Numerical analysis of alternating bars in straight channels

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Joint work with dr. P. Zegeling and T. van Leeuwen, Utrecht University

Motivation of the study

Dynamics of free bars: Theoretical analysis versus numerical analysis

'Natural rivers do not have long enough straight reaches for alternating bars to develop'

http://env-web.ceri.go.jp/alternate-bars/tokachi-river1.jpg

http://env-web.ceri.go.jp/alternate-bars/toshibetsu-river1.jpg

Same, or different: point bars

River Meuse, the Netherlands

Motivation of the study

Theoretical analysis of free bars in straight channels
2 contributions:

1. Colombini et al (JFM 181 (1987), pp 213-232)
2. Schielen et al (JFM 252 (1993), pp 325-356)

Result: Amplitude equation (Ginzburg)-Landau equation for bed-patterns

Can analytical results be reproduced numerically ?

B. Federici, thesis (2002): Topics on fluvial Morphodynamics, University of Genoa

Numerical analysis

Reproduce analytical results and
-‘Proof’ spontaneous pattern formation
-Show modulation behaviour

Existing models: black box, different phenomena and difficult to comprehend

Solution: Develop own numerical model

Benefit: Little tuning parameters and concentrate on phenomena

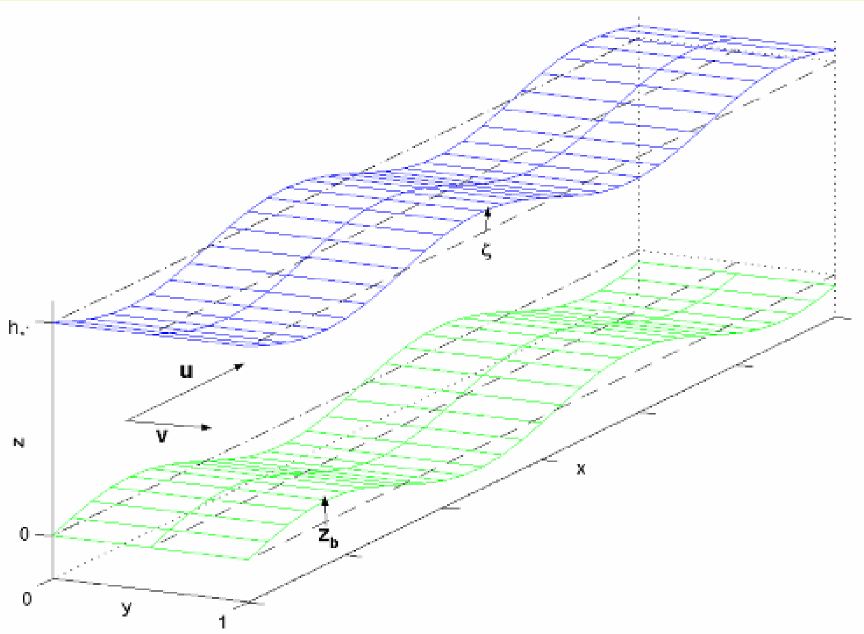
Analytical approach

-Simple model: Shallow water equations and bed-evolution
-Linear theorie: Neutral stability curve, critical width to depth ratio and critical wave number
-Nonlinear theorie: Amplitude equation

$$z_b(x,y,t) = A(\xi,\tau) cos(\pi y) e^{ik_c x + \omega_c t}$$

$$\frac{\partial A}{\partial t} = rA + \alpha \frac{\partial^2 A}{\partial \xi^2} + \beta |A|^2 A$$

