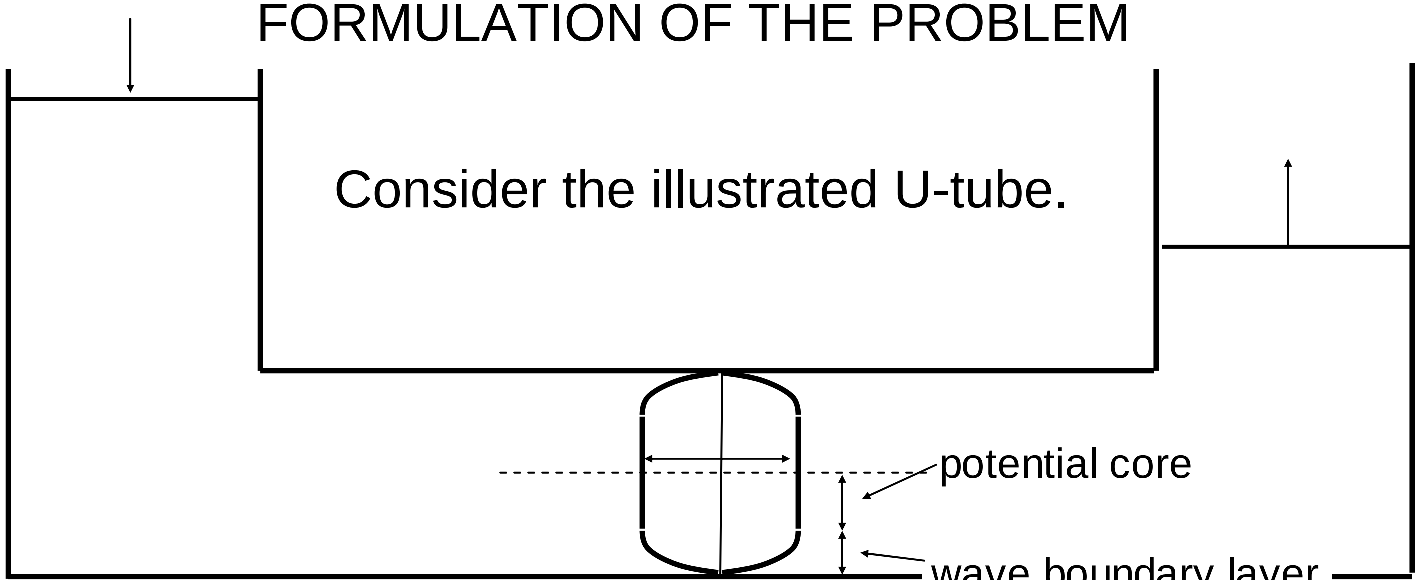
SEDIMENT TRANSPORT IN WAVE BOUNDARY LAYERS:

FORMULATION OF THE PROBLEM

Consider the illustrated U-tube.



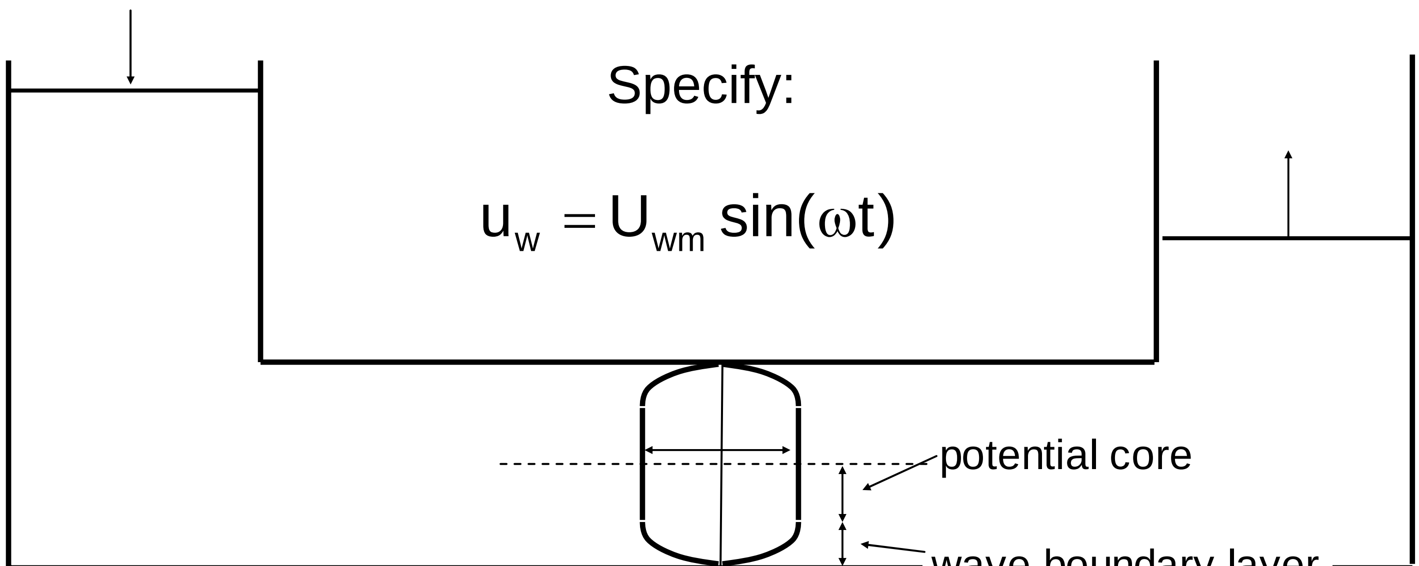
$$\frac{\partial \bar{u}}{\partial t} + \bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{w}\frac{\partial \bar{u}}{\partial z} = -\frac{1}{\rho}\frac{dp_w}{dx} + \frac{1}{\rho}\frac{\partial \tau}{\partial z} - g\frac{\partial \eta}{\partial x}$$

