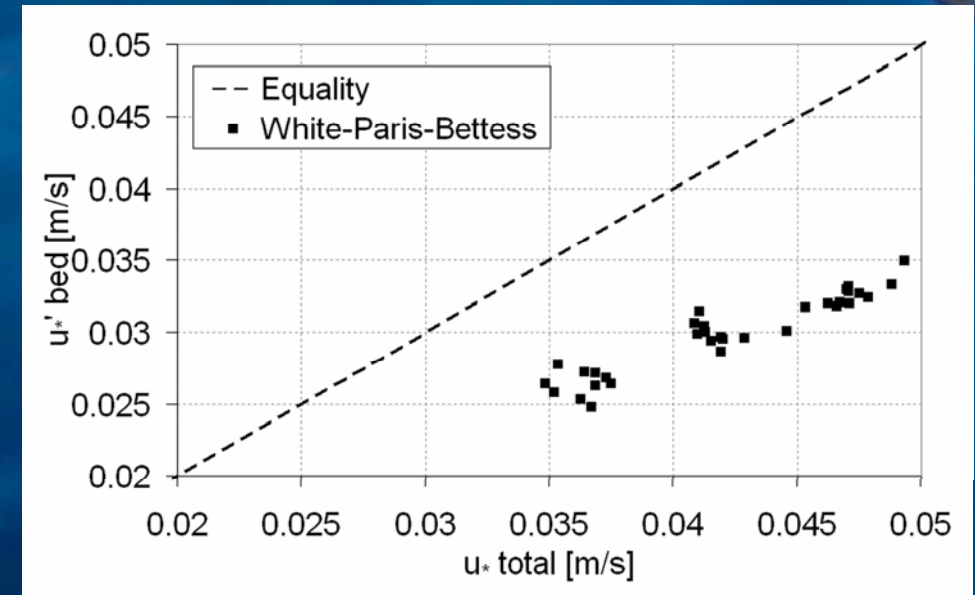
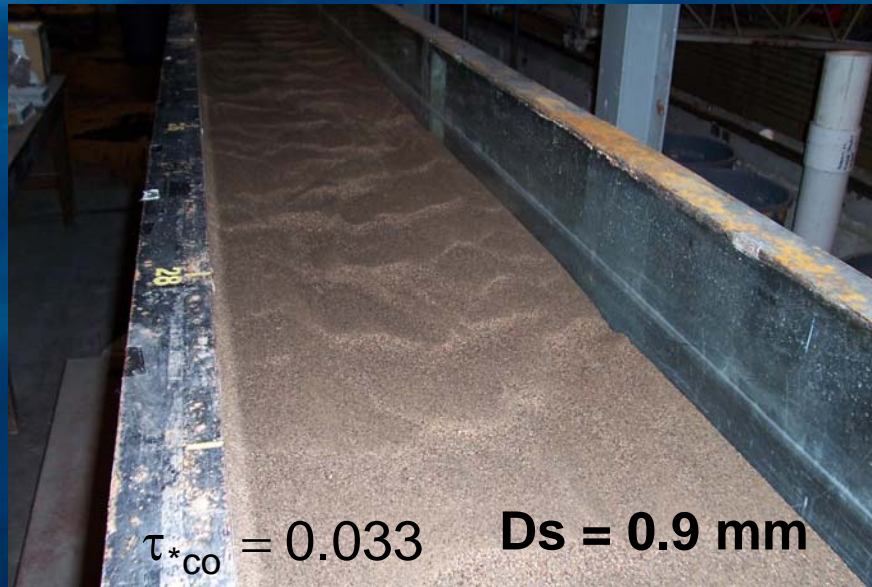
In a sediment-recirculating flume the total amount of sediment is conserved. In addition, a downstream weir controls the downstream elevation of the bed. The combination of these two conditions constrains the bed slope at mobile-bed equilibrium



- ➡ **Solid discharge** measured on the recirculating line.
- ➡ **Bed elevation:** five points were measured for each transverse cross-section and the average value was assumed as the cross-sectional mean value of the bottom elevation for the purpose of characterizing long profiles.
- ➡ **Flow velocity:** velocity profiles were measured using a micro-propeller with a diameter of 14 mm.

# Sediment transport relationship

A simple sediment transport relationship has been applied in the case of hydrostatic pressure distribution, from all values of measured solid discharge





## Experimental activity

Run code	$Q_w$ [l/s]	$Q_{\text{seepage}}$ [l/m]					
		20	40	60	80	100	140
1*	20.6	20	40	60	80	100	140
2*	16	20	40	60	80	100	140
3*	12	20	40	60	80	100	140
4**	20.6	30		60		90	
5**	16	30		60		90	
6**	12	30		60		90	

Preliminary experiments with the same hydraulic conditions in the flume and different seepage flow, showed a quick answer of the bed to the seepage flow.

