CHAPTER 3
RELATIONS FOR BEDLOAD AND SUSPENDED LOAD

3.1 INTRODUCTION

Alluvial rivers are authors of their own geometry. They create their own channel, with a characteristic bankfull width and depth. Although slope is partly imposed by geological constraints, rivers are nevertheless free to change slope substantially by means of aggradation and degradation.

Alluvial rivers can be broadly categorized into two types; sand bed streams and gravel bed streams. Sand bed streams typically have values of bed material median size D_{50} between 0.1 and 0.8 mm; the range of sizes is usually rather modest. Gravel bed streams have values of surface sediment D_{50} ranging from 10 to 200 mm; substrate D_{50} is usually finer by a factor of 1.5 to 2 due to a surface armor present at low flow. The range of sizes is usually quite wide, including up to 30% sand in the interstices of the gravel.

The relative lack of streams in the intermediate range is due to both geologic and mechanistic factors. Weathered granite, however, can yield sediment in the range 1 – 10 mm, so such streams are occasionally observed. This notwithstanding, the dichotomy between the two types is best represented in terms of a dimensionless regime diagram, here presented as Figure 2.1. The horizontal axis of the diagram is an explicit particle Reynolds number, given by

$$\mathbf{Re}_p = \frac{\sqrt{R g D}\, D}{\nu} \tag{3.1}$$

where D is characteristic grain size and ν is the kinematic viscosity of water, and the vertical axis is Shields stress τ^*. Shown on the diagram are three lines. The first of these describes the critical condition for the onset of motion; where τ_c^* denotes a critical Shields stress,

$$\tau_c^* = 0.5\,[0.22\ \mathbf{Re}_p^{-0.6} + 0.06 \cdot 10^{(-7.7\,\mathbf{Re}_p^{-0.6})}] \tag{3.2}$$

The above relation represents my modification of Brownlie's (1981) fit to the original Shields curve. The second of these corresponds to the criterion for the onset of significant suspension (2.31), reduced with the Dietrich (1982) relation for fall velocity;

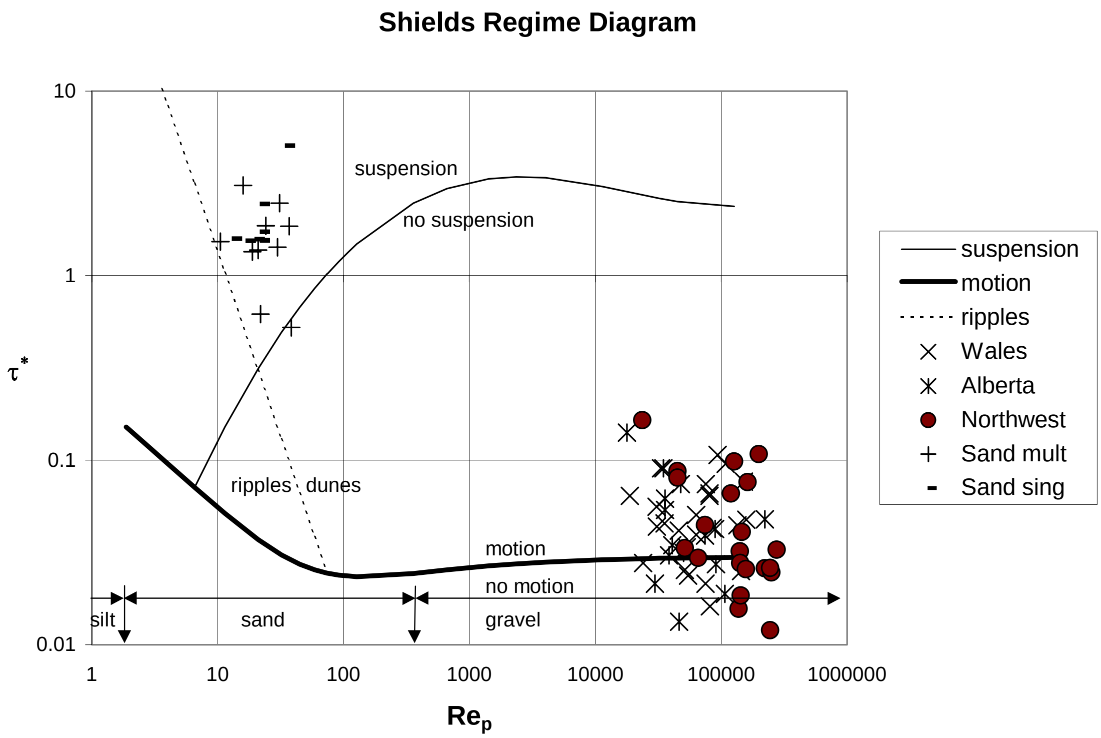

Figure 2.1 Dimensionless regime diagram for streams.

