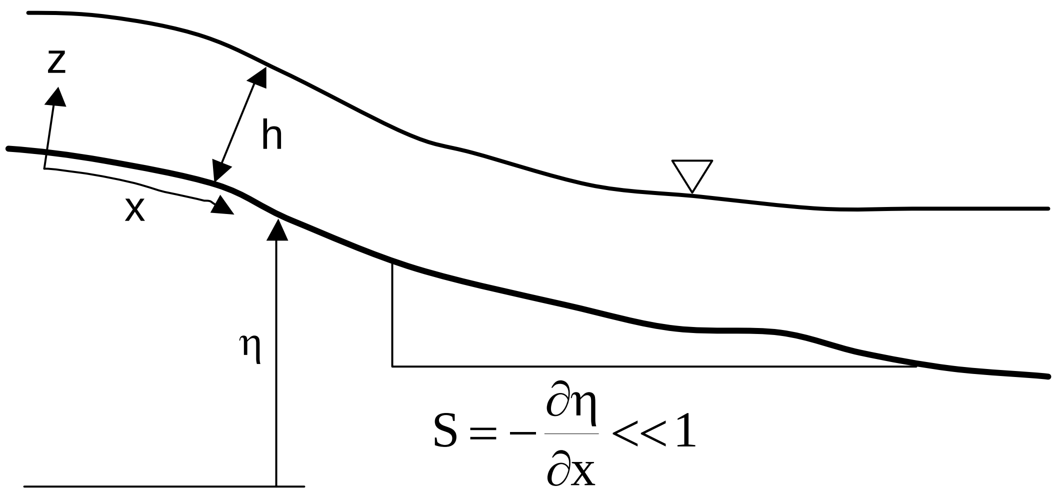
Short Course: Geomorphological Fluid Mechanics
Saint-Oyen, Italy, 2000

CHAPTER 1
INTRODUCTION TO MORPHODYNAMICS



River problems are formulated in *boundary layer coordinates*:

x is an arc-length coordinated imbedded in the stream bottom

z is an upward normal coordinate

η is bed elevation

$$S = -\frac{\partial \eta}{\partial x}$$ is bed slope << 1 for most river problems

1-D St. Venant shallow water equations

Flow momentum balance

$$\frac{\partial}{\partial t}(uh) + \frac{\partial}{\partial x}(u^2 h) = -gh\frac{\partial h}{\partial x} + ghS - \frac{\tau_b}{\rho}$$

where u is depth-averaged flow velocity, τ_b denotes boundary shear stress, g is the acceleration of gravity and ρ is water density

Flow mass balance

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(uh) = 0$$

Closure for boundary shear stress:

$$\tau_b = \rho C_f u^2$$

where C_f is a friction coefficient

Relations for friction coefficient

Keulegan relation

$$C_f^{-1/2} = \frac{u}{u_*} = 2.5\,\ell n(11\frac{h}{k_s})$$

Manning-Strickler relation

$$C_f^{-1/2} = 8.1\,(\frac{h}{k_s})^{1/6}$$

where k_s is a roughness height and

$$u_* = \sqrt{\frac{\tau_b}{\rho}}$$

Normal flow

Steady, uniform equilibrium flow

$$C_f u^2 = ghS \qquad uh = q_w$$

$$\tau_b = \rho C_f u^2 = \rho ghS$$

2-D generalization

y is transverse coordinate, v is depth-averaged transverse velocity

$$\frac{\partial}{\partial t}(uh) + \frac{\partial}{\partial x}(u^2 h) + \frac{\partial}{\partial y}(uvh) =$$

$$-gh\frac{\partial h}{\partial x} - gh\frac{\partial \eta}{\partial x} - C_f(u^2 + v^2)^{1/2}\frac{u}{h}$$

$$\frac{\partial}{\partial t}(vh) + \frac{\partial}{\partial x}(uvh) + \frac{\partial}{\partial y}(v^2 h) =$$

$$-gh\frac{\partial h}{\partial y} - gh\frac{\partial \eta}{\partial y} - C_f(u^2 + v^2)^{1/2}\frac{v}{h}$$

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(uh) + \frac{\partial}{\partial y}(vh) = 0$$

$$\tau_{bx} = \rho C_f \sqrt{u^2 + v^2}\, u$$

$$\tau_{by} = \rho C_f \sqrt{u^2 + v^2}\, v$$

Bedload and suspended load

Modes of sediment transport:
wash load or **bed material load**

Modes of bed material transport
bedload or **suspended load**

bedload:
sliding, rolling or saltating just above bed
role of turbulence is indirect

suspended load:
feels direct dispersive effect of eddies
may be wafted high into the water column

driving parameter:
portion of boundary shear stress associated with
skin friction

$$\tau_b = \tau_{bs} + \tau_{bf}$$

shear stess = skin friction + form drag (bedforms)

bedload: *transport rate* = fn(τ_{bs})
suspended load: *entrainment rate* = fn(τ_{bs})