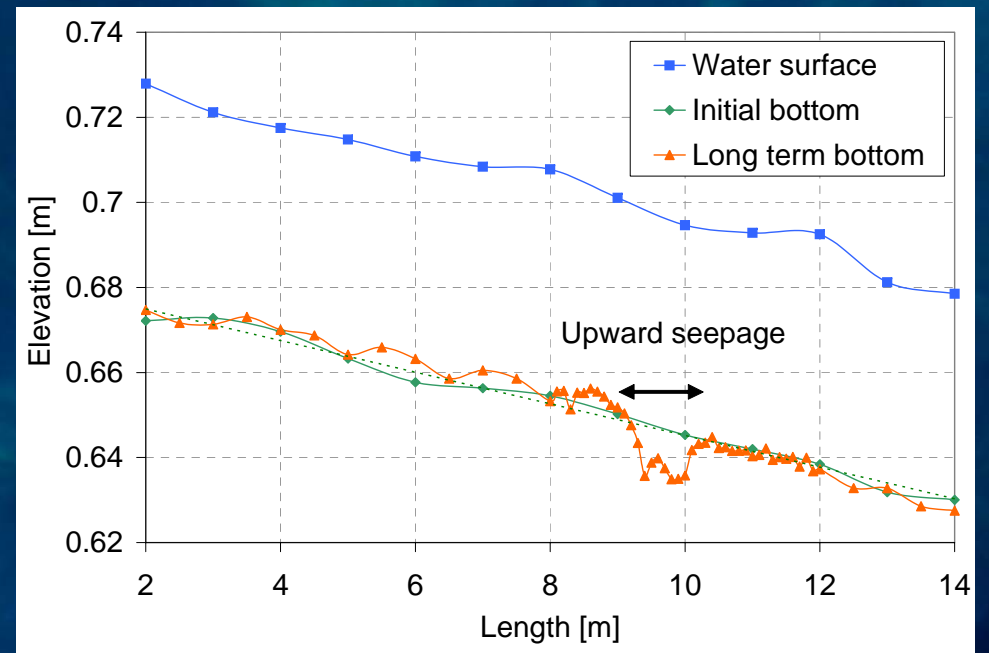
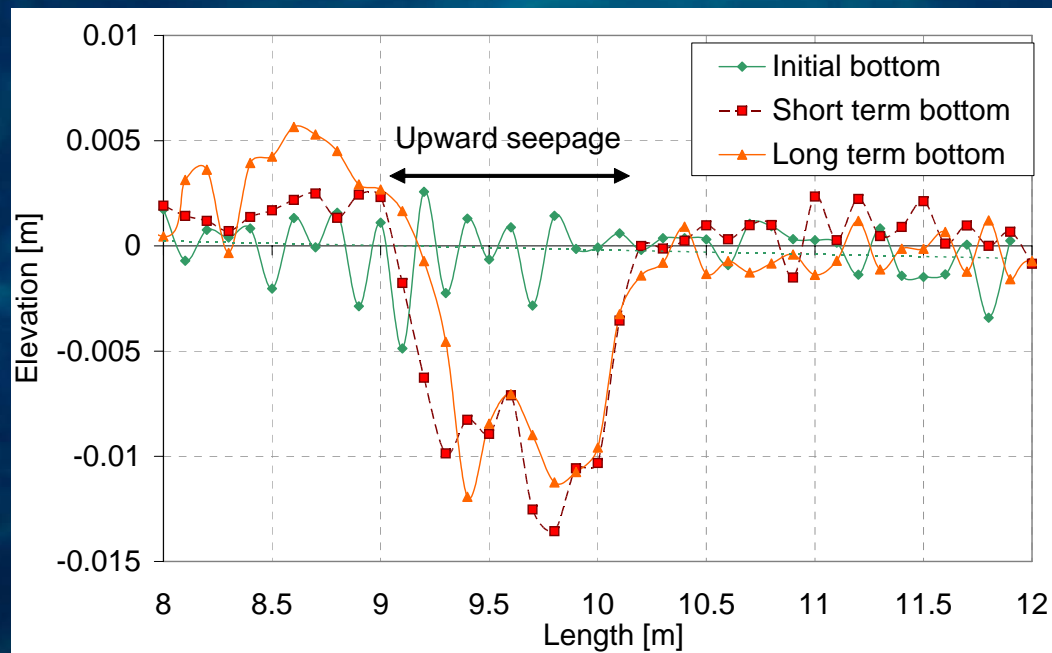
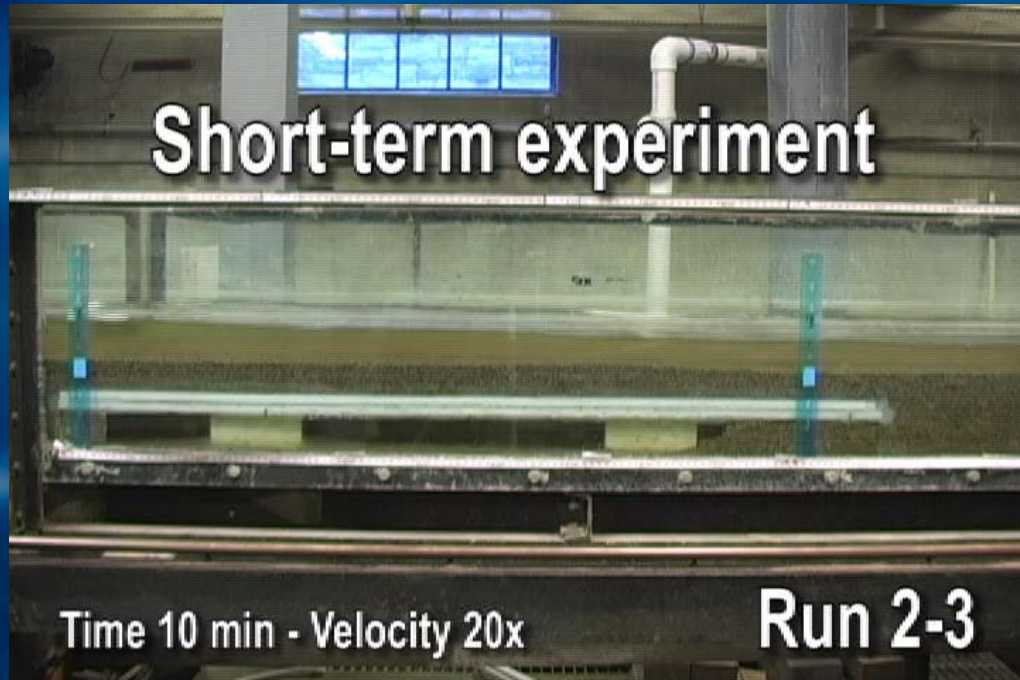