Situation and model

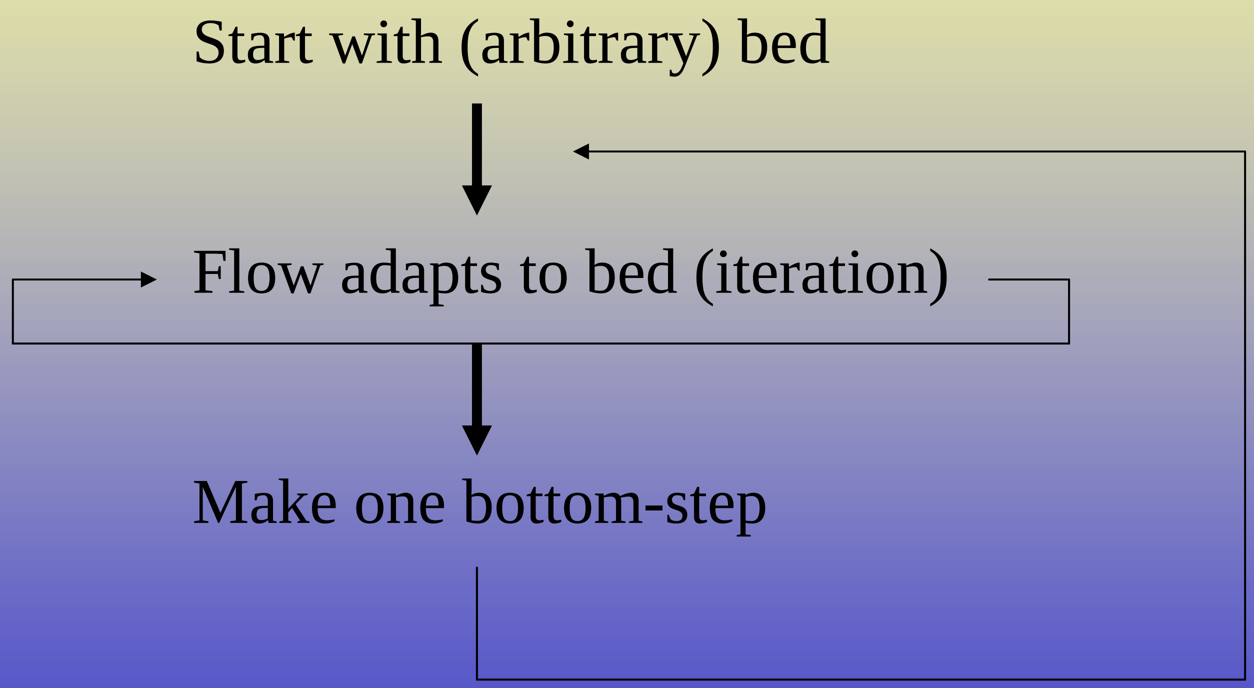


$$
\begin{aligned}
\kappa\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\frac{\partial \zeta}{\partial x} &= -CR(\frac{u|\mathbf{U}|}{h}-1),\\
\kappa\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+\frac{\partial \zeta}{\partial y} &= -CR\frac{v|\mathbf{U}|}{h},\\
\kappa\frac{\partial h}{\partial t}+\frac{\partial uh}{\partial x}+\frac{\partial vh}{\partial y} &= 0,\\
\frac{\partial z_b}{\partial t}+\frac{\partial q_x}{\partial x}+\frac{\partial q_y}{\partial y} &= 0. \qquad (4)
\end{aligned}
$$

Method

-Decoupling of flow and bed
-Flow: Lax-Friedrichs
-Bed: FSCT
-Periodic boundary conditions (!)

Decoupling of flow and bed



Flow: Lax Friedrichs

-(Adapted) explicit method (one matrix-vector mult. per step)
-Restriction on timestep (Von Neumann-analysis)

$$\frac{\partial f}{\partial t} + c_x \frac{\partial f}{\partial x} + c_y \frac{\partial f}{\partial y} = 0.$$

$$f_{i,j}^{n+1} = F(f_{i\pm 1, j\pm 1}^{n}; \Delta x, \Delta y, \Delta t)$$

$$f_{i,j} = \tilde{f_{i,j}} + \epsilon_{i,j}$$

$$\epsilon_{i,j} = g^n e^{\imath(i\omega_x \Delta x + j\omega_y \Delta y)}$$

$$g(\theta, \phi) = \frac{1}{2}(\cos(\theta) + \cos(\phi)) - \imath \Delta t \left(c_x \frac{\sin(\theta)}{\Delta x} + c_y \frac{\sin(\phi)}{\Delta y}\right),$$

Flow: Lax Friedrichs

$$\Delta t = \frac{\mu min(\Delta x, \Delta y)}{2max(u,v) + \sqrt{\frac{C}{i_o}}}, 0 < \mu < 1$$

(CFL-condition)

-Consistency: $\frac{\Delta x^2}{\Delta t}, \frac{\Delta y^2}{\Delta t} \rightarrow 0$

Bed: FTCS

- Von Neumann analysis:

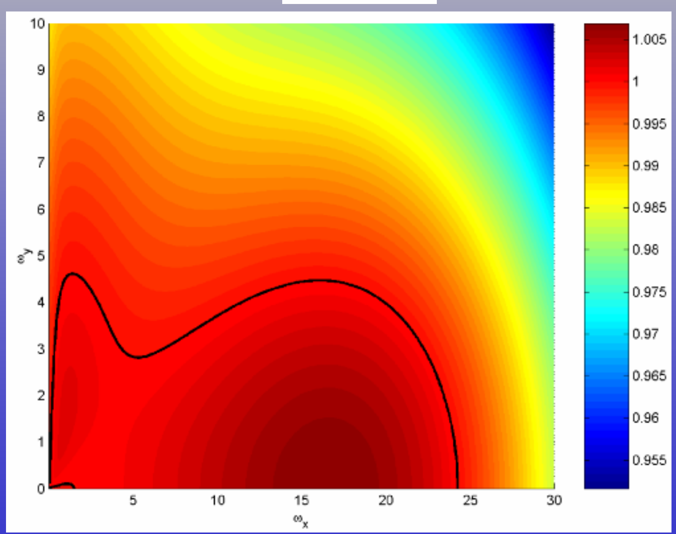
$$z_{i,j}^{n} = g^{n} e^{\imath(i\omega_x \Delta x + j\omega_y \Delta y)}$$

$$g(\theta,\phi) = 1 + \frac{2\Delta t}{R}\left(\frac{\cos(\theta)-1}{\Delta x^2} + \frac{\cos(\phi)-1}{\Delta y^2}\right) - \imath\Delta t\left(b\frac{\hat{u}}{\hat{z}}\frac{\sin(\theta)}{\Delta x} + \frac{\hat{v}}{\hat{z}}\frac{\sin(\phi)}{\Delta y}\right) = 0.$$

