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Modeling Channel-Floodplain Co - evolution in Sand-Bed Streams

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Abstract

The importance of including floodplain effects in one-dimensional river channel hydraulic computations is well known. However, in morphodynamic modeling, these are often partially ignored by using a single effective discharge to drive bed elevation changes. This neglects changes in channel capacity (and thus effective discharge) caused by deposition on or erosion from either the channel bed or the floodplain. This paper presents a model for reach-averaged channel bed and bank top elevation evolution that specifically accounts for changes in channel depth over time. The model considers two grain sizes: one for sand, which interacts primarily with the bed, and one for mud, which interacts only with the floodplain. The model also describes the evolution of the proportion sand and mud in the floodplain deposits. Sediment transport and floodplain deposition are driven by a simple gradually varied flow solution. Erosion from the floodplain is represented as a net loss associated with channel migration. Because overbank deposition is strongly affected by flow, effective floodplain deposition and in-channel sediment transport are obtained by integrating results from an entire flow duration curve. In the absence of bed elevation changes, the channel and floodplain co-evolve toward a stable bankfull geometry where overbank deposition just equals floodplain erosion.

Introduction

It is generally accepted that alluvial rivers are responsible for building their floodplains. To accomplish this, rivers must both deposit material on and erode material from their floodplains. Since floodplain deposits are usually much finer than the river bed sediment, correctly predicting the interaction between a river and its floodplain requires accounting for both bed material and fine sediment (Narinesingh et. al., 1999).

Most river bed aggradation/degradation models consider only sediment in the bed material size range. Furthermore, while many models allow floodplain effects to be included in the flow solution, the floodplain surface is usually not allowed to change over time as a function of the fine material supplied to the system.

This paper presents a model for floodplain evolution that allows a river bed and its floodplain to evolve together by specifically accounting for both bed material and finer grain size fractions. The model is based on relatively simple reach-averaged sub-models for both overbank deposition and floodplain erosion, and a simple 1-D model for flow and sediment transport within the river channel. The model is applied to an example long term prediction of bed and floodplain evolution.

Formulation

The model is intended to represent reach-averaged behavior over relatively large length and time scales. Consequently, it ignores all lateral and most vertical structure within the floodplain itself and simplifies the channel by assuming a wide rectangular section. The model enforces conservation of sediment mass in each of four completely-mixed sediment reservoirs at each cross section; a) the active layer of the bed (Hirano, 1971), b) a basal floodplain layer, c) an upper floodplain layer, and d) the water column (Figure 1). The model also allows material to be moved to or from a passive, vertically unmixed substrate as the channel aggrades or degrades, respectively.

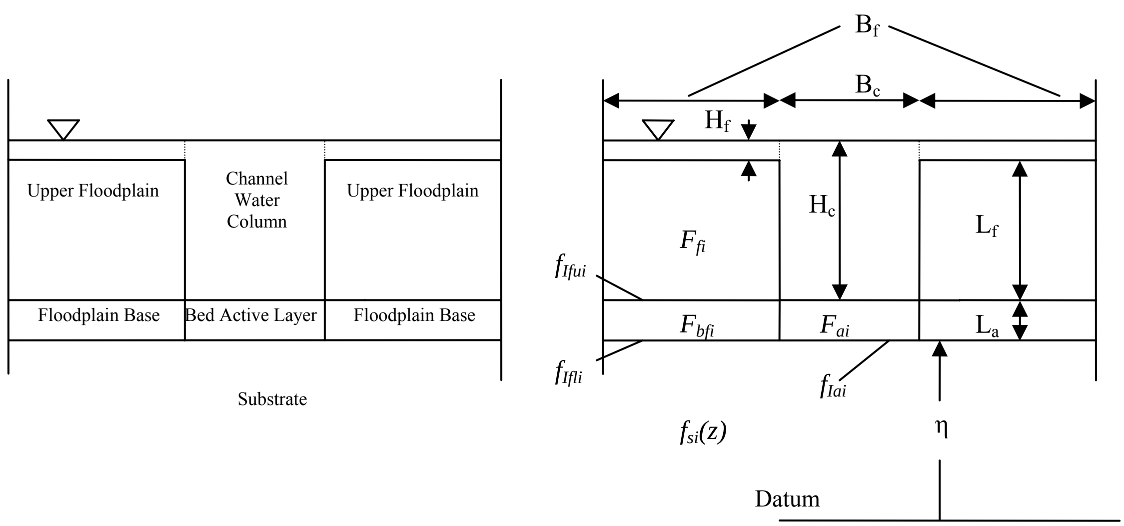

Figure 1. Conceptual cross sections showing sediment reservoirs and defining variables.