**Short term** experiment to catch the answer of the bed to the seepage flow (10 minutes)

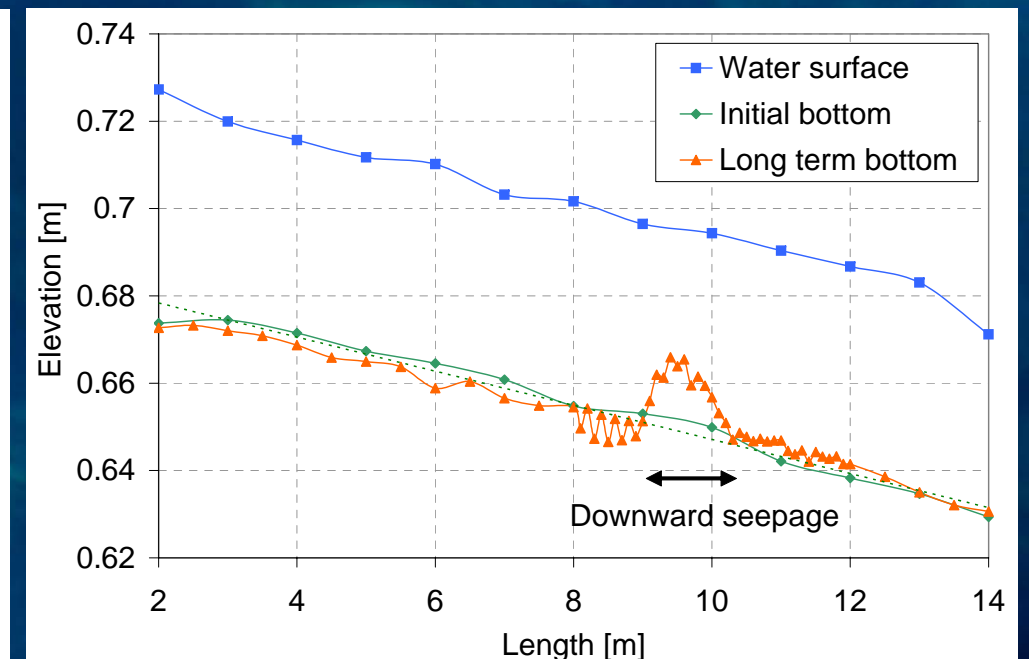
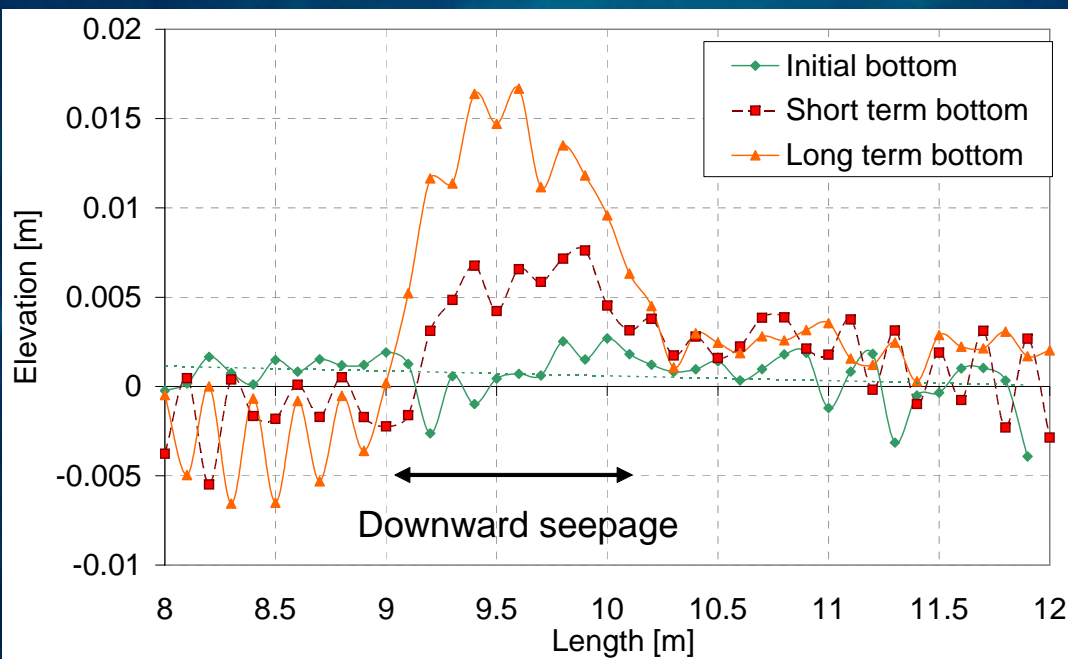
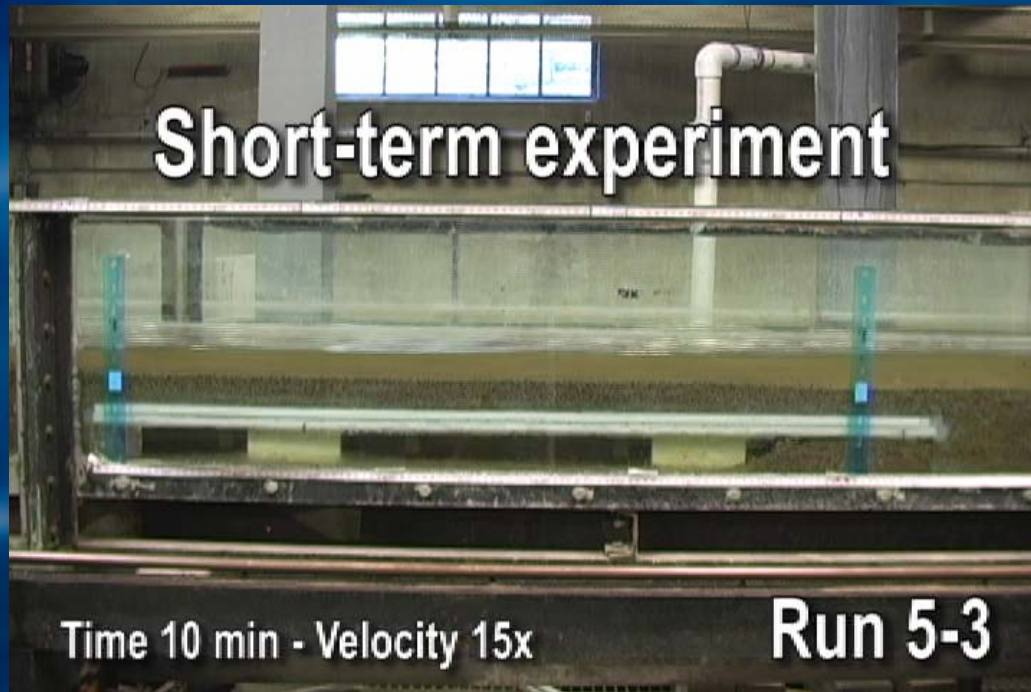