$$\Delta t = f(\nu, R, \Delta x, \Delta y)$$

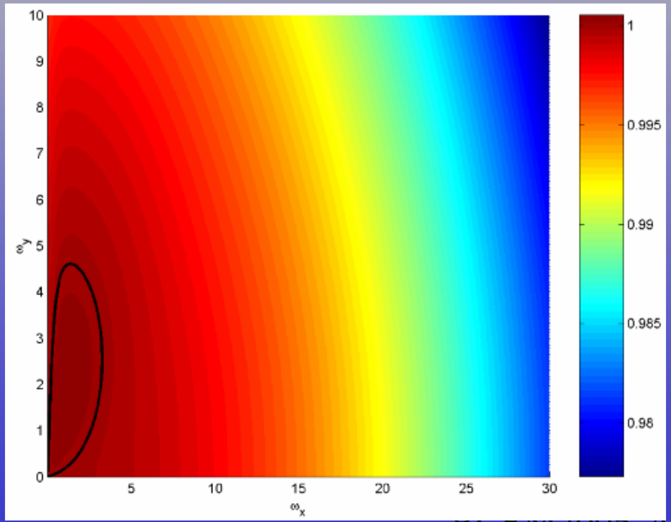
$R = 30$

$\nu = 0.15$



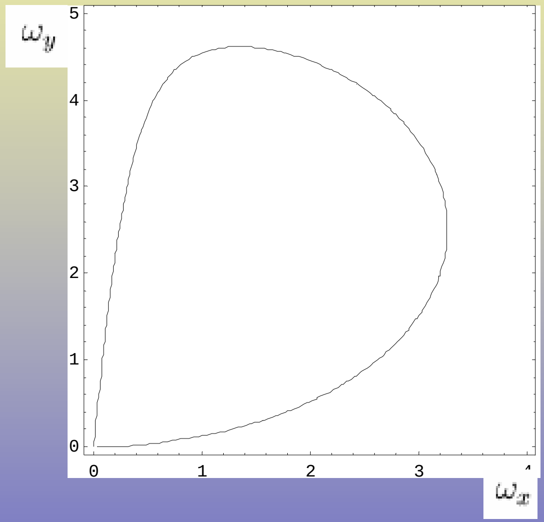
$R = 30$

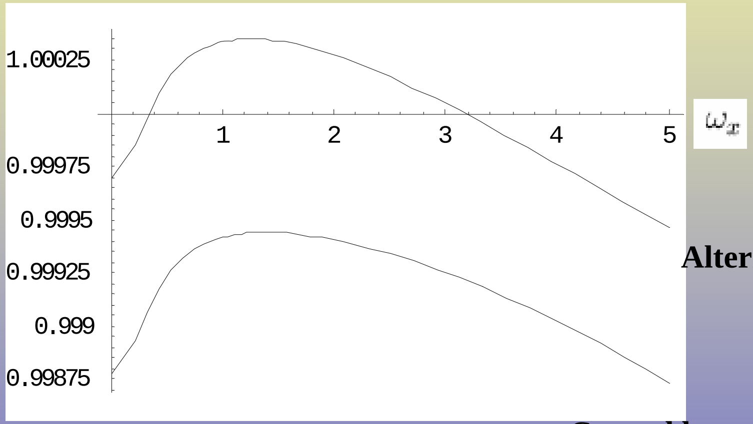
$\nu = 0.05$



- Decoupling of flow and bed
- Flow: Lax-Friedrichs
- Bed: FSCT
- Periodic boundary conditions (!)

Stability-criterion





Alternating bar-mode

Central bar-mode

$R = 30$

$\nu = 0.05$

- Decoupling of flow and bed
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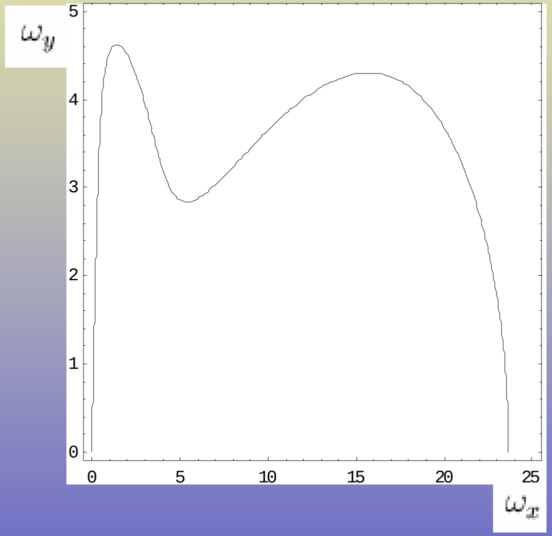
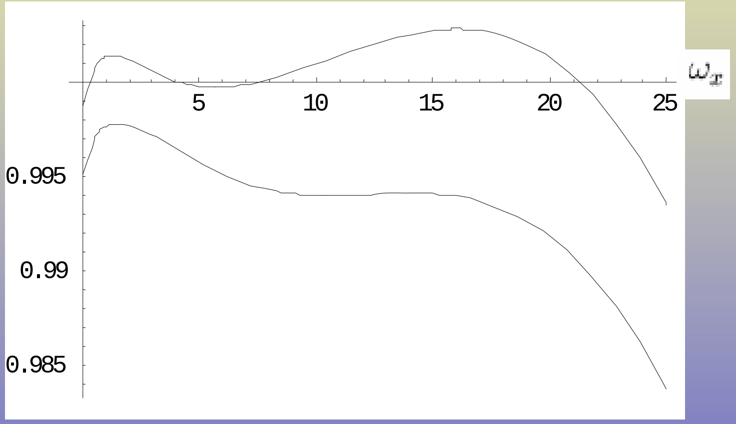
Stability-criterion