$$\frac{\partial \bar{c}}{\partial t} + \bar{u}\frac{\partial \bar{c}}{\partial x} + \left(\bar{w} - v_s\right)\frac{\partial \bar{c}}{\partial z} = -\frac{\partial F}{\partial z}$$

WAVE BOUNDARY LAYER IN U-TUBE

$$\frac{\partial \overline{u}}{\partial t} = -\frac{1}{\rho}\frac{dp_w}{dx} + \frac{1}{\rho}\frac{\partial \tau}{\partial z} \qquad \frac{\partial \overline{c}}{\partial t} - v_s \frac{\partial \overline{c}}{\partial z} = -\frac{\partial F}{\partial z}$$

$$(1-\lambda_p)\frac{\partial \eta}{\partial t} = -\frac{\partial q_b}{\partial x} + v_s\left(\overline{c}_b - E\right)$$



FLOW FIELD

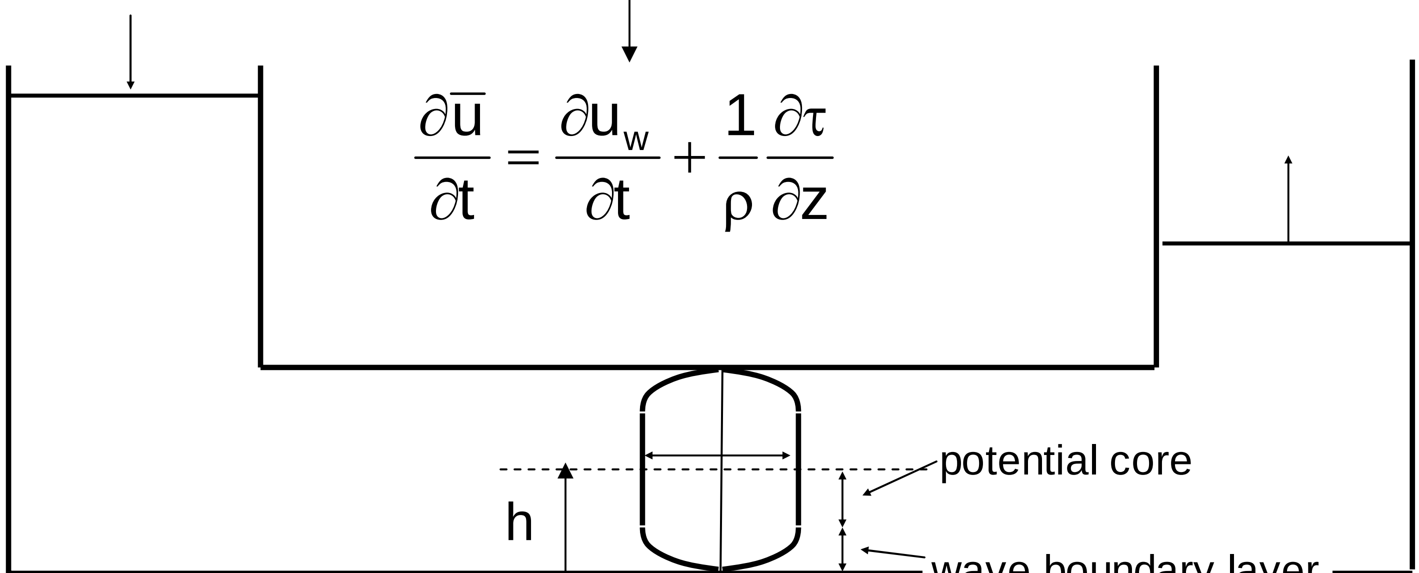
Potential core:
$\delta_w < z < h$

$$\frac{\partial u_w}{\partial t} = -\frac{1}{\rho}\frac{dp_w}{dx} \qquad \bar{c} = 0$$