Geometric variables used in the formulation are as follows: η = elevation of the base of the bed's active layer, B_c = channel width, B_f = floodplain width, L_a = thickness of the bed's active layer, L_f = thickness of the upper floodplain = channel bankfull depth, H_f = depth of flow on the floodplain, and H_c = depth of flow in the channel. Grain size fractions in size class i are represented in the upper floodplain by F_{fi}, in the floodplain base by F_{bfi}, and in the active layer by F_{ai}. Several local grain size fractions are also required; f_{Ifui} = fraction moving across the upper interface of the floodplain base layer, f_{Ifli} = fraction moving across the lower interface of the floodplain base layer, and f_{Iai} = fraction moving across the interface between the channel bed active layer and the substrate. These variables depend on the direction the boundaries are moving. Finally, $f_{si}(z)$ = fraction in class i at any level z in the substrate.

The model is simplified for the present application to two grain sizes: one for sand, which interacts primarily with the bed but can be present anywhere, and one for mud, which can be present anywhere except in the active layer of the bed. While sand can be suspended in the water column, the sand concentration is always assumed to be in equilibrium with the entrainment rate from the bed (Garcia and Parker, 1991). Figure 2 shows the sediment flux pathways of the model. Accounting for the flux along each pathway results in a description of the evolution of the channel bed and floodplain elevation and the fraction sand and mud in each reservoir.

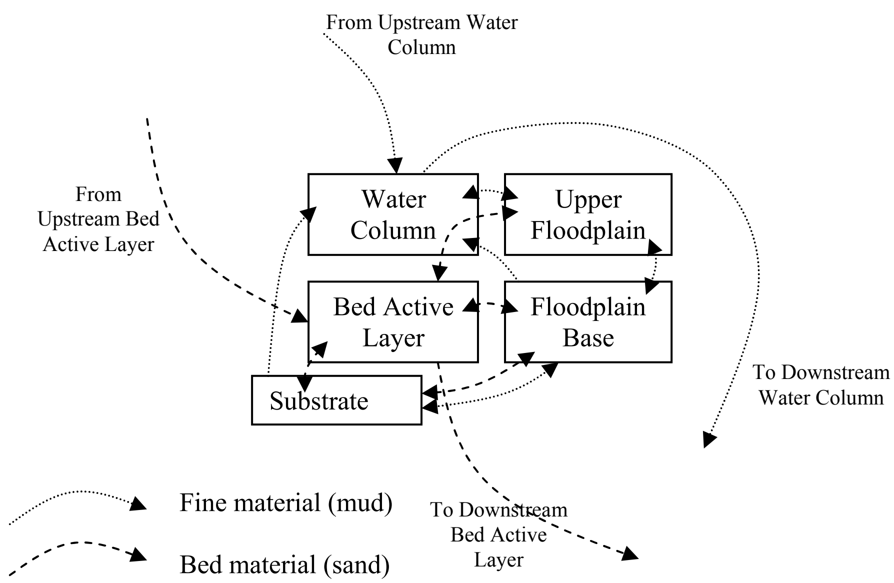

Figure 2. Sediment flow pathways considered by the model at a given cross section. The flow of bed material (sand) is denoted by a dashed line while flow of material finer than that on the bed (< 62.5 µm; mud) is denoted by a dotted line Note that the concentration of bed material suspended in the water column is fully specified by the flow, so any overbank deposition of sand on the upper floodplain is effectively taken directly from the bed's active layer, and any sediment of bed material size eroded from the floodplain is effectively moved directly to the bed's active layer.

The fundamental well-mixed layer is the active layer of the channel bed, which by definition contains only the bed material size. As the channel migrates, the relatively coarse material on the bed is abandoned in the floodplain. It is not realistic to assume that this material immediately becomes mixed across the entire floodplain thickness, since it enters floodplain at a relatively low elevation. Instead, our model forces it to enter the floodplain base layer, which is defined geometrically based on the thickness of the bed's active layer.

