CHAPTER 4
ALTERNATE BARS AND ALLUVIAL FANS

4.1 INTRODUCTION

At least as interesting as the mechanism of sediment transport itself is the ability of rivers to construct morphologies. Primary among these morphologies is the river itself, with a definable self-formed channel maintaining consistent bankfull geometry, as well as a floodplain. Within the river there can be bedforms such as ripples, dunes and antidunes, and bar forms such as alternate bars and the point bars of bends. The river can meander or braid, and is the agent of construction of larger morphologies such as drainage basins and alluvial fans. In the interest of space, only alternate bars and alluvial fans are treated here.

4.2 ALTERNATE BARS

In streams of sufficient width B the bed of the channel does not remain plane, but rather is subject to bar instability. The governing parameter here is the ratio of width to depth B/H. When this number exceeds a value between 12 and 20, the flow becomes unstable and creates the single-row alternate bars shown in Figure 4.1 below. Single-row alternate bars can be considered to be precursors to meandering. If the banks are inerodible, the bars grow to a finite-amplitude equilibrium and then migrate downstream. If the banks are erodible, the bars induce enough sinuosity to allow the primary mechanism for meandering to take hold. At larger values of B/H, multiple-row alternate bars can form. These are precursors to braiding.

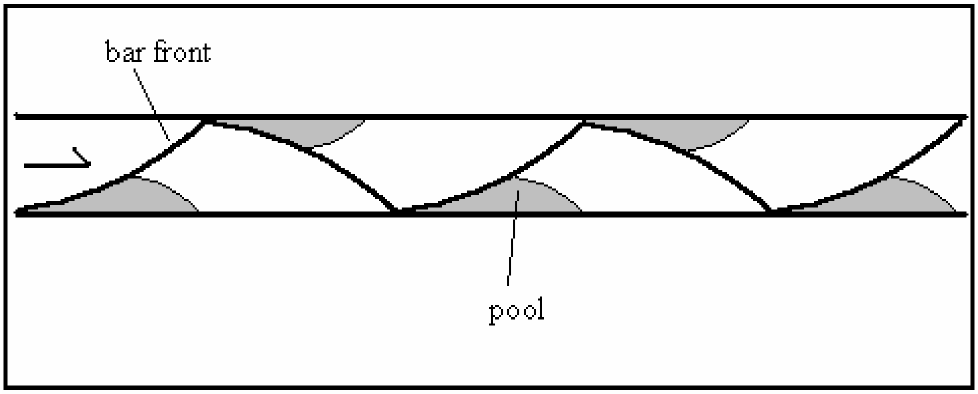

Figure 4.1 Alternate bars.

4.3 FORMULATION FOR ALTERNATE BARS

As in most sediment transport problems, the quasi-steady approximation can be employed. That is, the flow is treated as steady, with the time variation appearing in the Exner equation only. Thus (1.41) – (1.43) become

$$\frac{\partial}{\partial x}(UH)+\frac{\partial}{\partial y}(VH)=0 \tag{4.1}$$

$$\frac{\partial}{\partial x}(U^2H)+\frac{\partial}{\partial y}(UVH)=-gH\frac{\partial H}{\partial x}-gH\frac{\partial \eta}{\partial x}-C_f(U^2+V^2)^{1/2}\frac{U}{H} \tag{4.2}$$

$$\frac{\partial}{\partial x}(UVH)+\frac{\partial}{\partial y}(V^2H)=-gH\frac{\partial H}{\partial y}-gH\frac{\partial \eta}{\partial y}-C_f(U^2+V^2)^{1/2}\frac{V}{H} \tag{4.3}$$

In the formulation given here alternate bars in a gravel-bed river are considered. The Exner equation of sediment continuity (2.9) takes the form

$$(1-\lambda_p)\frac{\partial \eta}{\partial t}=-\frac{\partial q_{bx}}{\partial x}-\frac{\partial q_{by}}{\partial y} \tag{4.4}$$

The Meyer-Peter Muller relation (2.1) is used for gravel transport. The relation is generalized to two dimensional vectorial transport using (3.20). For simplicity the friction coefficient C_f is assumed to be constant. The analysis is easily generalized to arbitrary resistance relation, as shown in Parker and Johannesson (1989).