**Long term** experiment to catch an equilibrium steady state of the bed (2-4 hours)

# Effect of an upward seepage flow on bed dynamics



# Effect of a downward seepage flow on bed dynamics

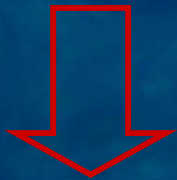


## Results – the total shear stress

The evaluation of the shear stress at the bed is done through the momentum integral equations in the case of a boundary seepage, using the water surface slope, the bed slope, the seepage velocity and other flow parameters, depth-averaged velocity and water depth.

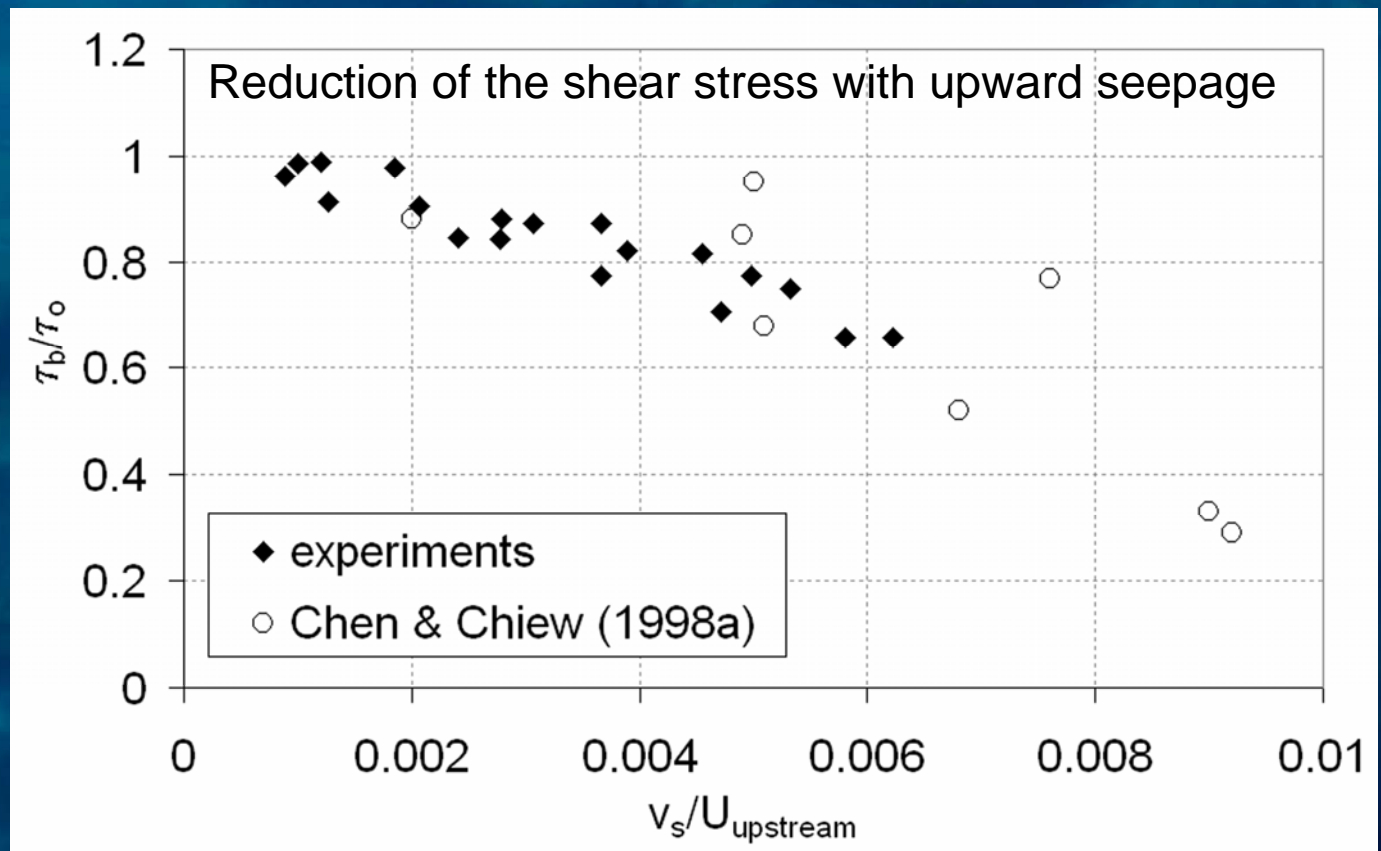
$$\tau_b = \rho u^2 = \rho gh \sin \phi_b - \rho gh \left( \frac{dh}{dx} \right) \left( \cos \phi_b - \frac{\beta U^2}{gh} \right) - 2\beta \rho U v_s$$

Chen & Chiew (1998a)