Conservation of Mass For Bed Region

The conservation of mass equation for the channel bed, which is here assumed to include both the well mixed active layer and a substrate with a potentially arbitrary vertical grain size structure, can be stated as follows for each grain size range:

> Rate of change of sediment volume below the bed surface = Transfer rate from the upper flooplain to the channel bed through floodplain erosion – Transfer rate to the upper floodplain from the bed through overbank deposition + net streamwise exchange rate of bedload and suspended bed material load with adjacent nodes along the channel profile + net exchange rate with the floodplain base layer as the channel migrates.

$$\left(1-\lambda_p\right)\left[\frac{\partial}{\partial t}\left[\int_0^{\eta+L_a}\Delta x_c B_c F_i(z)dz\right]\right] = q_{efpi}\Delta x_c - q_{obi}\Delta x_c + B_c q_{ti}\big|_{x_c} - B_c q_{ti}\big|_{x_c-\Delta x_c} + \left(1-\lambda_p\right)cL_a(F_{bfi}-F_{ai})\Delta x_c \quad (1)$$

Here λ_p = porosity of bed and substrate (assumed constant), x_c = down-channel coordinate, q_{efpi} = transfer per unit channel length of material in size class i from floodplain to channel due to bank erosion, q_{obi} = transfer per unit channel length of material in size class i from channel to floodplain due to overbank deposition, c = specified lateral stream migration rate, and the index i ranges from 1 (mud) to 2 (sand). Using the relationship

$$\frac{valley \quad length}{channel \quad length} = \frac{\Delta x}{\Delta x_c} = \Sigma = \text{channel sinuosity}, \quad (2)$$

taking the limit $\Delta x \rightarrow 0$, applying Leibnitz' rule and assuming constant channel width B_c, the following conservation equation results:

$$f_{Iai}\frac{\partial \eta}{\partial t} + \frac{\partial L_a}{\partial t}F_{ai} + \frac{\partial F_{ai}}{\partial t}L_a = \frac{1}{\left(1-\lambda_p\right)}\left(\frac{q_{efpi}-q_{obi}}{B_c} - \frac{1}{\Sigma}\frac{\partial q_{ti}}{\partial x}\right) + \frac{cL_a}{B_c}(F_{bfi}-F_{ai}) \quad (3)$$

For i = 2, F_a = 1, and the equation reduces to a form that describes the time evolution of the change in elevation of the bottom of the channel bed active layer.

$$\frac{\partial \eta}{\partial t} = F_{bf2}\left(\frac{cL_a}{B_c f_{Ia2}}\right) + \frac{1}{f_{Ia2}}\left[\frac{q_{efp2}-q_{ob2}}{\left(1-\lambda_p\right)B_c} - \frac{1}{\left(1-\lambda_p\right)\Sigma}\frac{\partial q_{t2}}{\partial x} - \frac{cL_a}{B_c} - \frac{\partial L_a}{\partial t}\right] \quad (4)$$

Conservation of Mass for Floodplain Base

Mass conservation in the basal floodplain layer for each grain size is as follows:

> Rate of change of sediment volume in the floodplain base layer = Net exchange rate with the floodplain as the upper boundary of the basal layer moves + Net exchange rate with the substrate as the lower boundary of the basal layer moves – Net exchange rate with the active layer of the channel bed as the channel migrates.

$$\frac{\partial}{\partial t}\int_{\eta}^{\eta+L_a}(1-\lambda_p)F_{bfi}B_f\Delta x dz = (1-\lambda_p)B_f\left(\frac{\partial}{\partial t}(\eta+L_a)\Delta x_c f_{Ifui} - \frac{\partial}{\partial t}(\eta)\Delta x_c f_{Ifli}\right) + c\Delta x_c L_a(F_{ai}-F_{bfi}) \quad (5)$$

Taking the limit as Δx → 0, assuming no time variation in B_f, setting i = 2, and making the assumption that the basal layer is well mixed, the following form results for the time evolution of the fraction sand in the floodplain base layer:

$$\frac{\partial F_{bf2}}{\partial t} = \frac{\partial \eta}{\partial t}\left(\frac{f_{Ifu2}-f_{Ifl2}}{L_a}\right) - F_{bf2}\left(\frac{c\Sigma}{B_f}+\frac{1}{L_a}\frac{\partial L_a}{\partial t}\right) + \left(\frac{f_{Ifu2}}{L_a}\frac{\partial L_a}{\partial t}+\frac{c\Sigma}{B_f}\right) \quad (6)$$

Conservation of Mass for Floodplain

For the upper floodplain layer, mass conservation becomes:

> Rate of change of sediment volume in the floodplain layer = Net exchange rate of sediment with the channel – net exchange rate with the floodplain base as the upper boundary of the floodplain base layer moves.

