Saltating or rolling stones?

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RCEM conference, 5 October 2005

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Content

- Theoretical computation of the solid discharge: Einstein's approach
- Experimental facility: 2D flume
- Experimental observations

[Ancey *et al.*, *Phys. Rev. E* **66** (2002) 036306; Ancey *et al.*, *Phys. Rev. E* **67** (2003) 011303; Boehm *et al.*, *Phys. Rev. E* **69** (2004) 061307; Ancey *et al.*, in preparation]

Theoretical derivation of bed-load discharge

- Bagnold (1966) $q_s(x,t) = \int_0^h c(x,z,t)\bar{u}_s(x,z,t)\mathrm{d}z$
- Einstein (1950) $\frac{\partial q_s(x,t)}{\partial x} = E(x,t) - D(x,t)$

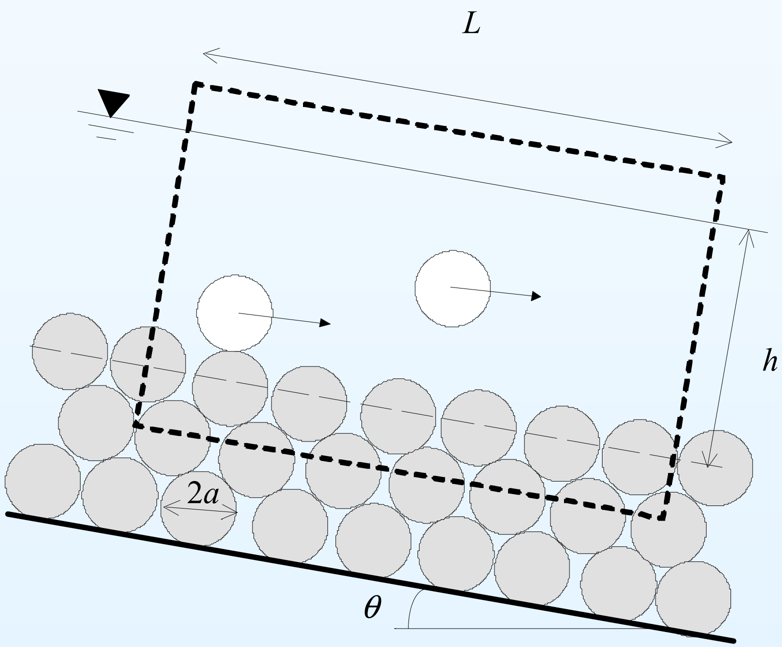
Exchange with the bed

$$B \rightleftarrows R \text{ and } R \rightleftarrows S,$$

Kinetic equation for the rolling fraction

$$\frac{dn_r}{dt} = \frac{n_{b\to r}}{t_b} + \frac{n_{s\to r}}{t_s} - \frac{n_{r\to s}}{t_r} - \frac{n_{r\to a}}{t_r},$$

(similar equation obtained for the saltating fraction)



Einstein's theory revisited

In a steady regime, we obtain

$$n_{b\to r} = \frac{t_b}{t_r} n_{r\to b}, \quad \text{and} \quad n_{r\to s} = \frac{t_r}{t_s} n_{s\to r}.$$

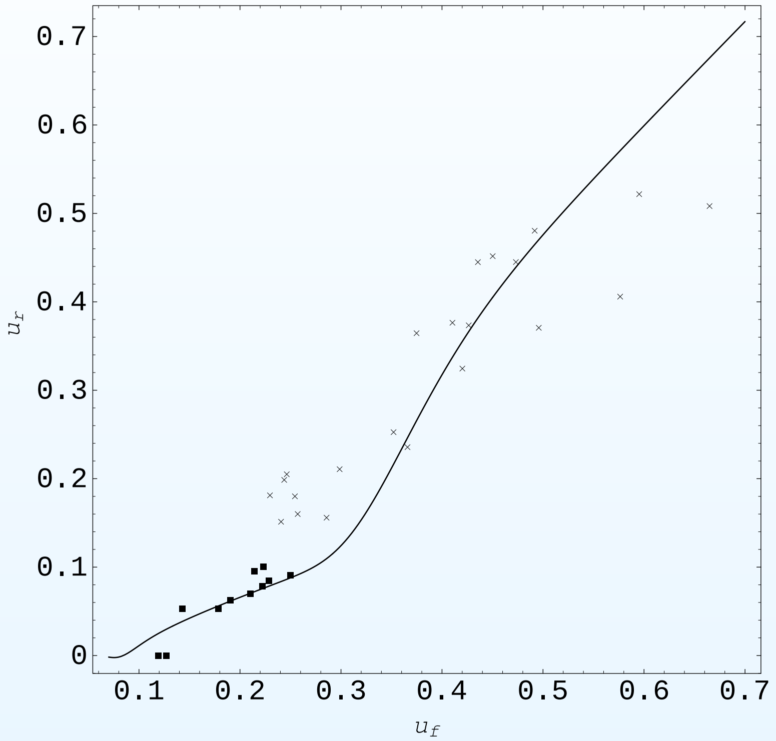
with $t_r = 2a/\bar{u}_r$ and $t_s = \ell_s/\bar{u}_s$.
How to compute the velocity of a rolling particle