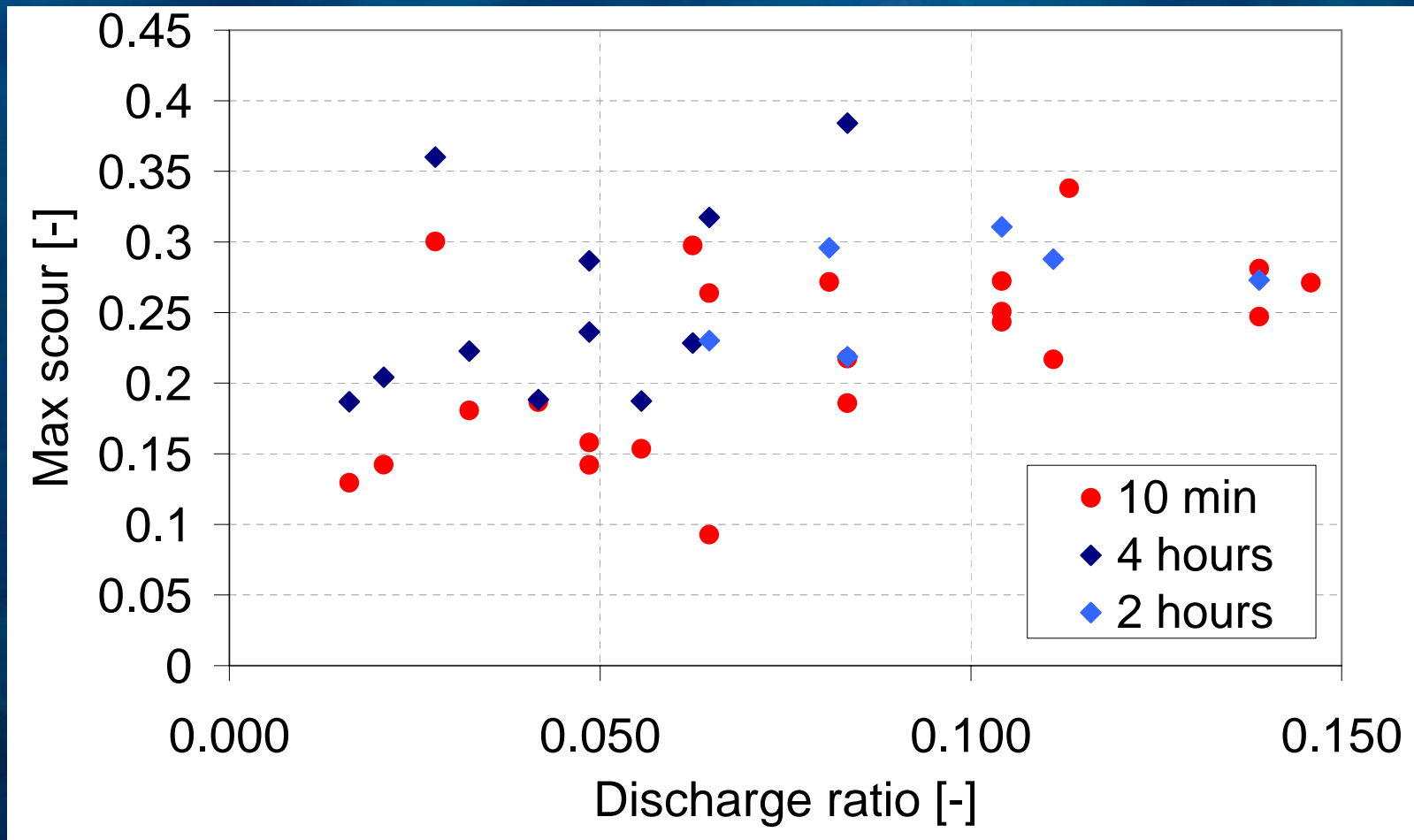
Two combined effects  
on Shields parameter

$$\tau^* = \frac{\tau_b}{\left( \rho_s - \rho \left( 1 + \frac{v_s}{K} \right) \right) gD}$$



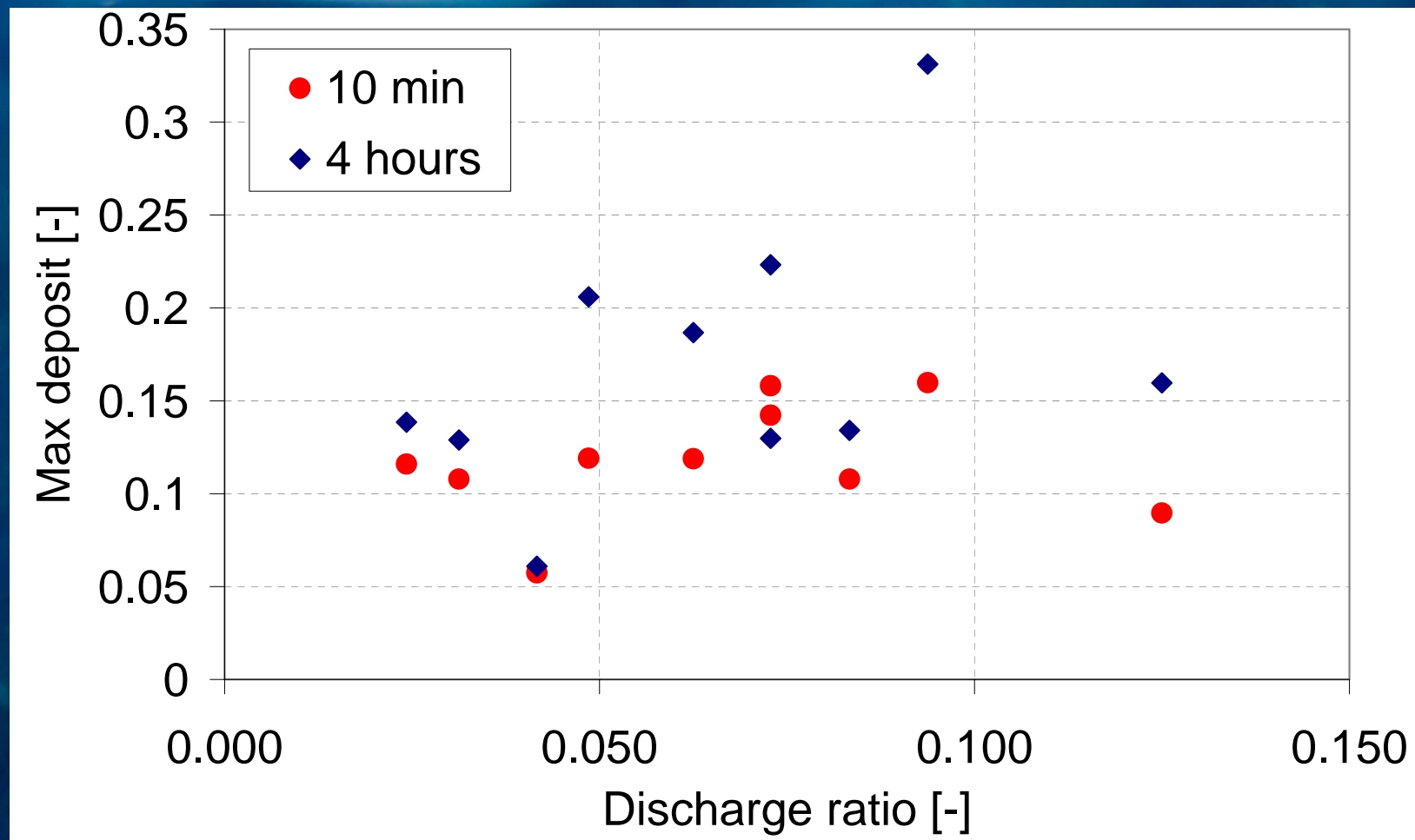
## ***Results – maximum dimensionless scour and deposition depth***