$$\mathbf{R}_f = \exp\{-b_1 + b_2\,\ell n(\mathbf{Re}_p) - b_3[\ell n(\mathbf{Re}_p)]^2 - b_4[\ell n(\mathbf{Re}_p)]^3 + b_5[\ell n(\mathbf{Re}_p)]^4\} \tag{3.3}$$

where

$$\mathbf{R}_f = \frac{v_s}{\sqrt{RgD}} \tag{3.4}$$

and $b_1 = 2.891394$, $b_2 = 0.95296$, $b_3 = 0.056835$, $b_4 = 0.002892$ and $b_5 = 0.000245$. The third of these takes the forms

$$\frac{D}{\delta_v} = 1 \qquad \delta_v = 11.6\frac{\nu}{u_*} \tag{3.5}$$

where δ_v denotes a scale for the thickness of the viscous sublayer. Empirical evidence indicates that small-scale ripples form when $D/\delta_v < 1$; for larger values ripples give way to larger dunes, the characteristics of which are independent of viscosity.

Finally, data pertaining to the Shields stress τ^* at bankfull conditions in a variety of sand bed streams (both single channel and multiple channel) and gravel bed streams (Wales, U.K., Alberta, Canada and Pacific Northwest, USA). The Shields stress was estimated assuming normal flow; according to (1.36). (1.40) and (2.3),

$$\tau^* = \frac{HS}{\mathit{R}D} \tag{3.6}$$

where H refers to bankfull depth.

The very different regimes for sand bed streams and gravel bed streams at bankfull conditions are readily apparent. Bankfull flow typically corresponds to a flood with a mean return period of 1 to 5 years. Sand bed streams can carry significant suspensions of the bed material; in gravel bed streams the gravel is not suspended. It is also the case that sand bed streams are typically in a regime for which dunes form readily at all but the highest flows (where they give way to upper regime plane bed or antidunes), whereas gravel bed streams often do not have prominent dunes even at flood flows.

3.2 RELATIONS FOR HYDRAULIC RESISTANCE

In order to compute sediment transport it is necessary to solve for the boundary shear stress τ_b, either from equilibrium normal flow or from the governing equations for disequilibrium flow, i.e. the de St. Venant equations for phenomena with a streamwise length scale that is large compared to the depth and the Reynolds equations with an appropriate turbulent closure for phenomena that scale with the depth or smaller.

Here it is assumed that the shallow water equations are applicable. In a gravel bed stream at flood conditions, form drag can often be neglected, and the appropriate resistance relations are (1.36) and (1.38), where

$$k_s = n\,D_{s90} \qquad n = 2 \sim 3 \tag{3.7}$$

and D_{s90} denotes a surface size such that 90% is finer.

In the case of a sand bed stream a somewhat more complicated approach must be taken to account for the effect of the dunes. The following method, developed by Engelund and Hansen (1967) using the Einstein decomposition for skin friction and form drag, takes the following form for normal (steady uniform) flow;

$$\tau_b = \tau_{bs} + \tau_{bf} \qquad \tau_{bs} = \rho C_{fs} U^2 = \rho g H_s S \qquad \tau_{bf} = \rho C_{ff} U^2 = \rho g H_f S$$

(3.8a,b,c)

$$C_{fs}^{-1/2} = 2.5\,\ell n(11\frac{H_s}{k_s}) \qquad k_s = 2.5\,D \tag{3.9}$$

$$\tau_s^* = 0.06 + 0.4(\tau^*)^2 \tag{3.10}$$

where

$$\tau_s^* = \frac{\tau_{bs}}{\rho R g D} \tag{3.11}$$

In the above relations, τ_{bs}, C_{fs}, H_s and τ_s^* denote the components of τ_b, C_f, H and τ^* associated with skin friction, and τ_{bf}, C_{bf} and H_f denote the corresponding components due to form drag. In point of fact (3.10) applies only to lower regime flow, i.e. flows with sufficiently low Froude numbers so that antidunes or upper regime plane beds do not form. This is, however, the most common case in sand bed streams.