4.4 THE BASE STATE FOR ALTERNATE BARS

It is easily verified that the governing equations admit a base solution corresponding to steady, uniform flow in the x direction for which $U = U_o$, $V = 0$, $H = H_o$, $\tau_{bx} = \tau_{bo}$, $\tau_{by} = 0$, $q_{bx} = q_{bo}$, $q_{by} = 0$ and

$$\eta = \eta_o(x) = \eta_r - Sx \qquad \xi = \xi_o(x) = \eta_r + H_o - Sx \tag{4.5}$$

where

$$\xi = \eta + H \tag{4.6}$$

denotes water surface elevation. Specifically, once C_f, water discharge per unit width q_w, mean bed slope S, grain size D and sediment submerged specific gravity *R* are specified, the base solution can be obtained from the following.

$$0 = gS - C_f \frac{U_o^2}{H_0} \qquad U_o H_o = q_w$$
$$\tau_{bo} = \rho C_f U_o^2 \qquad \frac{q_{bo}}{\sqrt{RgD}\,D} = 8\left(\frac{\tau_{bo}}{\rho R g D} - 0.047\right)^{1.5} \tag{4.7}$$

4.5 NORMALIZATION AND PERTURBATION

Here a linear stability analysis of alternate bars is performed. The alternate bar perturbation is assumed to have streamwise wavelength λ, and thus dimensionless wavenumber

$$k = \frac{2\pi b}{\lambda} \tag{4.8}$$

where $b = B/2$ denotes the half-width of the channel. It is useful to introduce the parameters γ and ε defined as

$$\gamma = \frac{b}{H_o} \qquad \varepsilon = \gamma C_f \tag{4.9}$$

The parameter γ becomes the bifurcation parameter of the problem, with a plane bed giving way to alternate bars when γ is sufficiently large. The parameter ε takes a value on the order of 0.1 for typical meandering streams, and can thus be treated as small.

Wavenumber k also tends to be on the order of 0.1 for typical meandering streams, so an o(1) normalized dimensionless wavenumber r can be defined such that

$$r = \frac{k}{\varepsilon} \qquad (4.10)$$

The equations are made dimensionless in the following way: U and V are normalized with U_o, x and y are normalized with half-width b, H is made dimensionless with H_o, q_{bx} and q_{by} are normalized with q_o and the deviation of η and ξ from the base state, i.e. $\eta_d = \eta - \eta_o$ and $\xi_d = \xi - \xi_o$ are made dimensionless with H_o. Time is made dimensionless in such a way as to obtain the most compact formulation. Here it is assumed that all subsequent parameters have been made dimensionless.

Further normalization of the dimensionless parameters can be performed as follows;

$$kx = \varepsilon r x \equiv \phi \qquad \tilde{V} = \frac{V}{\varepsilon} \qquad (4.11a.b)$$

In the above ϕ is a phase varying from 1 to 2π over one wavelength, and $\tilde{V}$ is the transverse velocity normalized in a form so as to be o(1). The following perturbations are inserted into the normalized dimensionless governing equations;

$$\begin{aligned} &U = 1 + u' \quad \tilde{V} = 0 + v' \quad H = 1 + h' \quad q_{bx} = 1 + q'_x \quad q_{by} = 0 + q'_y \\ &\eta_d = 0 + \eta' \quad \xi_d = 0 + \xi' \end{aligned} \qquad (4.12)$$

In a linear stability analysis the primed quantities are assumed to be arbitrarily small; note from (4.6) that

$$\xi' = \eta' + h' \qquad (4.13)$$

Upon linearization and neglect of terms of $o(\varepsilon^2)$, (4.1) – (4.4) reduce to

$$r\frac{\partial u'}{\partial \phi} + r\frac{\partial(\xi' - \eta')}{\partial \phi} + \frac{\partial v'}{\partial n} = 0 \qquad (4.14)$$

$$r\frac{\partial u'}{\partial \phi} = -r\,\mathbf{Fr}^{-2}\frac{\partial \xi'}{\partial \phi} - 2u' + h' \qquad (4.15)$$

$$0 = -\mathbf{Fr}^{-2}\frac{\partial \xi'}{\partial n} \qquad (4.16)$$

$$\frac{\partial \eta'}{\partial t} + Mr\frac{\partial u'}{\partial \phi} + \frac{\partial v'}{\partial n} - \Gamma\frac{\partial^2 \eta}{\partial n^2} \qquad (4.17)$$

where

$$\mathbf{Fr} = \frac{U_o}{\sqrt{gH_o}} \qquad \Gamma = \frac{\beta}{\gamma\varepsilon}$$
$$M = \frac{3}{(1 - \frac{0.047}{\tau_o^*})} \qquad \tau_o^* = \frac{\tau_{bo}}{\rho R g D} \tag{4.18}$$

In the above relations **Fr** denotes the Froude number of the base state, β is a parameter specified in (3.21) (and here approximated as constant), Γ is another dimensionless parameter found to be o(1) for typical meandering streams and τ_o^* is the Shields number at the base state, here assumed to be in excess of 0.047 in order to insure sediment transport.