The maximum scour depth is evaluated as the maximum difference between the initial and the final bed, made dimensionless with the water depth. The maximum scour depth was observed to increase with the discharge ratio, and thus with higher groundwater flow.



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## *Theoretical and numerical model*

A one-dimensional morphodynamic model that considers the continuity equation, the momentum equation and the Exner equation has been implemented in order to study the effect of seepage on bed morphodynamics.

$$\frac{\partial H}{\partial t} + \frac{\partial UH}{\partial x} = v_s$$

Conservation of flow mass

$$\frac{\partial UH}{\partial t} + \frac{\partial U^2 H}{\partial x} = -gH \frac{\partial H}{\partial x} + gHS - \frac{\tau_b}{\rho}$$

Equation of flow momentum

$$(1 - \lambda_p) \frac{\partial \eta}{\partial t} = - \frac{\partial q_t}{\partial x}$$

Conservation bed sediment (Exner)

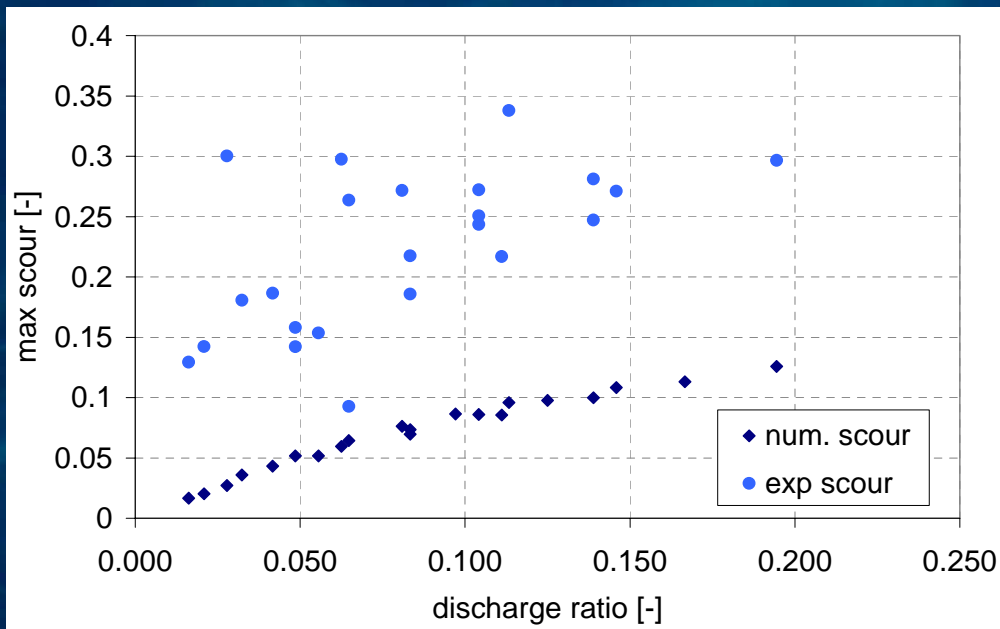
The pressure distribution is hydrostatic inside the water and non-hydrostatic inside the sand layer, to drain water.

The system of equations was solved using the quasi-steady approximation and a decoupling scheme between flow dynamics and morphodynamics. Two input parameters for the model have been set as follows: porosity  $\lambda_p = 0.3$ ,  $K = 0.0025$  m/s.