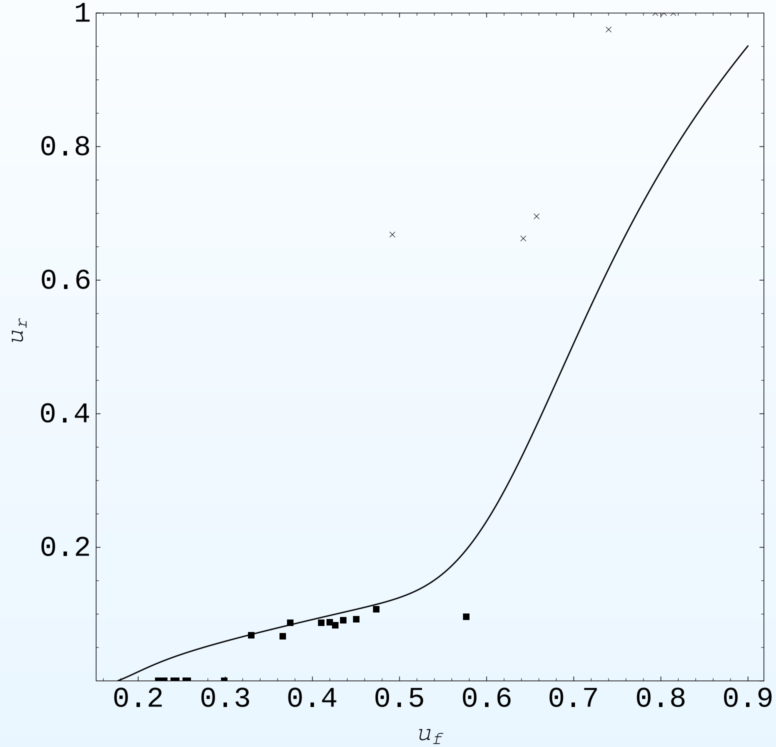
$$m' g \sin\theta \bar{u}_p + \bar{P}_d = \bar{P}_c,$$

$m' = m - 4\pi\rho_f a^3/3$ buoyant mass, $\bar{P}_d = \bar{\mathbf{F}}_d \cdot \bar{\mathbf{u}}_p$ power of drag forces supplied to the particle, power P_c lost in contacts. We derive