4.6 STABILITY ANALYSIS

Note from the relation of transverse momentum balance that the perturbation in water surface elevation drops out of the problem, reducing (4.14). (4.15) and (4.17) to three equations in three unknowns. The approximation to the case for which the water surface does not respond the presence of alternate bars is not without consequences, as pointed out below.

The following sinusoidal forms for the perturbations are assumed;

$$u' = u_1 e^{\alpha t} \cos(\phi - ct)\sin(\frac{\pi}{2}n) \qquad \eta' = \eta_1 e^{\alpha t}\cos(\phi - ct)\sin(\frac{\pi}{2}n)$$
$$v' = v_1 e^{\alpha t}\cos(\phi - ct)\cos(\frac{\pi}{2}n) \tag{4.19}$$

where α denotes a dimensionless growth rate and c denotes a dimensionless streamwise migration speed. That is, the flow is unstable to alternate bars when $\alpha > 0$; the bars migrate downstream when $c > 0$.

Substituting (4.19) into (4.14), (4.15) and (4.17) yields three linear homogeneous algebraic equations in the three unknowns u_1, η_1 and v_1. The condition for the existence of solutions is that the determinant of the associated matrix vanish. This condition yields a characteristic relatioin which can be solved to yield forms for α and c.

$$\alpha = \frac{[(M-1)-(\frac{\pi}{2})^2\Gamma]r^2 - 4(\frac{\pi}{2})^2\Gamma}{r^2+1} \qquad (4.20)$$

$$c = r\frac{r^2+6-2M}{r^2+1}$$

These relations predict stable and unstable regimes, and regions of downstream and upstream migration of bars as follows;

$$\alpha > 0 \quad \text{for} \quad \Gamma < \frac{M-1}{(\frac{\pi}{2})^2}\frac{r^2}{r^2+4} \quad \text{or} \quad \gamma = \frac{b}{H_o} > \frac{\pi}{2}\sqrt{\frac{\beta}{C_f(M-1)}}\frac{\sqrt{r^2+4}}{r} \qquad (4.21)$$

$$c > 0 \quad \text{for} \quad r > 2(M-3) \quad \text{or} \quad k > 2\varepsilon(M-3)$$

A sample case is considered here for which β = 0.5, τ_o^* = 0.10 and C_f =0.005. The smallest value of b/H_o that admits alternate bar instability is found to be 7.3, corresponding to ratio of width to depth of 14.6. This number is in general agreement with observations. The neglect of the transverse momentum equation simplifies the analysis to the point where it is not possible to identify a characteristic wavenumber k_c associated with the lowest value of γ for the onset of instability. This notwithstanding, the analysis predicts that alternate bars of any reasonable wavelength migrate downstream, not upstream. Rephrasing, only bars with unrealistically long wavelengths are predicted to migrate upstream.

4.7 ALLUVIAL FANS

Alluvial fans tend to form where sediment is available to deposit in zones undergoing tectonic subsidence. They are often seen at the base of mountains. In the absence of subsidence the fan continues to prograde outward. Subsidence, which creates accomodation space for the storage of the sediment, can arrest fan growth and create an equiilibrium in which aggradation due to sediment deposition is perfectly balanced by subsidence in the long term.

The depositive mechanism on fans can be either debris flows or river flow. Here only the latter case is considered. The equilbrium state sought is a long-term one. At any given time the river is likely to be either dry or morphologically inactive. With this in mind an intermittency I is introduced characterizing the fraction of the time that the river is morphologically active. On alluvial fans the river gradually deposits sediment, becomes higher than the surrounding fan and then avulses. The average fan morphology is created by the repetition of aggradation and avulsion. The characterization sought here is one that is averaged over a time window that is large compared to the amount of time necessary to rework the entire fan surface. Downfan slope S is assumed to be identical to the slope of the channel that deposited the sediment.

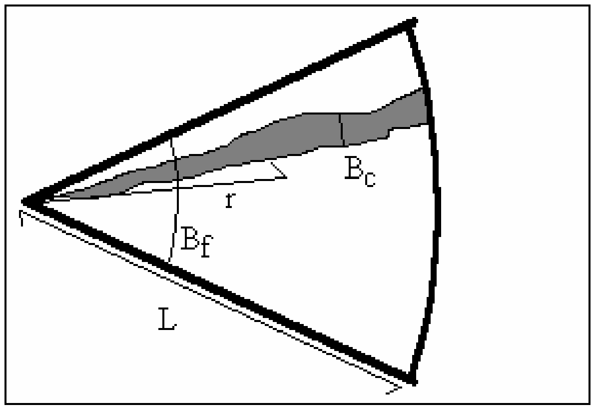

Figure 4.2 Definition diagram for alluvial fan.