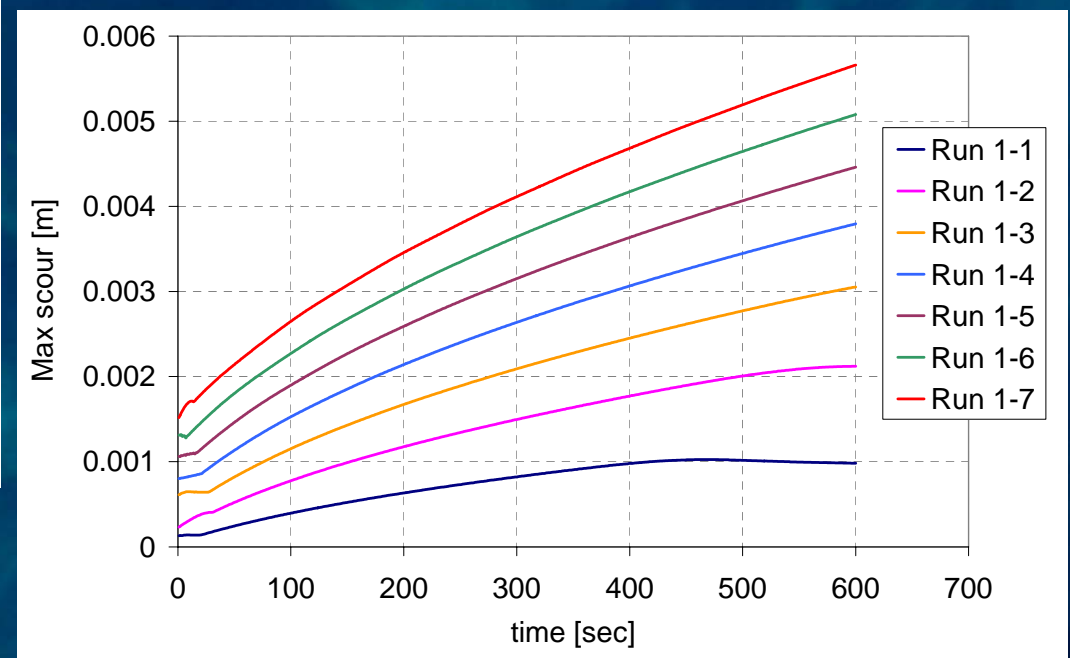
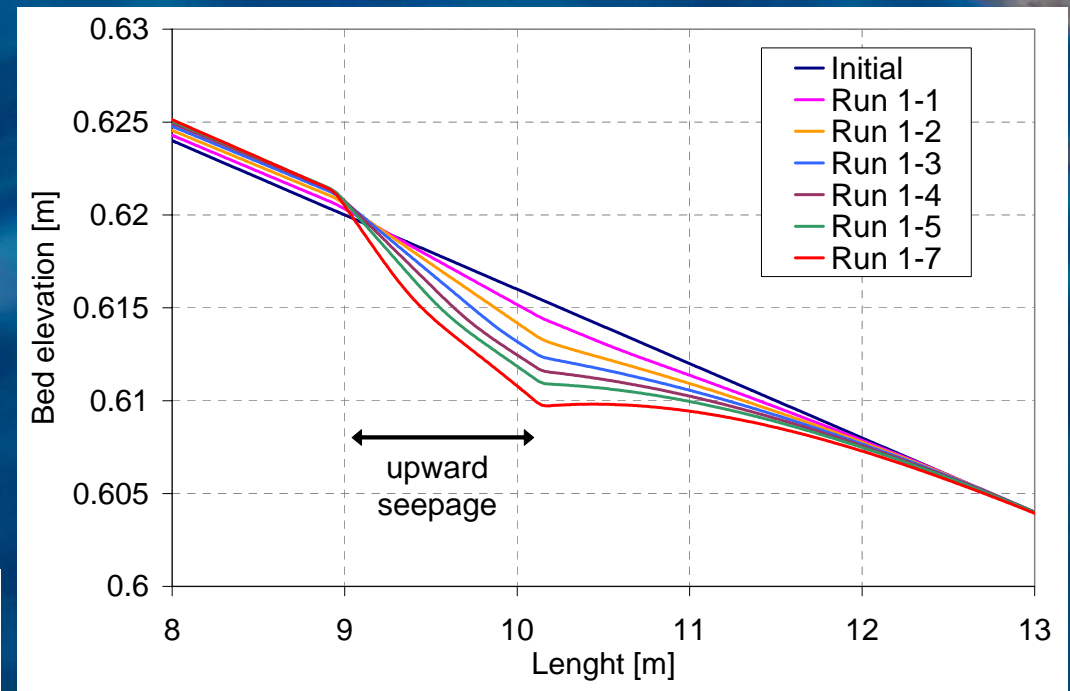
# Theoretical and numerical model

Numerical results of short term experiments (10 min) with upward seepage flow, reduction of shear stress and no-correction in the Shields parameter