Sample bedload relation

Meyer Peter and Muller (1948)

$$q_b^* = 8(\tau^* - \tau_c^*)^{1.5}$$

Einstein number:

$$q_b^* = \frac{q_b}{\sqrt{RgD}\,D}$$

Shields number:

$$\tau^* = \frac{\tau_b}{\rho RgD}$$

q_b =volume transport rate of bedload/width/time
(m^2/s: corrsponding mass transport rate = $\rho_s q_b$)
D =grain size
$R = (\rho_s/\rho - 1)$ ρ_s = sediment density
τ_c^* = critical Shields number (M.P.M.: 0.047)
$\tau_b \rightarrow \tau_{bs}$ in presence of bedforms

Sample entrainment relation for suspension

Smith and McLean (1977)

$$E = 0.65 \frac{\gamma_o \left(\frac{\tau^*}{\tau_c^*} - 1 \right)}{1 + \gamma_o \left(\frac{\tau^*}{\tau_c^*} - 1 \right)} \qquad \gamma_o = 0.0024$$

where

$$E = \frac{E_s}{v_s}$$

and

E_s = volume entrainment rate/bed area/time
(m/s: corresponding mass rate = $\rho_a E_s$)
v_s = fall velocity in quiescent water

Again $\tau_b \rightarrow \tau_{bs}$ ($\tau^* \rightarrow \tau_s^*$) with bedforms

Equation of continuity of bed sediment

(Exner equation)

$$(1-\lambda_p)\frac{\partial \eta}{\partial t} = -\frac{\partial q_b}{\partial x} + D_s - E_s$$

λ_p = bed porosity
t = time

$$E_s = v_s E \qquad D_s = v_s c_{lb}$$

(volume entrainment and deposition rates)

$$(1-\lambda_p)\frac{\partial \eta}{\partial t} = -\frac{\partial q_b}{\partial x} + v_s(c_{lb} - E)$$

c_{lb} = near-bed volume suspended sediment concentration

If length scale of interest >> u h/v_s, global:

$$(1-\lambda_p)\frac{\partial \eta}{\partial t} = -\frac{\partial q_t}{\partial x} \qquad q_t = q_b + q_s$$

2-D generalization:

$$(1-\lambda_p)\frac{\partial \eta}{\partial t} = -\frac{\partial q_{bx}}{\partial x} - \frac{\partial q_{by}}{\partial y} + v_s (c_{lb} - E)$$

Generalization for bedload transport
(in absence of direct gravity effects)

scalar form:

$$q_b = f(\tau_b)$$

vector form:

$$q_{bx} = f(|\tau_b|)\frac{\tau_{bx}}{|\tau_b|} \qquad q_{by} = f(|\tau_b|)\frac{\tau_{by}}{|\tau_b|}$$

$$|\tau_b|^2 = \tau_{bx}^2 + \tau_{by}^2$$

$$\tau_{bx} = \rho C_f (u^2 + v^2)^{1/2} u$$

$$\tau_{by} = \rho C_f (u^2 + v^2)^{1/2} v$$

Generalization for entrainment into suspension:
E = function of $|\tau_b|$
($\tau_b \rightarrow \tau_{bs}$ in presence of bedforms)

Equation of continuity of suspended sediment

Let $c_l(x, y, z, t)$ denote the local volume concentration of suspended sediment.
Here we assume a *dilute* suspension, i.e. $c_l << 1$

Balance equation for suspended load:

$$\frac{\partial c_l}{\partial t} + \frac{\partial}{\partial x}(u_l\, c_l) + \frac{\partial}{\partial y}(v_l\, c_l) + \frac{\partial}{\partial z}(w_l\, c_l)$$

$$- v_s \frac{\partial c_l}{\partial z} = - \frac{\partial}{\partial x}\overline{u'c'} - \frac{\partial}{\partial y}\overline{v'c'} - \frac{\partial}{\partial z}\overline{w'c'}$$

where (u_l, v_l, w_l) denote local flow velocities in the (x, y, z) directions averaged over turbulence,

$$\overline{w'c'}$$

etc. denote Reynolds fluxes of suspended sediment and v_s denotes the fall velocity of the suspended sediment