$$\bar{u}_p = K(Sh)\bar{u}_f.$$

Velocity of a single particle in a rolling regime

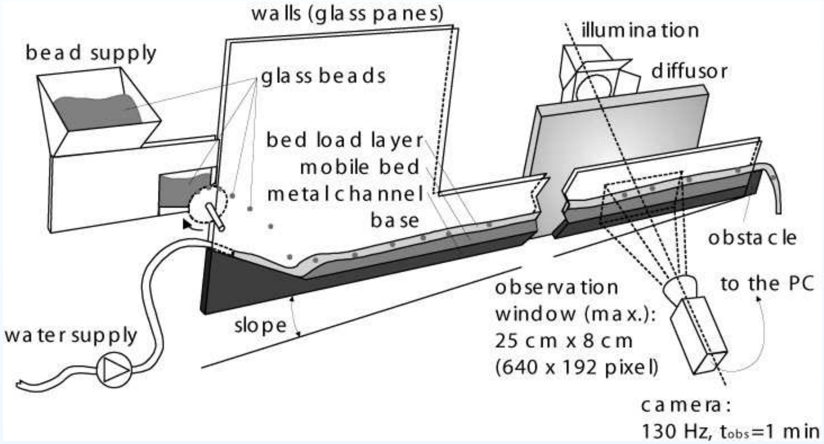




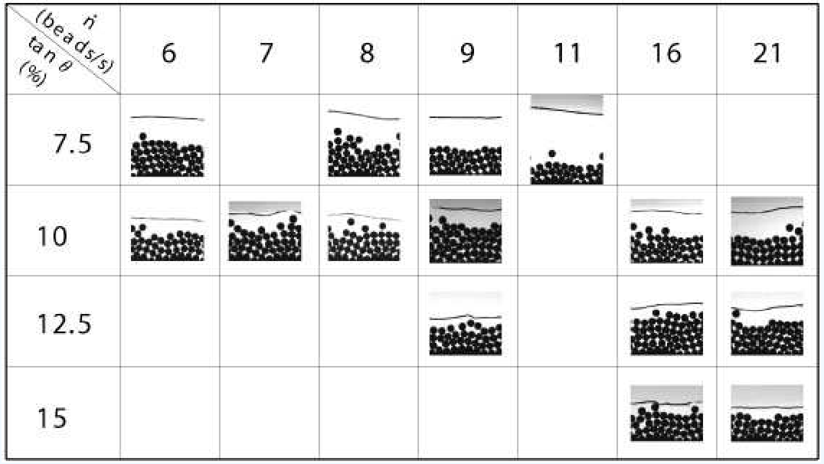
glass beads steel beads

[Ancey *et al.*, *Phys. Rev. E* **67** (2003) 011303]