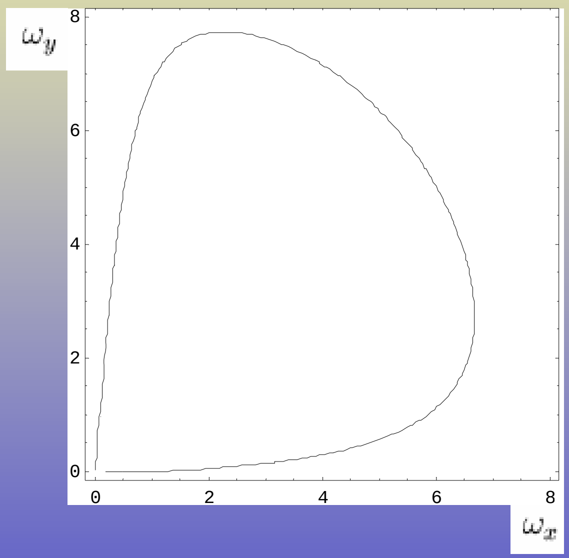


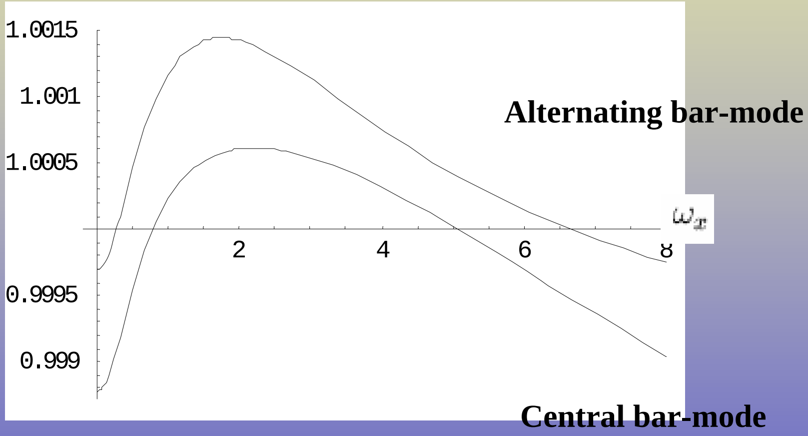
$R = 30$

$\nu = 0.15$

- **Decoupling of flow and bed**
- **Flow: Lax-Friedrichs**
- **Bed: FSCT**
- **Periodic boundary conditions (!)**