$$\frac{\partial}{\partial t}(1-\lambda_p)\Delta x B_f F_{fi} L_f = q_{obi}\Delta x_c - q_{efpi}\Delta x_c - (1-\lambda_p)\frac{\partial}{\partial t}(\eta+L_a)\Delta x B_f f_{ifui}\,. \quad (7)$$

Summing over all grainsizes,

$$q_{obT} = \sum_{i=1}^{n} q_{obi}\,,\; q_{efpT} = \sum_{i=1}^{n} q_{efpi}\,, \quad (8)$$

assuming no time variation in B_f, and taking the limit as Δx → 0 yields:

$$\frac{dL_f}{dt} = \frac{\Sigma}{(1-\lambda_p)B_f}(q_{obT}-q_{efpT}) - \frac{d\eta}{dt} - \frac{dL_a}{dt}\,. \quad (9)$$

This result, which describes the evolution of floodplain thickness, can be substituted back into 7 to give, for i = 2:

$$\frac{\partial F_{f2}}{\partial t} = \frac{F_{f2}-f_{Ifu2}}{L_f}\left(\frac{\partial \eta}{\partial t}+\frac{\partial L_a}{\partial t}\right) + \frac{\Sigma}{(1-\lambda_p)B_f L_f}(q_{ob2}-F_{f2}q_{obT}), \quad (10)$$

which describes the grain size evolution of the upper floodplain deposit.

Equations 4, 6, 9 and 10 represent a system of four equations and four unknowns (η, F_{bf2}, L_f and F_{f2}). However, it requires the independent specification several additional terms, specifically the floodplain erosion terms q_{efp1} and q_{efp2}, the overbank deposition terms q_{ob1} and q_{ob2}, the streamwise rate of change of total bed material load $\partial q_t/\partial x$, the thickness of the active layer L_a, and the time rate of change of the active layer thickness $\partial L_a/\partial t$. The other terms, c, Σ, B_c, and B_f can be arbitrarily specified based on observed data.

Erosion Model

Erosive transfer of material from floodplain to channel can occur by several processes ranging from direct erosive stripping of the floodplain surface (Nanson and Croke, 1992) to several kinds of losses associated with river bank migration. One of the migration-related processes is the tendency of river systems to migrate until an avulsion or cutoff occurs. The average volume of the abandoned channel divided by the average time between cutoffs represents a loss rate of floodplain material. Another migration-related loss process is the tendency for channels to build point bars that are somewhat lower in elevation than the cut bank on the opposite side of the channel. If the top of the point bar represents the floodplain surface that is being regenerated, then there is a net loss of material since less is redeposited that is eroded. Only the third erosive process is included in the present version of the model. For a given system, we assume a net elevation difference, Δη, between the cut bank and the top of the opposite point bar. Assuming the point bar composition is not significantly different than the cut bank,

$$q_{efpi} = F_{fi}\left(1-\lambda_p\right)\left(c\Delta\eta\right). \tag{11}$$

Assuming that c and Δη are constant in time results in a model that describes the erosion rate from the floodplain as constant in time but allows the fraction sand and mud eroded to vary as the floodplain grain size distribution evolves.

Deposition Model

Narinesingh et. al. (1999) and Parker et al. (1996) provide similar deposition models that characterize reach-average deposition as a function of flow on the floodplain and suspended sediment concentration in the channel. Both are based on advective transport of suspended sediment onto a floodplain which is treated as an array of stream tubes that act as individual settling basins. The form of Parker, (1996) is used here since it provides several coefficients that allow the model to be calibrated.

$$q_{obi} = F_l \frac{C_{0i} Q_f}{B_f}\left(1-\exp\left(\frac{-\alpha v_{si} B_f^{\ 2}}{Q_f}\right)\right) \tag{12}$$

Here, F_l and α are dimensionless coefficients that must be obtained by calibration, C_{0i} = average sediment concentration in the water column above the floodplain level (obtained using the Rouse profile for sand, Rouse, 1939), v_{si} = settling velocity of size class i in quiescent water, and Q_f = volumetric flow rate of water on the floodplain.