Since fans typically have a fan-like planform, the problem is formulated in terms of a radial coordinate r, as shown in figure 4.2. Where θ denotes fan angle, fan width B_f is given as

$$B_f = r\theta \tag{4.22}$$

Here solutions are sought for an equilibrium fan for which subsidence and aggradation are in balance, so that fan length L is constant. The width of the active channel(s) coursing the fan at any given time is taken to be $B_c < B_f$, where B_c can vary down the fan. All sediment transport occurs in the channels. This sediment is, however, spread across the entire width of the fan by channel avulsion.

4.8 FORMULATION

The governing equation for sediment continuity is (2.14) reduced with (4.22), i.e.

$$(1-\lambda_p)r\theta\left(\frac{\partial\eta}{\partial t}+\sigma\right) = -\frac{\partial Q_{st}}{\partial r} \tag{4.23}$$

where where σ denotes the vertical tectonic subsidence rate and

$$Q_{st} = B_c q_t \tag{4.24}$$

It can be assumed that subsidence is continuous, whereas morphologically active floods areonly active in accordance with the intermittency I. Averaging (4.23) and assuming a perfect balance between sediment deposition and subsidence (so that the elevation profile $\eta(r)$ of the fan does not change in time) yields the result

$$\frac{dQ_{st}}{dr} = -(1-\lambda_p)\frac{\sigma}{I}\theta r \qquad (4.25)$$

In the above relation Q_{st} refers to the sediment transport rate during floods, not an average value including time periods for which the channel(s) are inactive and the sediment transport rate is vanishing. The boundary conditions on this relation are

$$Q_{st}\big|_{r=0} = Q_{so} \qquad Q_{st}\big|_{r=L} = 0 \qquad (4.26)$$

That is, the sediment feed at the apex of the fan is specified, and the sediment transport rate at the end of the fan vanishes, all of the sediment having been consumed by deposition upstream.

Integration of (4.25) subject to (4.26) yields the results

$$\frac{Q_{st}}{Q_{so}} = 1-(\frac{r}{L})^2 \qquad \frac{\sigma}{I} = \frac{Q_{so}}{\frac{1}{2}(1-\lambda_p)\theta L^2} \qquad (4.27a,b)$$

The first of these relations indicates a sediment transport rate that declines parabolically down the fan. The second of these determines the length L of the fan at which the subsidence has created just enough accomodation space to store the sediment delivered from the uplands.

4.9 SOLUTION FOR FAN MORPHOLOGY

A solution for fan slope S as a function of r can be obtained by tying (4.27a) to closure relations for sediment transport, hydraulic resistance and channel geometry. At the scale of an alluvial fan, it is not necessary to consider factors associated with gradually varying flow, and the assumption of normal flow, i.e. (1.40) can be used. It is assumed that water is conserved on the fan during floods, so that where Q_w is the constant water duscharge at flood conditions,

$$B_c UH = Q_w \qquad (4.28)$$

Here a simplified form of the Engelund-Hansen relation for total load (3.26), here written as

$$q_t^* = \alpha_s (\tau^*)^{5/2} \tag{4.29}$$

where α_s is taken to be a constant, is used in conjunction with a Manning-Strickler formulation for resistance (1.39), here slightly modified to

$$C_f^{-1/2} = \alpha_r \left(\frac{H}{D}\right)^{1/6} \tag{4.30}$$

where α_r is a dimensionless coefficient. In addition, it is assumed that the channel(s) develop so that the bankfull Shields stress τ_b^* obtains a specified value τ_{bo}^*;

$$\tau_b^* = \frac{HS}{RD} = \tau_{bo}^* \tag{4.31}$$

For example, Parker et al. (1998) have found that an appropriate value for τ_{bo}^* for sand-bed streams is near 1.8.

Substitution of the above relations into (4.27a) allows for a complete solution to the problem, including the variation of depth H, slope S, channel width B_c and mean fan elevation η as functions of r. The solutions for slope S and channel width B_c take the form

$$S = [R^{-1/2}\alpha_s^{-1}\alpha_b^{(3+2p)/2}\alpha_r\left(\frac{\alpha_b}{R}\right)^{-n}\left(\frac{Q_{st}}{Q_w}\right)]^{1/(1+p)}$$
$$\frac{B_c}{D} = \alpha_s^{-1}\left(\frac{\alpha_b}{R}\right)^{-n}\frac{Q_{st}}{\sqrt{RgD}\,D^2} \tag{4.32a,b}$$

where in the above relations Q_{st} is given by (4.27a), p = 1/6, n = 5/2 and

$$\alpha_b = R\tau_{bo}^* \tag{4.33}$$

The depth profile in the channel(s) can be determined from (4.32a) and (4.31). The elevation profile is obtained by integrating the following relation;

$$-\frac{d\eta}{dr} = S \tag{4.34}$$