Stability-criterion

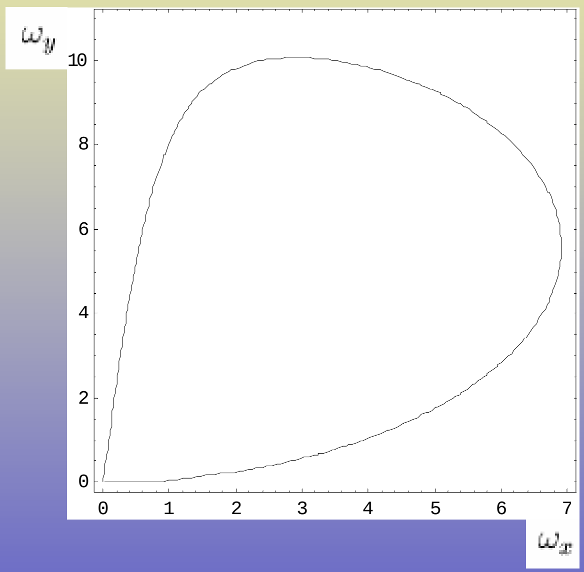


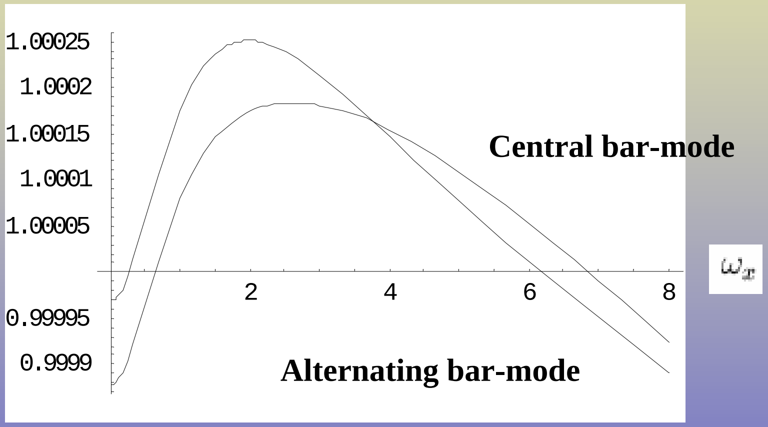


$R = 50$

$\nu = 0.05$

Stability-criterion



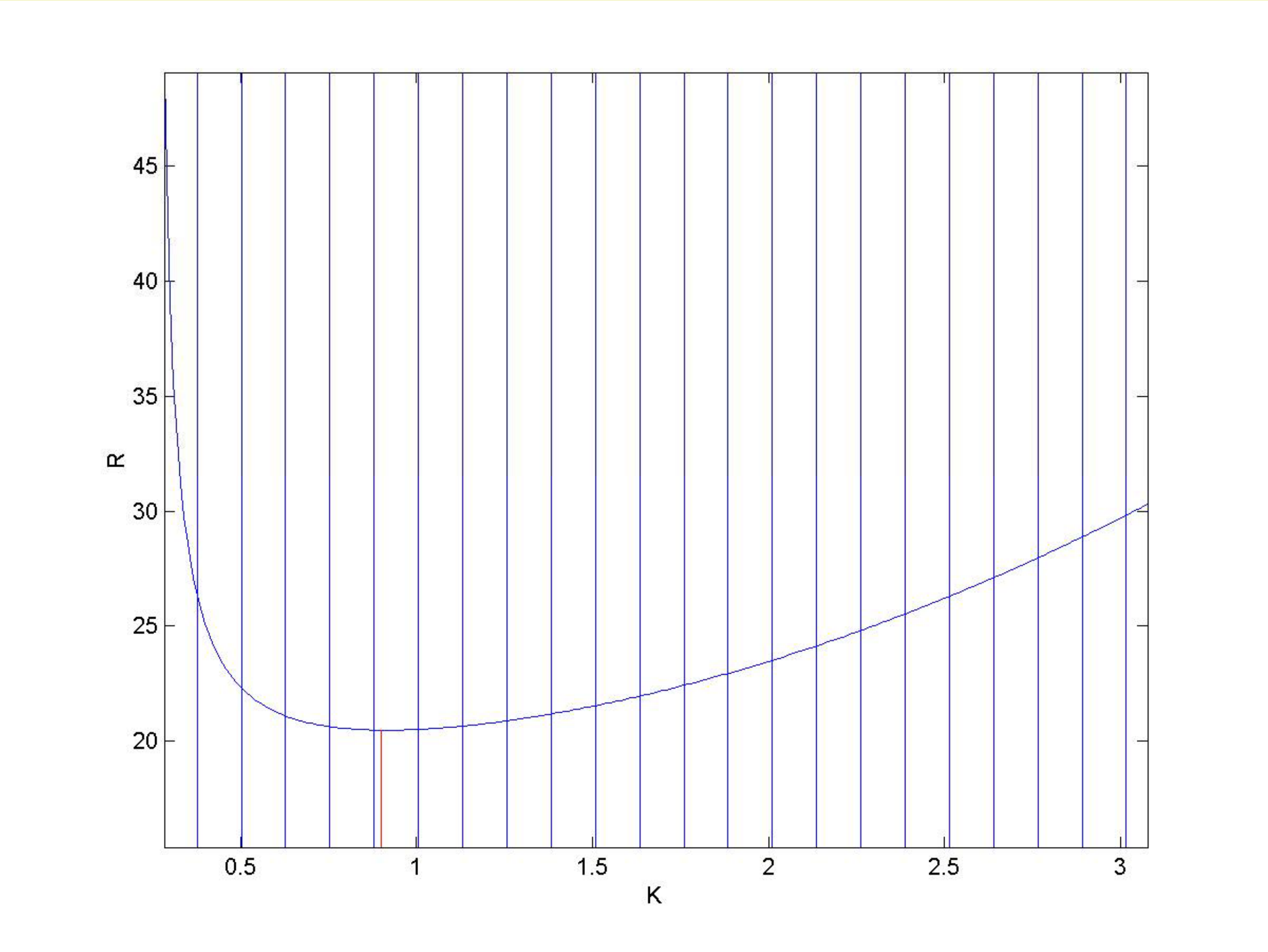


$$R = 65$$

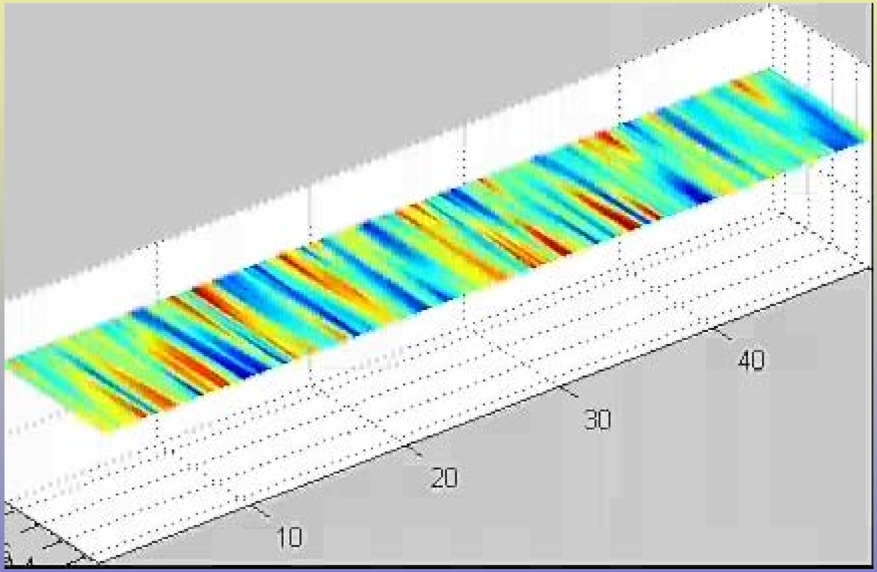
$$\nu = 0.005$$

- **Decoupling of flow and bed**
- **Flow: Lax-Friedrichs**
- **Bed: FSCT**
- **Periodic boundary conditions (!)**