Channel Hydraulics and Sediment Transport

The deposition model requires a description of 1-D flow on the channel floodplain and the concentration of suspended sand in the channel above the level of the floodplain. The morphodynamic model also requires a total load sediment transport model, which is used to compute $\partial q_t/\partial x$. A simple 1-D gradually varied flow model is used to partition flow between the channel and floodplain. The hydraulic model is conceptually straightforward, using the standard form of the energy equation to compute a gradually varied flow water surface elevation profile (Sturm 2001). The hydraulic model accounts for form drag associated with dunes using the friction model of Wright and Parker, (in press), which also includes the effects of density stratification on channel bed friction. Manning's equation with a friction coefficient of 0.1 is used for the floodplain. The suspended sediment transport model of Wright and Parker, (in press) is used to predict suspended sediment transport rates, while the model of Ashida and Michiue, (1972) is used for bed load. The active layer thickness, L_a, is specified as an arbitrary fraction (10%) of bankfull depth L_f.

Because overbank deposition is driven by floodplain inundation while in-channel transport is primarily a function of flow below the bankfull level, it is necessary to drive the model using a representative set of flow rates taken from a flow duration curve. The time rate of change of any of the four sediment storage variables (η, L_f, F_{f2}, and F_{bf2}) computed for any given flow in the flow duration curve are multiplied by the fraction of time represented by the given flow and then summed to get the average change rate used to step forward in time;

$$\frac{\partial \phi_j}{\partial t} = f(Q_j)\,; \qquad \frac{\partial \phi}{\partial t} = \sum_{j=1}^{p} P_j \frac{\partial \phi_j}{\partial t} \tag{13}$$

In the above equations, ϕ is one of the four sediment storage variables, Q_j is the flow rate for the jth bin in the flow duration curve, P_j is the fraction of the time that flow Q_j occurs, p is the number of bins, $\partial\phi_j/\partial t$ is the computed value of the time rate of change of one of the sediment storage variables at flow Q_j, and $\partial\phi/\partial t$ is the value of the rate of change of a sediment storage variable used to step forward in time.

Conservation of Mass for Water Column

The overbank deposition model requires suspended sediment concentrations for both sand and mud size classes. For sand, the concentration is specified by the channel hydraulics. For mud, the concentration is independent of hydraulics, which is why suspended mud is often called washload. For relatively short reaches, it may be appropriate to assume that the washload concentration is simply specified by a rating curve constructed from observed data. However, over longer reaches, we would expect washload concentrations to vary in the downstream direction, particularly where floodplain erosion is not in equilibrium with overbank deposition. Since we are primarily interested in long term, large scale effects, we have balanced the conservation of mass equation for a whole time step by summing over all flows and not considering individual bins of the flow duration curve. The equation for washload concentration C_1 at a given flow rate j is:

$$\frac{\partial C_{1,j}}{\partial x} = \frac{1}{H_{c,j} B_c}\left(q_{efp1} - q_{ob1} + F_{bf1}(1-\lambda_p)cL_a - \frac{\partial \eta}{\partial t} B_c f_{Ia1} \right) \tag{14}$$

where

$$q_{efi} = \sum_{j=1}^{p} P_j q_{efpi,j} \;;\; q_{obi} = \sum_{j=1}^{p} P_j q_{obi,j} \;.\; \text{ and } f_{Ia1} = \begin{cases} 0, & \frac{\partial \eta}{\partial t} \geq 0 \\ f_{s1}(\eta), & \frac{\partial \eta}{\partial t} < 0 \end{cases} \tag{15}$$

Model Application

The model is applied to the Minnesota River, a tributary of the Mississippi with a drainage area of approximately 43,000 km^2 at the confluence. The model is driven by a set of 20 flow rates that characterize the upper half of the observed flow duration curve. Wash load concentrations at the upstream end are taken from a suspended sediment rating curve from which suspended sand concentration has been removed using the Wright and Parker (2003) suspended load transport relationship. The downstream end of the model has a fixed bed elevation and a normal depth water surface boundary condition. The upstream sediment feed in the sand size range is adjusted so that it is in equilibrium with the computed capacity of the reach.

The coefficient $\Delta\eta$ required by the erosion model was obtained from cross sections taken at several locations along the river. Migration rates were obtained from sequential aerial photography. The coefficients for the deposition model were then calibrated base on the observed bankfull L_f of 4.6 m so that the resulting deposition rate was exactly equal to the erosion rate predicted by the erosion model and so that the proportion of sand and mud deposited on the floodplain is reasonable. We assumed a sand fraction in the floodplain of 30%. This calibration assumes that floodplain erosion and deposition are currently in equilibrium along the modeled river reach. The floodplain elevation was then reduced by 1 m along a 5-km reach of the valley, locally putting floodplain and channel erosion and deposition out of balance.