Experimental facility



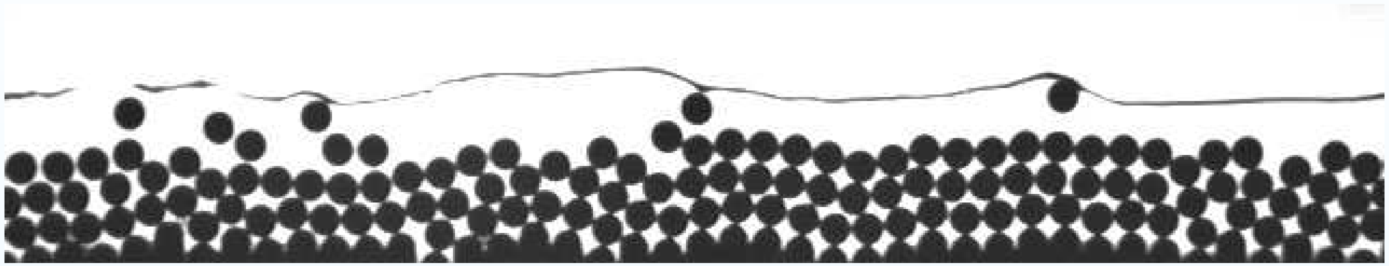
2D Flume

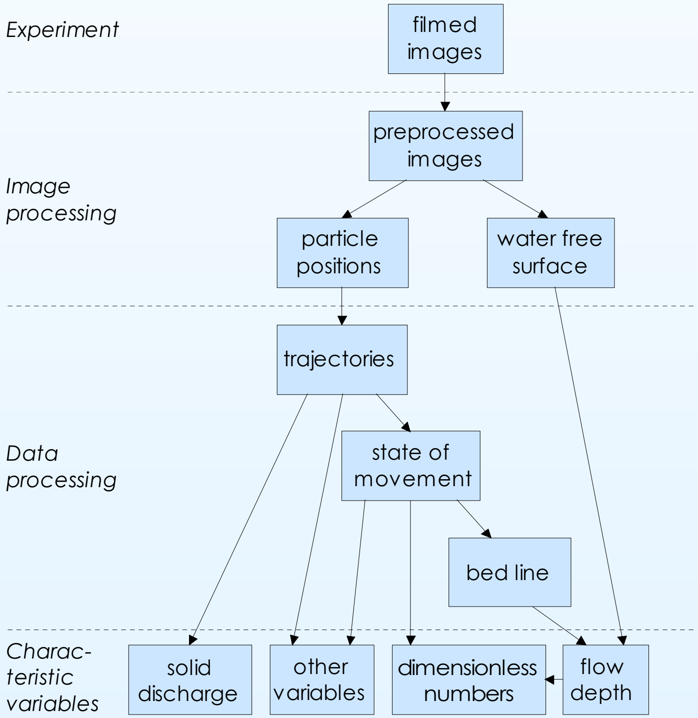


Experiments

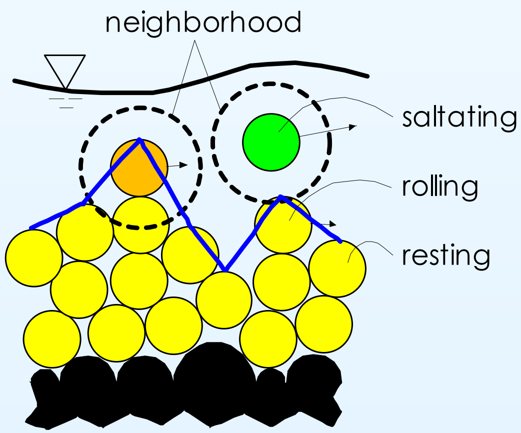
- Slope from 7% to 15%
- Flow rate range: 6–21 beads/s
- bed equilibrium
- 8000 images at 140 Hz (approx. 60 s.)

Example



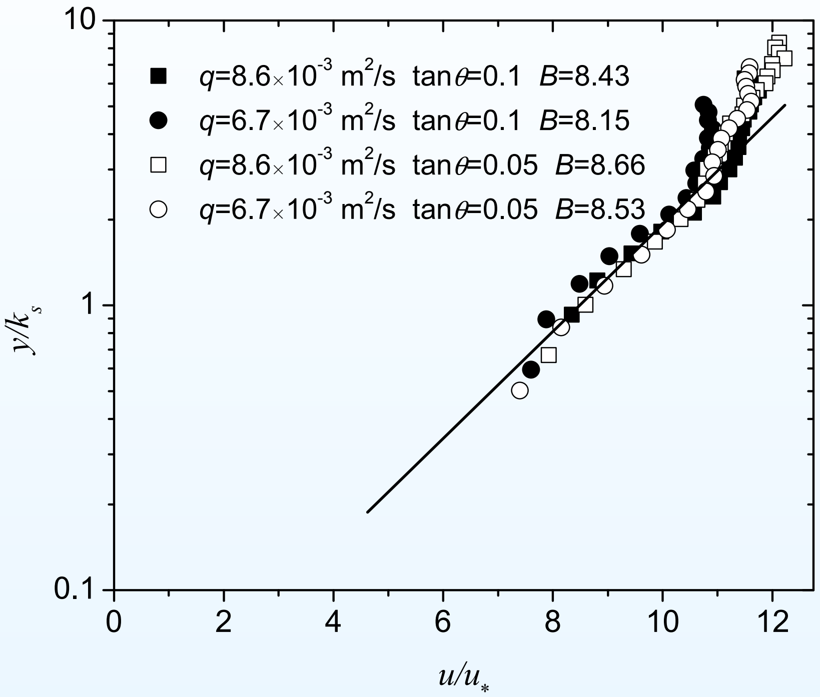


Experiment
filmed images
Image processing
preprocessed images
particle positions
water free surface
Data processing
trajectories
state of movement
bed line
Charac-teristic variables
solid discharge
other variables
dimensionless numbers
flow depth