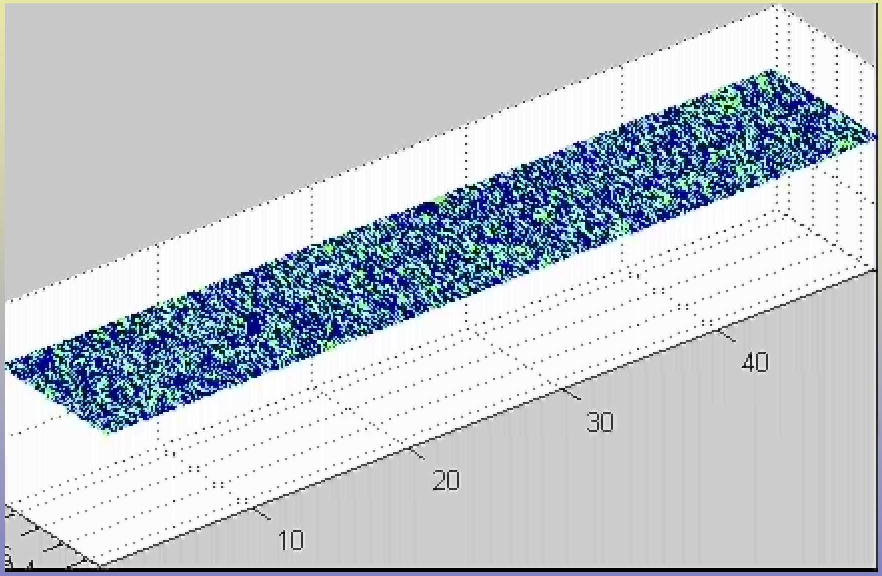
Periodic boundary conditions



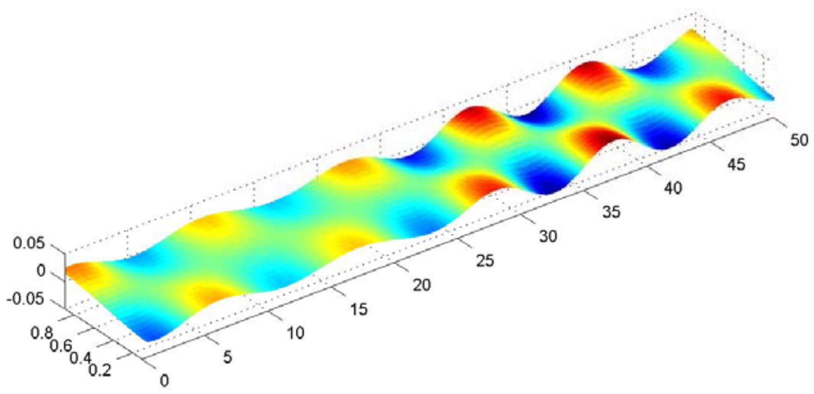
R=50, arbitrary bed profile

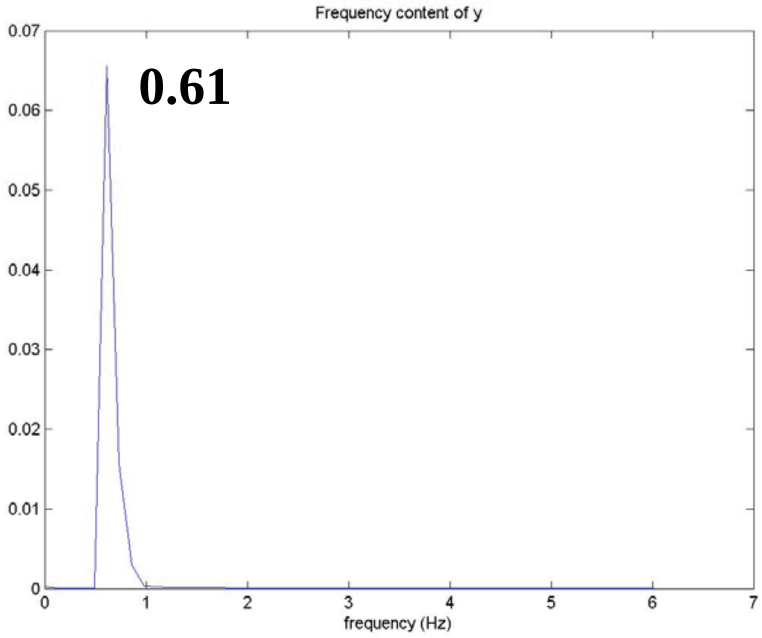


R=50, initial formation



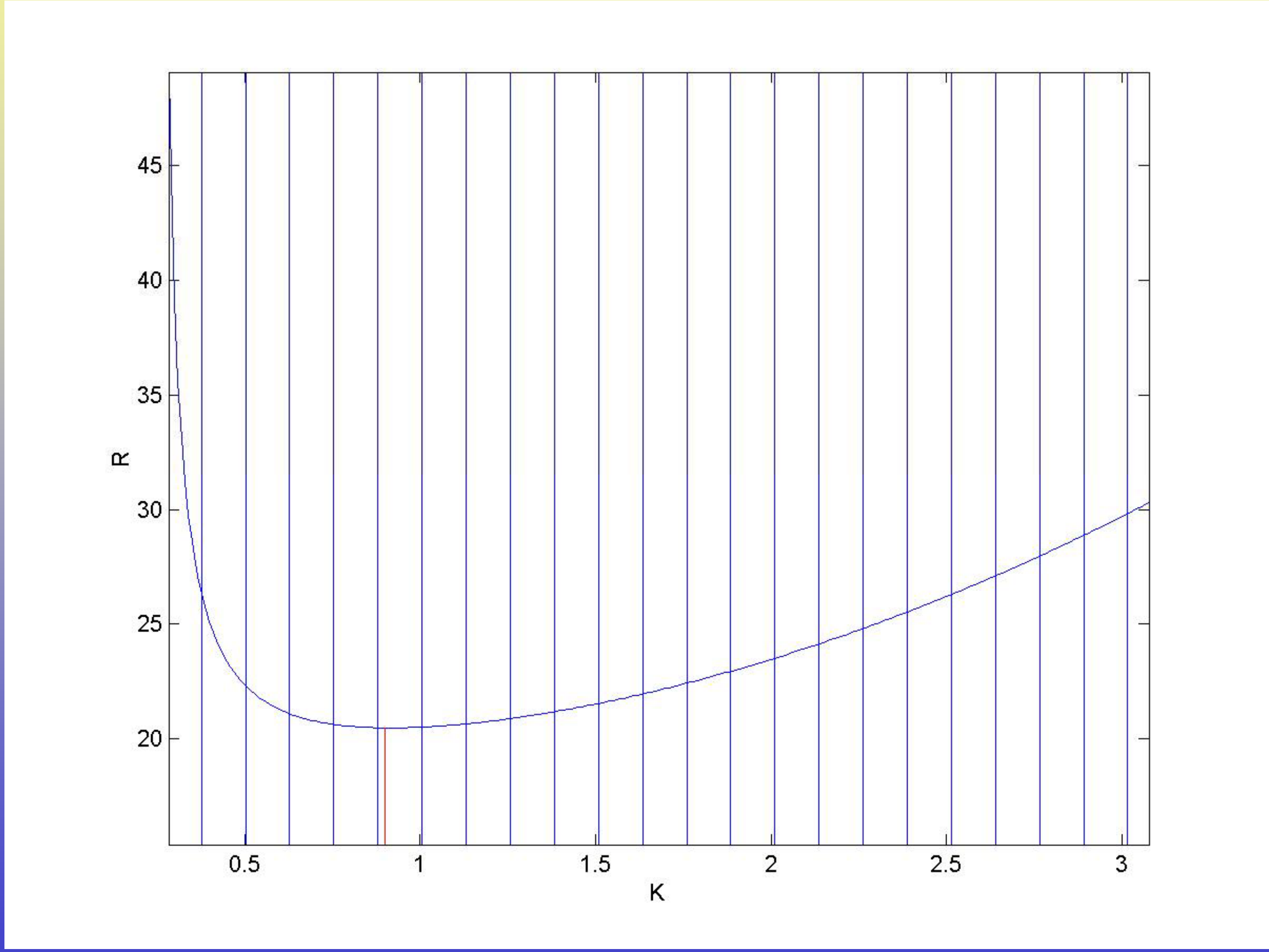