$$\tau^* = \frac{\tau_b}{(\rho_s - \rho)gD}$$

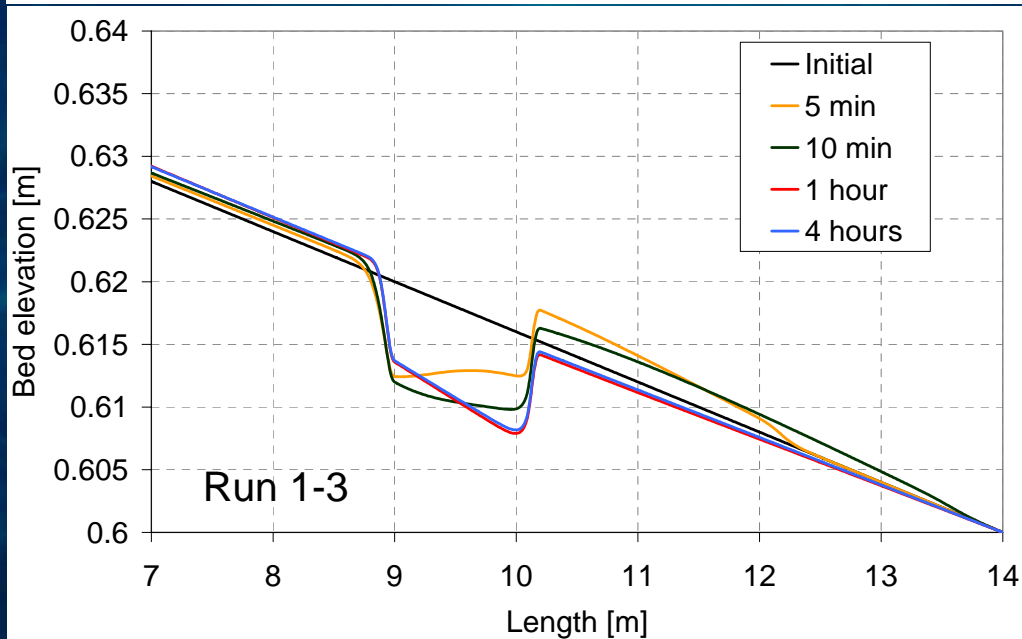
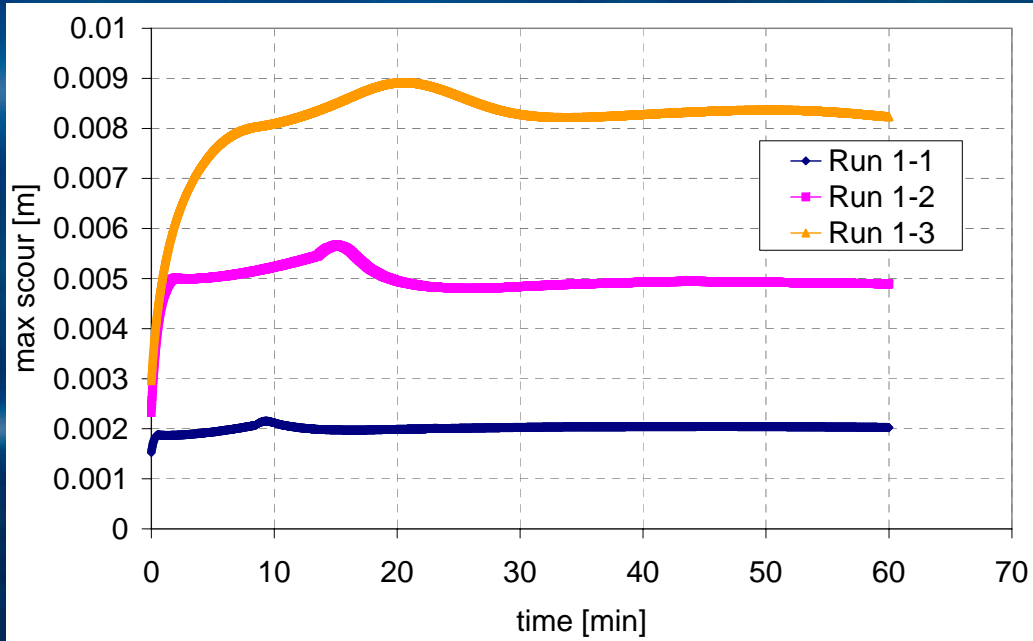


Under-estimation of local scour depth

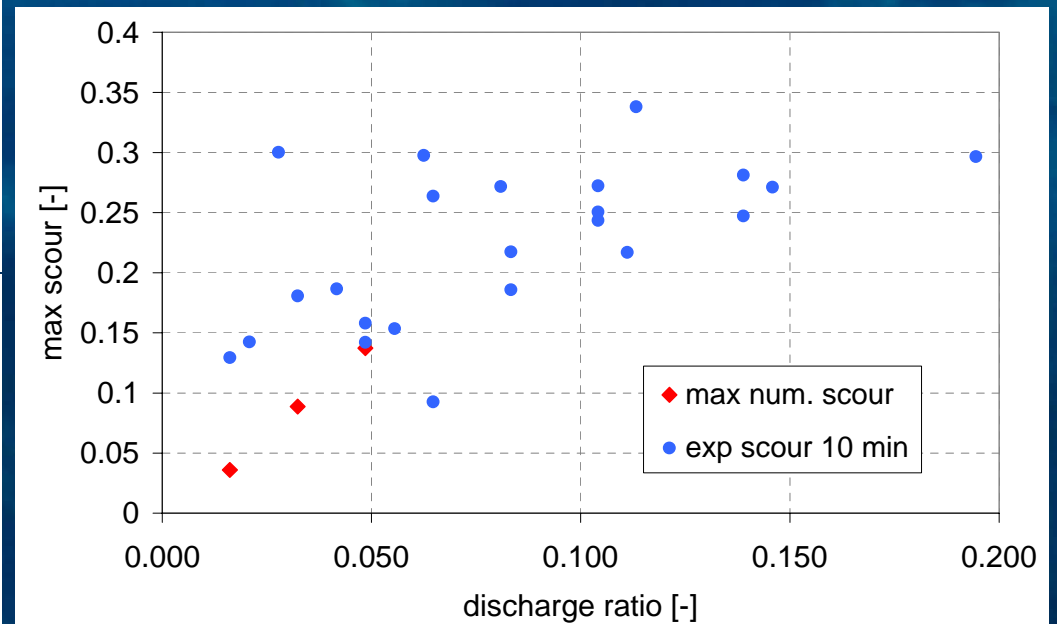




# Theoretical and numerical model



→ 
$$\tau^* = \frac{\tau_b}{\left( \rho_s - \rho \left( 1 + \frac{V_s}{K} \right) \right) gD}$$



## **Conclusions**

- ✓ An experimental activity on the effect of upward and downward seepage flow has been carried out to investigate the relation of non-hydrostatic pressure distribution and bedload transport.
- ✓ In the case of upward seepage net scour was observed in the zone of seepage; the opposite effect was observed in the case of downward seepage. Experimental results of the reduction of the bed shear stress and time evolution of the bed are consistent.
- ✓ A theoretical and numerical model has been implemented to analyze bed morphodynamics in the presence of seepage flow.
- ✓ Preliminary results from the numerical model show that a general formulation for bedload transport that has been validated in the case of a hydrostatic pressure distribution is able to capture the qualitative aspects of the effect of seepage on bed morphodynamics. The same formulation appears, however, to significantly underestimate the effect of seepage.

## ***Conclusions***

- ✓ Comparison of the experimental values of the maximum scour in the seepage area with the results of the numerical model show that a generalized expression of the Shields parameter is needed in the case of relevant non-hydrostatic pressure distribution.
- ✓ A future goal will be to develop a modified sediment transport relationship that does allow extension at least to the simple non-hydrostatic pressure distribution created by vertical seepage flow.

## ***Acknowledgements***

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