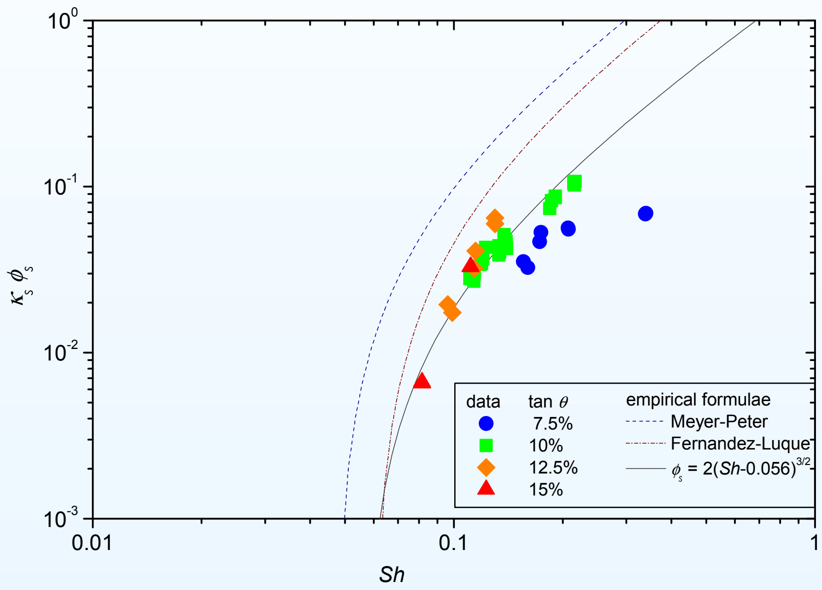


neighborhood
saltating
rolling
resting

Representative of flow conditions usually met?