R=50, end situation



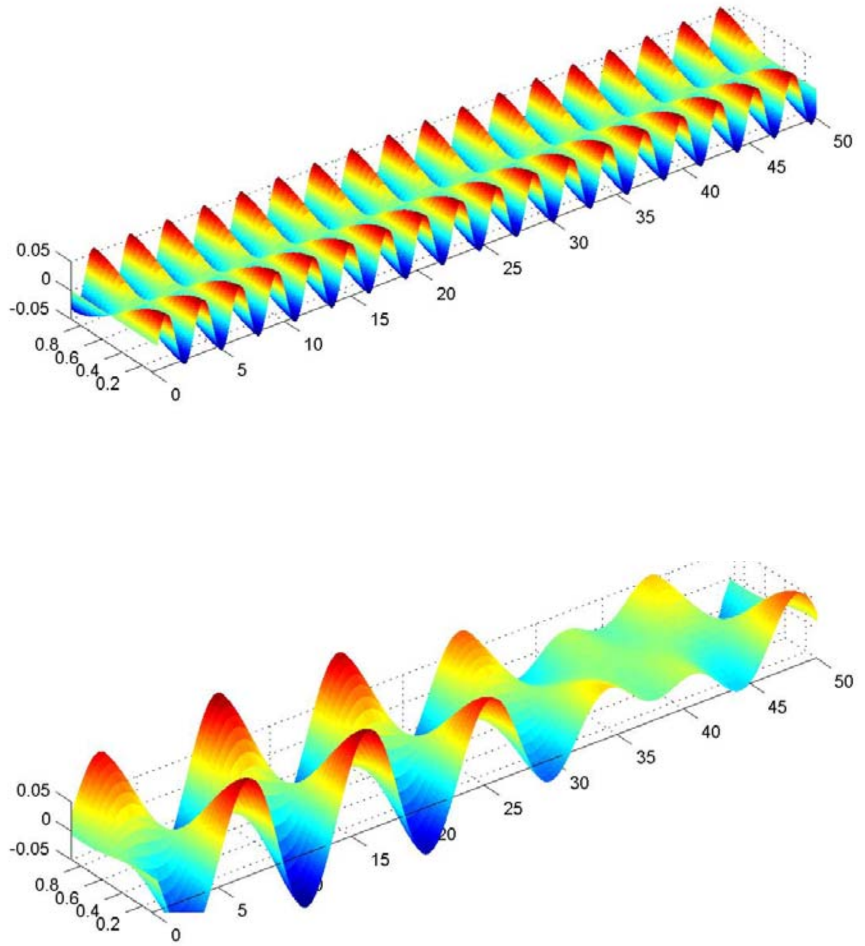


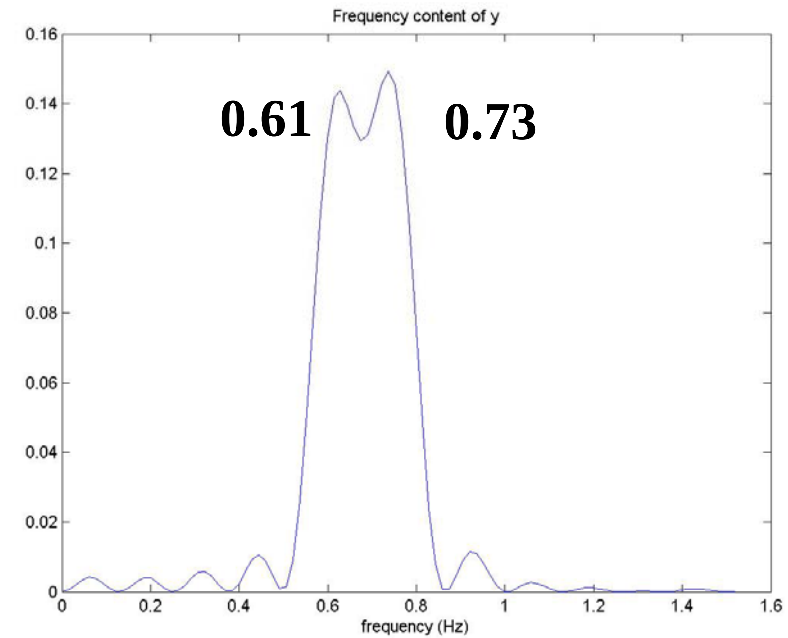
- **Decoupling of flow and bed**
- **Flow: Lax-Friedrichs**
- **Bed: FSCT**
- **Periodic boundary conditions (!)**