Wave boundary layer:
$z < \delta_w$

$$\frac{\partial \bar{u}}{\partial t} = -\frac{1}{\rho}\frac{dp_w}{dt} + \frac{1}{\rho}\frac{\partial \tau}{\partial z} \qquad \frac{\partial \bar{c}}{\partial t} - v_s\frac{\partial \bar{c}}{\partial z} = -\frac{\partial F}{\partial z}$$

$$\frac{\partial \bar{u}}{\partial t} = \frac{\partial u_w}{\partial t} + \frac{1}{\rho}\frac{\partial \tau}{\partial z}$$



SETUP FOR EDDY VISCOSITY CLOSURE

$$\frac{\partial \bar{u}}{\partial t} = \frac{\partial u_w}{\partial t} + \frac{1}{\rho}\frac{\partial \tau}{\partial z} \qquad \frac{\partial \bar{c}}{\partial t} + v_s \frac{\partial \bar{c}}{\partial z} = -\frac{\partial F}{\partial z}$$

$$\tau = -\rho \overline{u'w'} = \nu_t \frac{\partial \bar{u}}{\partial z} \qquad F = \overline{c'w'} = -\nu_t \frac{\partial \bar{c}}{\partial z}$$

$$\frac{\partial \bar{u}}{\partial t} = \frac{\partial u_w}{\partial t} + \frac{\partial}{\partial z}\left(\nu_t \frac{\partial \bar{u}}{\partial z}\right) \qquad \frac{\partial \bar{c}}{\partial t} - v_s \frac{\partial \bar{c}}{\partial z} = \frac{\partial}{\partial z}\left(\nu_t \frac{\partial \bar{c}}{\partial z}\right)$$

$$\frac{\bar{u}\big|_{z=b}}{u_*} = \frac{1}{\kappa}\ell n\left(30\frac{b}{k_s}\right) \qquad -\nu_t \frac{\partial \bar{c}}{\partial z}\bigg|_{z=b} = v_s E$$

$$\nu_t \frac{\partial \bar{u}}{\partial z}\bigg|_{z=h} = 0 \qquad -\nu_t \frac{\partial \bar{c}}{\partial z}\bigg|_{z=h} = 0$$

Here h = half-width of U-tube, $u_* = \sqrt{\frac{\tau_b}{\rho}}$

SOLUTION FOR THE FLOW

$$\frac{\partial \bar{u}}{\partial t} = \frac{\partial u_w}{\partial t} + \kappa u_{*max} \frac{\partial}{\partial z}\left(z \frac{\partial \bar{u}}{\partial z} \right)$$

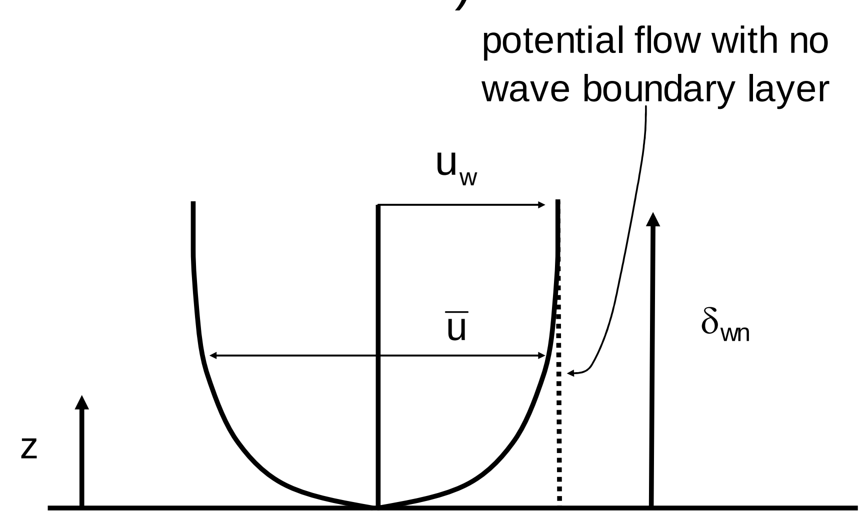
$$\tau_b = \rho u_*^2 = \rho \left(\nu_t \frac{\partial \bar{u}}{\partial z} \right)\Bigg|_{z=b} \rightarrow u_* = \kappa \left(z \frac{\partial \bar{u}}{\partial z} \right)\Bigg|_{z=b}$$