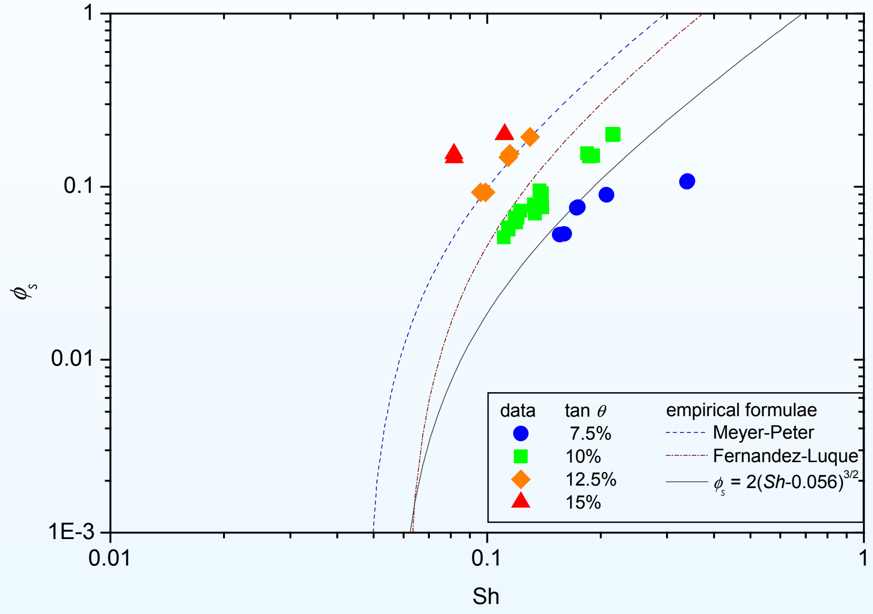
Velocity profile

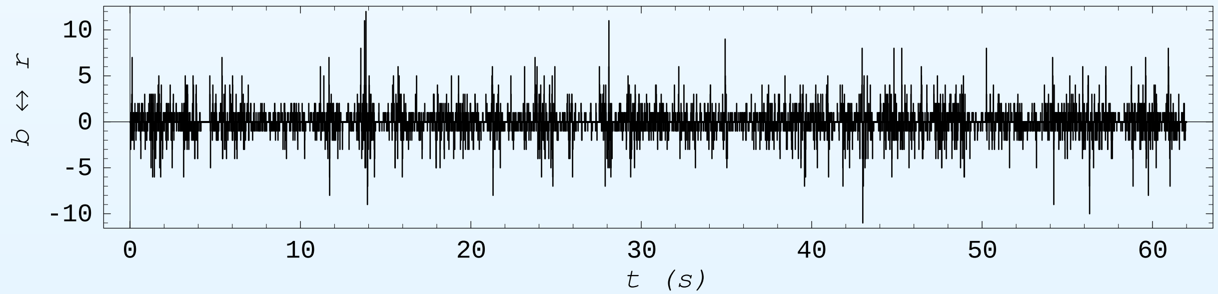


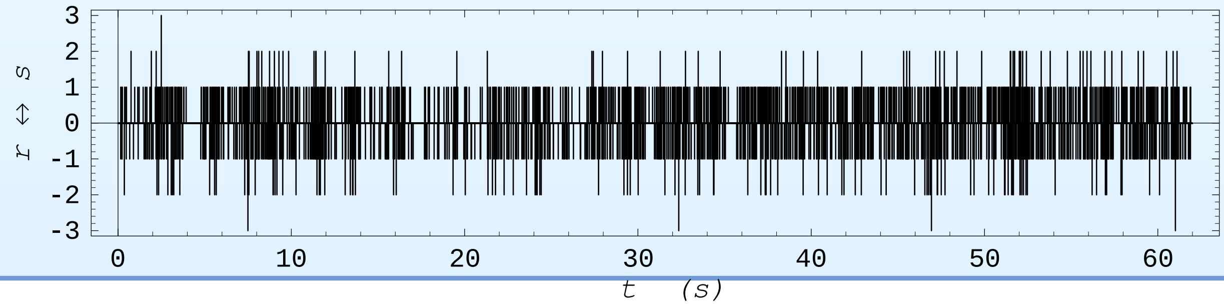
Saltating contribution

- Hydraulic conditions: flow controlled by bottom for shallow flow ($h/(2a) = O(1)$), otherwise by sidewalls
- When the bed-load discharge is computed with the saltating contribution alone, we retrieve empirical laws.

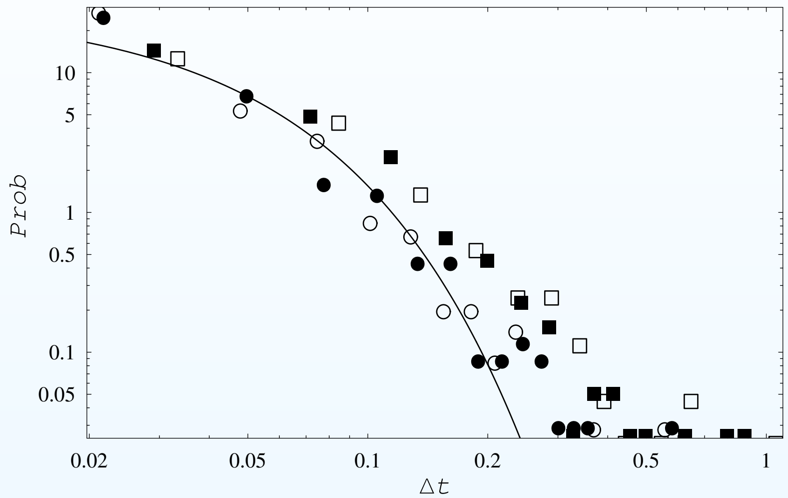
Contribution of the rolling particles

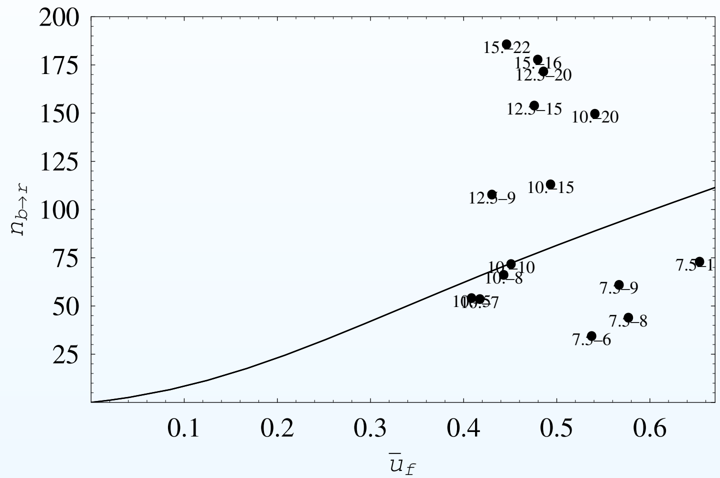






Statistics





- entrainment rate linearly proportional to the Shields number, with strong dependence on slope
- exchanges with bed: close to a Poisson process
- rolling velocity: nearly independent of slope and flow rate

A few words of conclusion

- Idealized hydraulic conditions, but not too irrealistic
- Increasing role of the rolling fraction with increasing slope
- Entrainment rate and frequency: close to Poisson process, but difficult to interpret the characteristic time