Results

Figure 3 shows several longitudinal profiles of the bed, floodplain, and one percent exceedance water surface taken at various points in time for this run (at 30, 300, and 3000 years). 30 years into the run, the bed in the zone of floodplain excavation has begun to aggrade due to the reduction in shear stress on the bed. During this time, the floodplain in the excavation area actually experiences a reduction in overbank deposition as sand that would have been deposited on the floodplain is instead deposited on the bed. During this initial time period, the upstream bed degrades in response to the additional energy slope caused by the reduced water levels in the excavated area. Eventually, however, deposition on the floodplain causes both the bed and floodplain to recover something close to the initial pre-excavation profile.

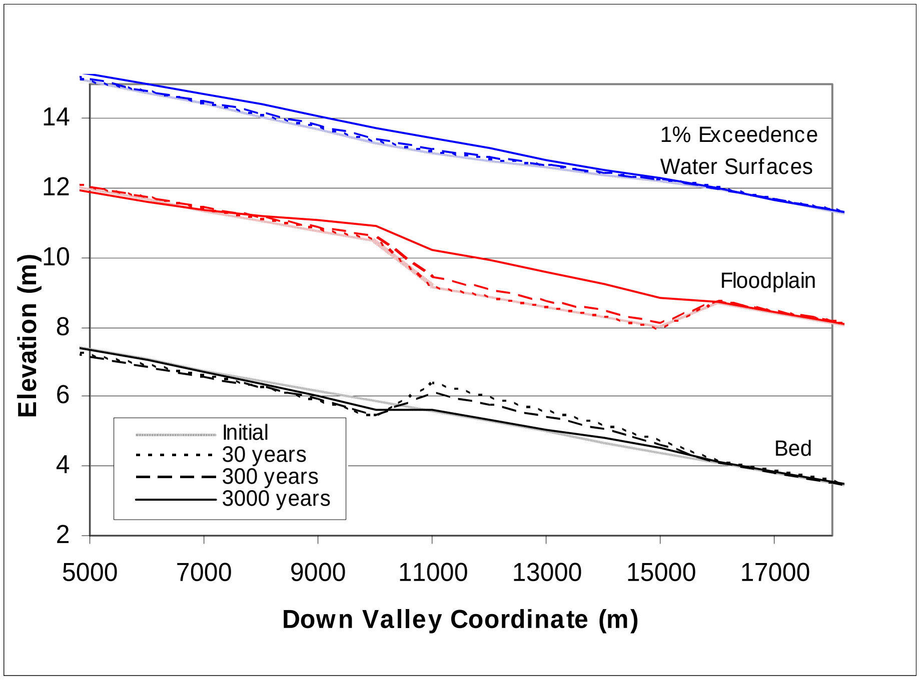

Figure 3. Evolution history of bed, floodplain, and one percent exceedence water surface elevation profiles. The model extends upstream an additional 5000 m and downstream an additional 12000 m.

Discussion

The relatively rapid bed elevation changes predicted by the model during the first 30 years of simulation are driven entirely by the additional conveyance that becomes available on the floodplain after the excavation occurs. While existing channel bed aggradation and degradation models are able to predict this, they are only able to do so if enough of the flood flow distribution is used to account for floodplain flow.

The eventual return to the initial profile, while perhaps not occurring in engineering time (and not quite complete even after 3,000 years of simulation), cannot be predicted using standard aggradation and degradation models that do not account for overbank deposition. The feedback that allows the model to return to a stable state is based on the tendency for the more frequent flooding in the low area to cause deposition, eventually increasing the floodplain's elevation. However, as the floodplain is built, it floods less frequently, eventually so infrequently that erosion and deposition come into equilibrium. A similar story can be told for a floodplain that is in some sense too high. As long as erosion from the floodplain is not a strong function of floodplain elevation, a high floodplain that floods infrequently should experience more erosion than deposition, which would tend to decrease its elevation until erosion comes into balance with deposition.

Since floodplain elevation - bed elevation = bankfull depth, our model can be thought of as a model for channel bankfull cross-sectional geometry. Our model differs in a fundamental way from the usual description of channel formation in which bankfull discharge is considered the channel-forming discharge. By definition, any flow, including bankfull, that is contained entirely within the banks can not, at least through deposition, build a channel. While the bankfull discharge provides a useful, observable parameter upon which to base alluvial channel geometry models, it can not by itself be responsible for a channel's formation. Rather, the range of flows near and above bankfull must play an important role. Our model, though conceptually rather simple, captures the essence of the feedback between erosion and deposition we think is driven by this range of flows and ultimately controls a channel's depth.

Acknowledgements

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