Periodic boundary conditions



Results: R=50, multiple bars

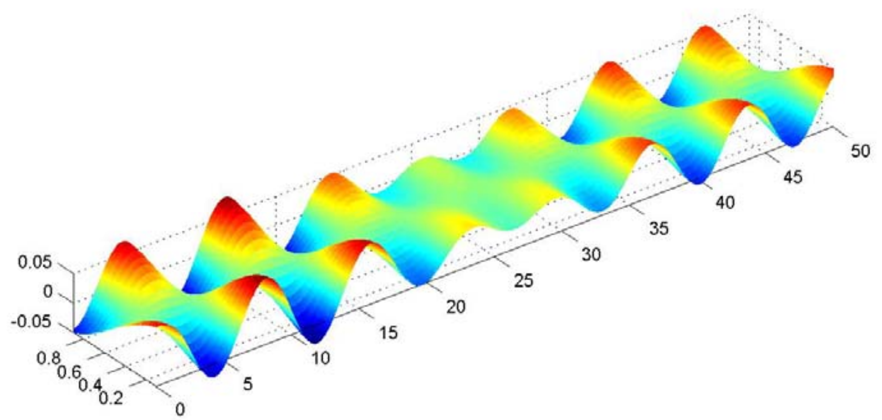


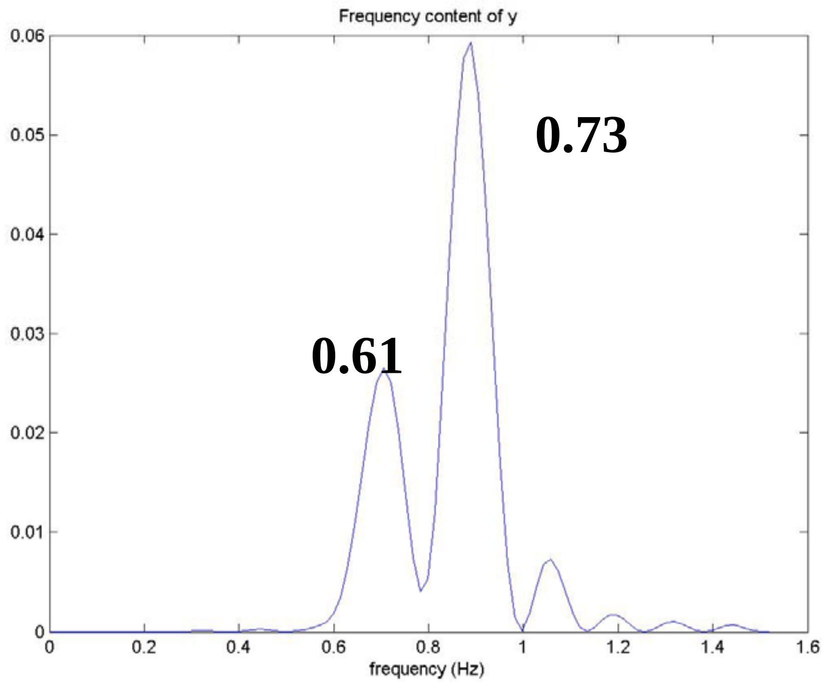


$$R = 50$$

$$\nu = 0.05$$

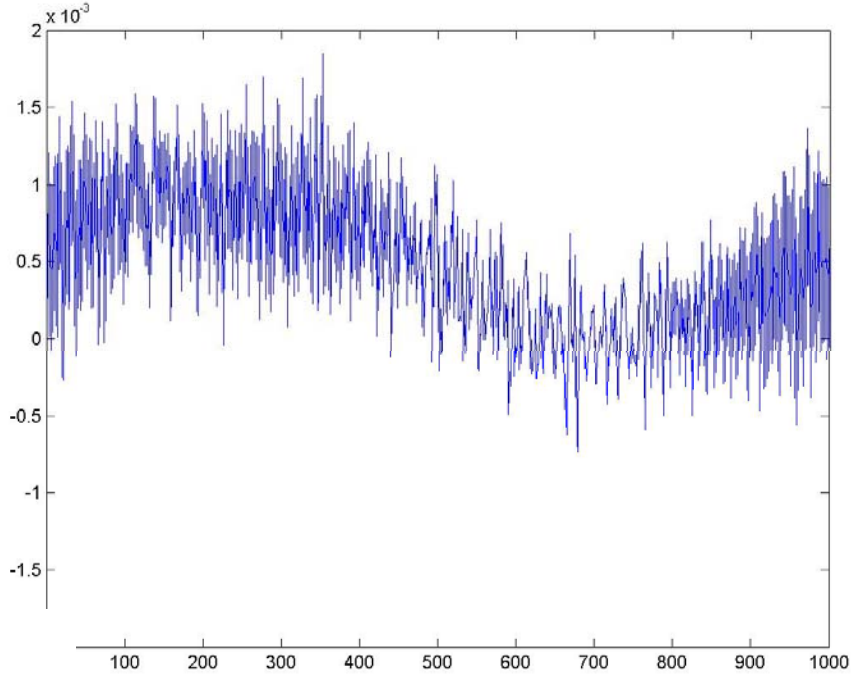
Results: R=60

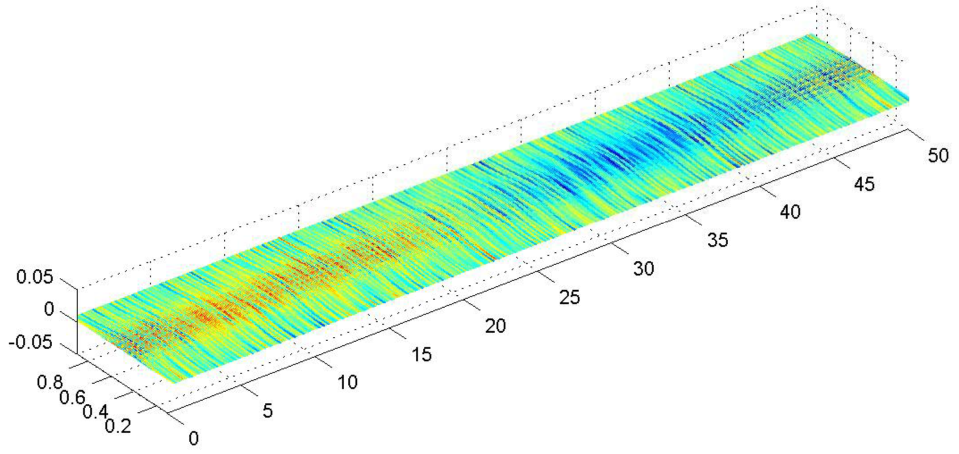




$$R = 60$$
$$\nu = 0.05$$

Central bars ?





C=0.07,
b=10

Conclusions

- Alternating bars can be reproduced adequately
- Typical bar-shape is missing
- Strongly nonlinear results are not present
- Selection of unstable modes is unclear

Further steps

- More physics such that more complex behaviour becomes possible
- Open boundaries
- Faster, mass-conserving schemes
- Bank-erosion
- Reproduce meander-characteristics
- Sediment distribution on confluents and bifurcations

Conclusions

- Critical wave number is reproduced
- For large values of R alternating bar pattern (singe and mixed modes)
- Selection of spectrum is unclear
- Typical bar pattern is not present
- Strongly nonlineair pattern is not present
- Central bars are not possible with this model