Implementation of the above scheme is iterative. If H_s is guessed for a flow in a river reach with known bed slope S and grain size D, C_{fs} can be found from (3.9), U from (3.8b), τ_s^* from (3.8b) and (3.11), τ^* from (3.10), H from (3.6) and water discharge per unit width q_w from (1.34). Total water discharge Q_w is then given by the approximate relation

$$Q_w = q_w B \tag{3.12}$$

where B denotes stream width at the flow in question. The depth H_f is given by H – H_s and C_{ff} is then given by (3.8c)

The above method is easily generalized for gradually varied flow. The method involves evaluating $C_f = C_{fs} + C_{ff}$ at a known value of H in the stepwise upstream integration of (1.35). Again, iteration is performed using H_s as an intermediary.

3.3 RELATIONS FOR BEDLOAD TRANSPORT

The theory behind bedload transport formulations has only recently reached mature form. While the treatment is fascinating (see e.g. Wiberg and Smith, 1989) there is no room in these notes for great detail. Suffice it to say that a rigorous analysis of the equations of motion of saltating particles leads to bedload relations that are of the general form of the relation of Meyer Peter and Muller (2.1)

Three other relations of this form are of interest. The relation of Ashida and Michiue (1972) provides excellent results for bedload transport in the presence of bedforms; it takes the form

$$q_b^* = 17(\tau_s^* - \tau_c^*)[(\tau_s^*)^{1/2} - (\tau_c^*)^{1/2}] \tag{3.13}$$

where τ_c^* takes a value of 0.05. The relation was verified with uniform bed material ranging from 0.3 to 7 mm, with most of the data in the sand range. The relation of Engelund and Fredsoe (1976) is quite similar;

$$q_b^* = 18.74(\tau_s^* - \tau_c^*)[(\tau_s^*)^{1/2} - 0.7(\tau_c^*)^{1/2}] \tag{3.14}$$

The relation of Parker (1990) is specifically designed for field gravel bed streams with a wide range of sizes. For this reason the relation is rather involved. The surface size distribution is divided into ranges i = 1..N, each with characteristic size D_i and fraction content in the surface layer F_i. Here F_i is normalized to include only sizes ≥ 2 mm, and must sum to unity. The geometric mean grain size D_{sg} and geometric standard deviation σ_{sg} of the surface material are given by

$$D_{sg} = 2^{\overline{\psi}} \qquad \sigma_{sg} = 2^{\sigma_{sa}} \tag{3.15}$$

where

$$\overline{\psi} = \sum_{i=1}^{N} F_i \psi_i \qquad \sigma_{sa} = \sum_{i=1}^{N} (\psi_i - \overline{\psi})^2 F_i \tag{3.16}$$

and

$$\psi_i = \ell n_2 (D_i) \tag{3.17}$$

A dimensionless bedload transport rate W_{si}^* is defined as

$$W_{si}^* = \frac{R g q_{bi}}{u_*^3 F_i} \tag{3.15}$$

The transport relation is specified as

$$W_{si}^* = 0.0218 G[\omega \phi_{sgo} (\frac{D_i}{D_{sg}})^{-0.0951}] \tag{3.16}$$

where q_{bi} is the bedload transport rate of the ith grain size range and

$$\phi_{sgo} = \frac{\tau^*_{sg}}{0.0386} \qquad \tau^*_{sg} = \frac{\tau_b}{\rho R g D_{sg}} \tag{3.17}$$

and

$$\omega = 1 + \frac{\sigma_{sa}}{\sigma_{sao}}(\omega_o - 1) \tag{3.18}$$

In (3.18), σ_{sao} and ω_o are specified functions of ϕ_{sgo} that can be found in the original reference. The function G is given as

$$G[\phi] = \begin{cases} 5474(1 - \dfrac{0.853}{\phi})^{4.5} & \text{for } \phi \geq 1.65 \\ \exp[14.2(\phi - 1) - 9.28(\phi - 1)^2] & \text{for } 1 \leq \phi < 1.65 \\ \phi^{14.2} & \text{for } \phi < 1 \end{cases} \tag{3.19}$$

Note that the relation includes both the geometric mean size of the surface material D_{sg} and the geometric standard deviation of the surface material σ_{sg} in the formulation. In the limit of uniform material it reduces to a form similar to Wiberg and Smith (1989). Software has been developed for easy application.