Upward normal flux of suspended sediment

$$F_{xz} = (\overline{w'c'} - v_s c_l)$$

Vertical flux at bed:

$$F_{xz}\Big|_{bed} = (\overline{w'c'} - v_s c_l)\Big|_{bed} = E_s - D_s =$$
$$v_s E - v_s c_{lb}$$

Thus

$$E_s = v_s E = \overline{w'c'}\Big|_{bed}$$

Close with eddy viscosity:

$$\overline{w'c'} = -\nu_e \frac{\partial c_l}{\partial z}$$

Thus bottom boundary condition is of flux form:

$$-\nu_e \frac{\partial c_l}{\partial z}\Big|_{bed} = v_s E$$

where E is a function of boundary shear stress

Equilibrium suspensions

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0 \qquad c_l = c_l(z)$$

$$\frac{d}{dz} F_{xz} = 0 \qquad F_{xz} = -v_s c_l + \overline{w'c'}$$

vanishing flux at water surface $\rightarrow$

$$F_{xz} = 0$$

introduce eddy viscosity

$$\nu_e \frac{dc_l}{dz} + v_s c_l = 0 \qquad \text{(a)}$$

applying flux boundary condition

$$-\nu_e \left.\frac{\partial c_l}{\partial z}\right|_{bed} = v_s E = v_c c_{lb} \therefore c_{lb} = E \qquad \text{(b)}$$

Rousean formulation for dilute suspensions
(not perfect)

Reynolds shear stress distribution for equilibrium flow

$$\tau_{Rxz} = \tau_b (1 - \frac{z}{h}) = \rho u_*^2 (1 - \frac{z}{h})$$

rough logarithmic law

$$\frac{u_l}{u_*} = \frac{1}{\kappa} \ell n(30 \frac{z}{k_s})$$

eddy diffusivity

$$\tau_{Rxz} = -\rho \overline{u'w'} = \rho \nu_e \frac{\partial u_l}{\partial z}$$

solve for eddy diffusivity

$$\nu_e = \kappa u_* z (1 - \frac{z}{h}) \qquad \text{(c)}$$

(note ν_e vanishes at z = 0)

Integrating (a) subject to (b) and (c),

$$c_l = c_{lb}\left[\frac{(1-\varsigma)/\varsigma}{(1-\varsigma_b)/\varsigma_b}\right]^{Z_r} = E\left[\frac{(1-\varsigma)/\varsigma}{(1-\varsigma_b)/\varsigma_b}\right]^{Z_r}$$

where

$\zeta = z/H$, $\zeta_b = z_b/H$

and the Rouse number Z_r is given by

$$Z_r = \frac{v_s}{\kappa u_*}$$

and $\kappa = 0.4$ is the Karman constant.

Criterion for onset of suspension:

$$\frac{u_*}{v_s} = 1$$

Relation for volume suspended sediment transport/width/time:

$$q_s = \int_0^H u_l c_l dz$$

Depth-integrated conservation of suspended sediment

Integrate

$$\frac{\partial c_l}{\partial t} + \frac{\partial}{\partial x}(u_l\, c_l) + \frac{\partial}{\partial y}(v_l\, c_l) + \frac{\partial}{\partial z}(w_l\, c_l)$$

$$- v_s \frac{\partial c_l}{\partial z} = - \frac{\partial}{\partial x}\overline{u'c'} - \frac{\partial}{\partial y}\overline{v'c'} - \frac{\partial}{\partial z}\overline{w'c'}$$

from z = 0 to z = h after applying boundary-layer approximations; obtain (1-D)

$$\frac{\partial}{\partial t}(ch) + \frac{\partial}{\partial x}(uch) = v_s (E - r_o c)$$

where c is depth-averaged suspended sediment concentration and

$$r_o = \frac{c_{lb}}{c}$$

1-D depth-integrated quasi-steady formulation for morphodynamics

Conservation of flow momentum

$$\frac{\partial}{\partial t}(uh) + \frac{\partial}{\partial x}(u^2 h) = -gh\frac{\partial h}{\partial x} + ghS - \frac{\tau_b}{\rho}$$

Conservation of flow mass

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(uh) = 0 \quad \therefore \quad uh = q_w$$

Conservation of suspended sediment

$$\frac{\partial}{\partial t}(ch) + \frac{\partial}{\partial x}(uch) = v_s(E - r_o c)$$