$$\frac{\bar{u}|_{z=b}}{u_*} = \frac{1}{\kappa} \ell n \left(30 \frac{b}{k_s} \right)$$

$$\frac{\partial \bar{u}}{\partial z}\Bigg|_{z=h} = 0$$

Recall that $u_w = U_{wm}\sin(\omega t)$. This problem can be solved e.g. iteratively using u_{*max} as the iteration parameter. (In the present case the equations are linear and a solution in closed form can be found.)

The nominal wave boundary layer thickness can be defined as the point $z = \delta_{wn}$ where at maximum excursion $\bar{u} = 0.99 u_p$. Here $\delta_{wn} \cong 2\kappa u_{*max}/\omega$



SOLUTION FOR SEDIMENT TRANSPORT FIELD

The flow solution gives $\tau_b = \rho u_*^2 = \tau_b(t)$.
The Shields number τ^* can thus be evaluated at all times, so q_b and E can be similarly evaluated at all times.
Evaluate the suspended sediment field by solving

$$\frac{\partial \overline{c}}{\partial t} - v_s \frac{\partial \overline{c}}{\partial z} = \kappa u_{*max} \frac{\partial}{\partial z}\left(z \frac{\partial \overline{c}}{\partial z} \right)$$

$$-\nu_t \left. \frac{\partial \overline{c}}{\partial z} \right|_{z=b} = v_s E$$

$$-\nu_t \left. \frac{\partial \overline{c}}{\partial z} \right|_{z=h} = 0$$

FORMULATION FOR WAVE-CURRENT BOUNDARY LAYERS

In general, for waves propagating parallel to the flow,

$$\frac{\partial \overline{u}}{\partial t} = -\frac{1}{\rho}\frac{dp_w}{dx} - g\frac{dH}{dx} + \frac{\partial}{\partial z}\left(\nu_t \frac{\partial \overline{u}}{\partial z}\right)$$

where u_{pw} is the potential velocity field associated with waves. Letting S = -dH/dx denote a water surface slope that drives the current,

$$\frac{\partial \overline{u}}{\partial t} = -\frac{1}{\rho}\frac{dp_w}{dx} + gS + \frac{\partial}{\partial z}\left(\nu_t \frac{\partial \overline{u}}{\partial z}\right)$$

Above the boundary layer, i.e. $z < \delta_w$,

$$\overline{u} = u_{pw} + \overline{u}_c$$

$$\frac{\partial u_{pw}}{\partial t} = -\frac{1}{\rho}\frac{dp_w}{dx} \qquad \nu_t = \kappa u_{*c} z \qquad gS + \frac{\partial}{\partial z}\left(\kappa u_{*c} \frac{\partial \overline{u}_c}{\partial z}\right)$$

where u_{*c} denotes a shear velocity characterizing the outer flow.

WAVE-CURRENT BOUNDARY LAYERS contd.

Within the boundary layer

$$\frac{\partial \bar{u}}{\partial t} = \frac{\partial u_{pw}}{\partial t} + gS + \frac{\partial}{\partial z}\left(\kappa u_{*cw\max} z \frac{\partial \bar{u}}{\partial z}\right)$$

where u_{*cwmax} denotes a shear velocity u_{*cw} at maximum excursion characterizing the effect of waves and currents on the inner flow. (Both u_{*cwmax} and u_{*c} of these must be solved for as part of the problem.)

$$u_d = u_{pw} - \bar{u}$$

$$-\frac{\partial u_d}{\partial t} = gS + \frac{\partial}{\partial z}\left(\kappa u_{*cw\max} z \frac{\partial u_d}{\partial z}\right)$$

$$\frac{u_d\big|_{z=b}}{u_{*cw}} = \frac{u_{pw}}{u_{*cw}} - \frac{1}{\kappa}\ell n\left(30\frac{b}{k_s}\right) \qquad u_d\big|_{z=\infty} = 0$$

THE ESSENTIAL RESULTS

•Treat both the inner and outer flow as within the constant stress layer, so that $\tau \cong \tau_b = \rho u_{*c}^2$.
•Wave-current boundary layer thickness $\delta_{wc} \cong 2\kappa u_{*cw}/\omega$
•Outer current profile:

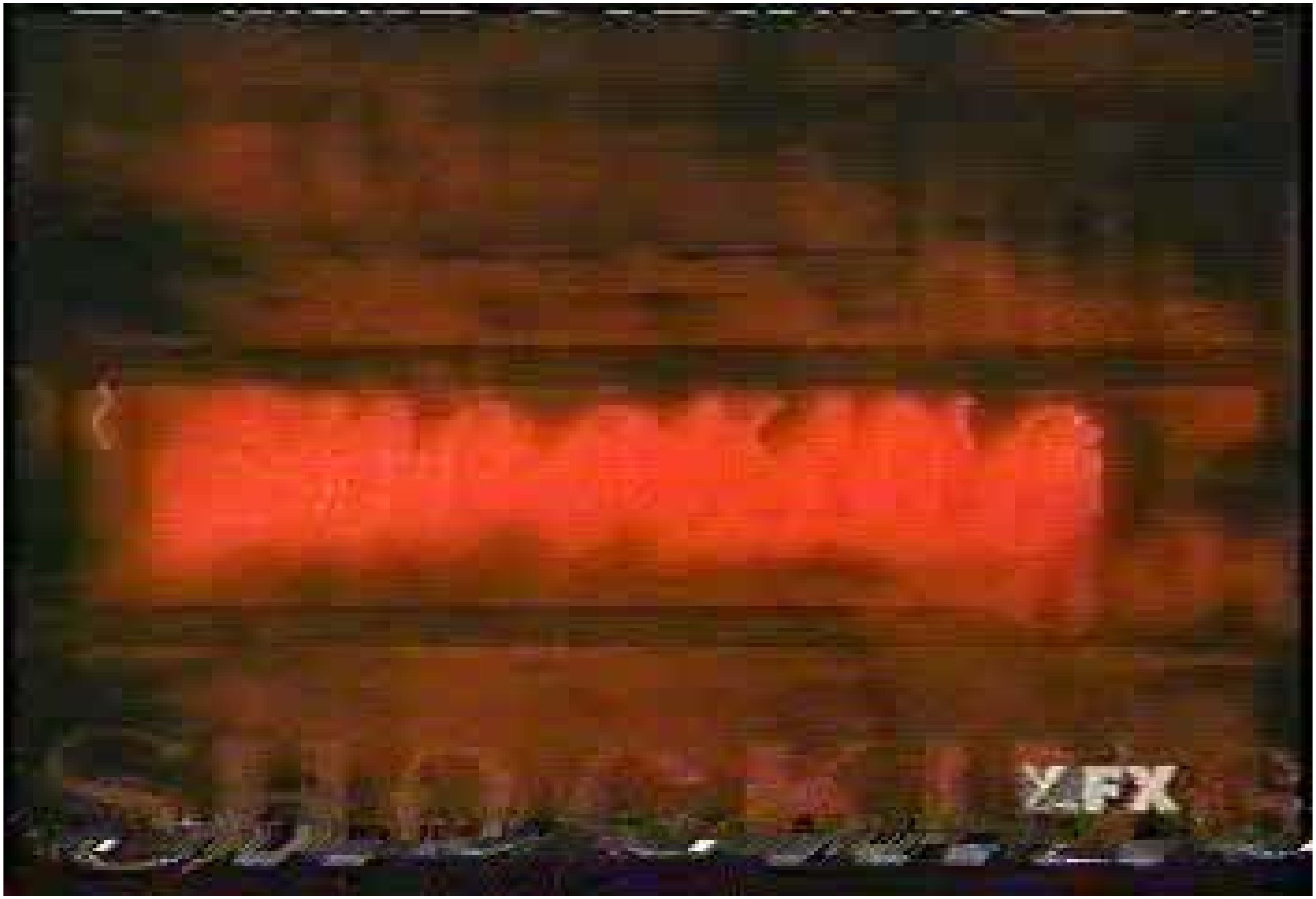
$$\frac{\overline{u}}{u_{*c}} = \frac{1}{\kappa}\ell n\left(30\frac{z}{k_c}\right), \quad z > \delta_{cw}$$

$$k_c > k_s$$

•The wave-current boundary layer yields a composite roughness k_c in the outer flow that exceeds the Nikuradse surface roughness k_s.
•This extra roughness of the outer flow in turn changes the net transport rate of sediment in the case of a current + waves as compared to that with a current alone.
•More interesting effects if waves propagate obliquely to the current.

I HOPE THAT THESE NOTES STIMULATE LIVELY BUT NOT TOO LIVELY DISCUSSION

video clip



AND I HOPE THAT THEY PROVOKE A POSITIVE RATHER THAN NEGATIVE REACTION

video clip