In some cases the effect of gravity impelling bedload particles down slopes must be included in the bedload formulation. This factor is almost negligible in terms of mean streamwise bed slope, but can become important on e.g. the lee side of dunes or on the side slopes of point bars. This element turns out to be critical to a correct analysis of dunes, alternate bars and meander bends. Many researchers have approached this problem. In the case of relatively small bed slopes, Parker and Andrews (1985) have suggested the following relation;

$$\begin{aligned} q_{bm} &= f(\sqrt{\tau_{bx}^2 + \tau_{by}^2}) \\ q_{bx} &= \frac{\tau_{bx}}{\sqrt{\tau_{bx}^2 + \tau_{by}^2}} q_{bm} - \beta \frac{\partial \eta}{\partial x} \\ q_{by} &= \frac{\tau_{by}}{\sqrt{\tau_{bx}^2 + \tau_{by}^2}} q_{bm} - \beta \frac{\partial \eta}{\partial y} \end{aligned} \tag{3.20}$$

where f denotes the bedload function for the unidirectional case in the absence of significant bed slopes and

$$\beta = \beta^* (\frac{\tau^*}{\tau_c^*})^{-1/2} \tag{3.21}$$

where β^* is an order one constant and τ^* is based on shear stress magnitude.

3.4 RELATIONS FOR ENTRAINMENT INTO SUSPENSION

The Smith-McLean relation has been introduced previously as (2.6). The value b at which the entrainment rate is to be evaluated corresponds to the top of the bedload layer, and is given by the relation

$$b = 26.3[(\tau_s^* / \tau_c^*) - 1]\, D + k_s \tag{3.22}$$

Garcia and Parker (1991) have developed the following relation for E;

$$E = \frac{AZ_u^5}{1 + \frac{A}{0.3} Z_u^5} \tag{3.23}$$

where A = $1.3\text{x}10^{-7}$ and

$$Z_u = \frac{u_{*s}}{v_s} \mathbf{Re}_p^{0.6} \qquad u_{*s} = \sqrt{\tau_{bs} / \rho} \tag{3.24}$$

In the above relation u_{*s} is meant to be evaluated with (3.10) for lower regime conditions, and the corresponding relation from Engelund and Hansen (1967) for upper regime conditions. The parameter b is evaluated simply as

$$b = 0.05\, H \tag{3.25}$$

Van Rijn (1984) has also offered a useful and accurate entrainment relation.

3.5 RELATIONS FOR TOTAL LOAD

A number of useful empirical predictors for total bed material load (bed load + suspended load) in sand bed streams. The relations of Ackers and White (1973) and Yang (1973) are considered to be reliable. The author's favorites for such relations are, however, those of Engelund and Hansen (1967) and Brownlie (1981).

The former relation takes the simple form

$$C_f q_t^* = 0.05(\tau^*)^{5/2} \qquad q_t^* = \frac{q_t}{\sqrt{RgD}\,D} \tag{3.26}$$

It should always be used in conjunction with theEngelund-Hansen resistance relation, as outlined above for lower regime conditions.

The Brownlie (1981) relation is rather more complicated. Let $Q_{st}=q_tB$ and

$$X = 1x10^6 \frac{\rho_s Q_{st}}{\rho Q_w + \rho_s Q_{st}} \tag{3.27}$$

denote the mass concentration of sediment in parts per million. The Brownlie relation takes the form

$$X = 7115 c_a \, (F_g - F_{go})^{1.978} S^{0.6601} (\frac{H}{D_{50}})^{-0.3301} \tag{3.28}$$

where

$$F_g = \frac{U}{\sqrt{RgD_{50}}}$$
$$F_{go} = 4.596(\tau_c^*)^{0.5293} S^{-0.1045} \sigma_g^{-0.1606} \tag{3.29a,b}$$

and

$$\tau_c^* = 0.22\,\mathbf{Re}_p^{-0.6} + 0.06 \cdot 10^{(-7.7\,\mathbf{Re}_p^{-0.6})} \tag{3.30}$$

In (3.28) σ_g denotes the geometric standard deviation of the bed sediment, and the parameter c_a takes the value 1.0 for flumes and 1.268 for field channels. If the channel is narrow mean depth H should be replaced by the hydraulic radius R_h.

The Brownlie relation should be used in conjunction with the Brownlie resistance relation, which takes the form

$$\frac{HS}{D_{50}} = 0.3724\,(\tilde{q}_w S)^{0.6539}\, S^{0.09188}\, \sigma_g^{0.1050} \tag{3.31}$$

where

$$\tilde{q}_w = \frac{q_w}{\sqrt{gD_{50}}\, D_{50}} \tag{3.32}$$