Conservation of bed sediment

$$(1-\lambda_p)\frac{\partial \eta}{\partial t} = -\frac{\partial q_b}{\partial x} + v_s(c_{lb} - E)$$

or thus from conservation of suspended sediment

$$(1-\lambda_p)\frac{\partial \eta}{\partial t} = -\frac{\partial q_b}{\partial x} - \frac{\partial q_b}{\partial x} = -\frac{\partial q_t}{\partial x}$$

where $q_s = uch$, $q_t = q_b + q_s$

Stream types

Sand-bed and gravel-bed alluvial streams

Sand-bed:
Surface median size D_{50} ~ 0.1 to 0.8 mm
little sediment stratification
dunes often prominent at flood flow
bed slope S is $1x10^{-5}$ ~ $2x10^{-3}$

Gravel-bed
surface D_{50} 10 ~ 200 mm
surface D_{50} ~ 1.5 to 3 x substrate D_{50}
dunes not common
bed slope is $5x10^{-4}$ to $3x10^{-2}$

Regime diagram: τ^* versus
explicit particle Reynolds number

$$\mathbf{Re}_p = \frac{\sqrt{RgD}\,D}{\nu}$$

Relations for the threshold of motion and fall velocity

Criterion for the threshold of motion
(based on Shields, 1936, Brownlie, 1981)

$$\tau_c^* = 0.5\,[0.22\ \mathbf{Re}_p^{-0.6} + 0.06 \cdot 10^{(-7.7\,\mathbf{Re}_p^{-0.6})}]$$

Criterion for the onset of suspension

$$\frac{u_*}{v_s} = \frac{u_*}{\sqrt{RgD}}\frac{1}{\mathbf{R_f}} = \frac{\sqrt{\tau^*}}{\mathbf{R_f}} = 1$$

$$\mathbf{R}_f = \frac{v_s}{\sqrt{RgD}}$$

Relation for fall velocity
(Dietrich, 1982)

$$\mathbf{R}_f = \exp\ \{-b_1 + b_2\ \ell n\ (\mathbf{Re}_p) - b_3\,[\ell n\,(\mathbf{Re}_p)]^{\,2} - b_4\,[\ell n\ (\mathbf{Re}_p)]^{\,3} + b_5\,[\ell n\ (\mathbf{Re}_p)]^{\,4}\}$$

b1 = 2.891394, b2 = 0.95296, b3 = 0.056835, b4 = 0.002892 and b5 = 0.000245

Criterion for viscous effects: bankfull flow

Criterion for viscous effects (ripples)

$$\frac{D}{\delta_v} \leq 1 \qquad \delta_v = 11.6\frac{\nu}{u_*}$$

or thus

$$\frac{1}{11.6}\frac{u_* D}{\nu} = \frac{1}{11.6}\sqrt{\tau^*}\,\mathbf{R_{ep}} = 1$$

*Data for rivers at **bankfull** conditions*

use normal flow approximation

$$\tau_b = \rho C_f u^2 = \rho g h S$$

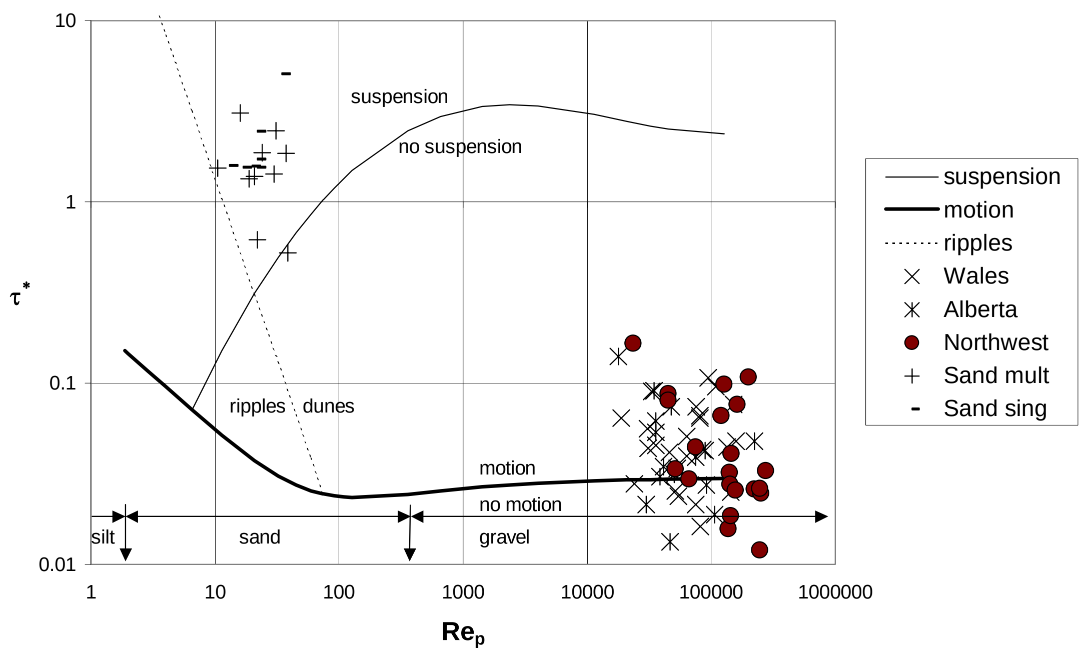
$$\therefore$$

$$\tau^* = \frac{hS}{RD}$$

where h refers to bankfull conditions

Regime diagram for stream type

Shields Regime Diagram



Relations for hydraulic resistance

Gravel-bed rivers in flood
(when sediment is moved)

Bedforms often not important:
$\tau_b = \tau_{bs}$

Use Keulegan relation:

$$C_f^{-1/2} = \frac{u}{u_*} = \frac{1}{\kappa}\ell n(11\frac{h}{k_s})$$

with $k_s = n\ D_{s90}$, n = 2 ~ 3
D_{s90} = surface D_{90} size

Sand-bed rivers

Bedforms often extremely important

Froude number **Fr** given by

$$\mathbf{Fr} = \frac{u}{\sqrt{gh}}$$

Relations for hydraulic resistance continued

Lower regime: dunes, prominent form drag

Fr $\leq$ about 0.6

Upper regime: plane bed – antidunes, little form drag

Fr $\geq$ about 0.6

Einstein (1950) decomposition for normal flow:
Split up τ_b C_f and h as

$\tau_b = \tau_{bs} + \tau_{bf}$, $C_f = C_{fs} + C_{ff}$, $h = h_s + h_f$

(skin friction + form drag)
at same u and S

$$\tau_{bs} = \rho C_{fs} u^2 = \rho g h_s S \qquad \tau_{bf} = \rho C_{ff} u^2 = \rho g h_f S$$

Engelund-Hansen (1967): skin friction & form drag:

$$C_{fs}^{-1/2} = 2.5 \ell n(11 \frac{h_s}{k_s}) \qquad k_s = 2.5 D$$

$$\tau_s^* = 0.06 + 0.4(\tau^*)^2$$

Relations for hydraulic resistance continued

Solution is iterative
Let S and D be given
Guess h_s, get τ_s^*, then τ^*, then h,
Get C_{fs}, then u, then

$$q_w = uh$$

then

$$Q_w = q_w B$$

where B denotes stream width

Method easily generalizes for
gradually varied flow:
for given h iterate to find C_f, use this value in

$$u\frac{du}{dx} = -g\frac{dh}{dx} + gS - C_f\,\frac{u^2}{h}$$

and integrate upstream
(subcritical flow: **Fr** < 1)

Useful relations for total load

Engelund-Hansen (1967)

$$C_f q_t^* = 0.05(\tau^*)^{5/2} \qquad q_t^* = \frac{q_t}{\sqrt{RgD}\,D}$$

Use in conjunction with Engelund-Hansen resistance relation

Brownlie (1981)

Let $Q_{st}=q_tB$ where B = channel width and

$$X = 1x10^6 \frac{\rho_s Q_{st}}{\rho Q_w + \rho_s Q_{st}}$$

denote mass concentration in ppm
Then

$$X = 7115 c_a \ (F_g - F_{go})^{1.978} S^{0.6601} (\frac{h}{D_{50}})^{-0.3301}$$

where c_a = 1 for laboratory flumes and 1.26 for natural streams

Useful relations for total load continued

where

$$F_g = \frac{u}{\sqrt{R g D_{50}}}$$

$$F_{go} = 4.596 (\tau_c^*)^{0.5293} S^{-0.1045} \sigma_g^{-0.1606}$$

$$\tau_c^* = 0.22 \, \mathbf{Re}_p^{-0.6} + 0.06 \cdot 10^{(-7.7 \, \mathbf{Re}_p^{-0.6})}$$

In the above σ_g = geometric standard deviation of bed sediment

Use in conjunction with the Brownlie resistance relation:

$$\frac{hS}{D_{50}} = 0.3724 (\tilde{q}_w S)^{0.6539} S^{0.09188} \sigma_g^{0.1050}$$

where

$$\tilde{q}_w = \frac{q_w}{\sqrt{g D_{50}} \, D_{50}}$$

and q_w = water discharge per unit width