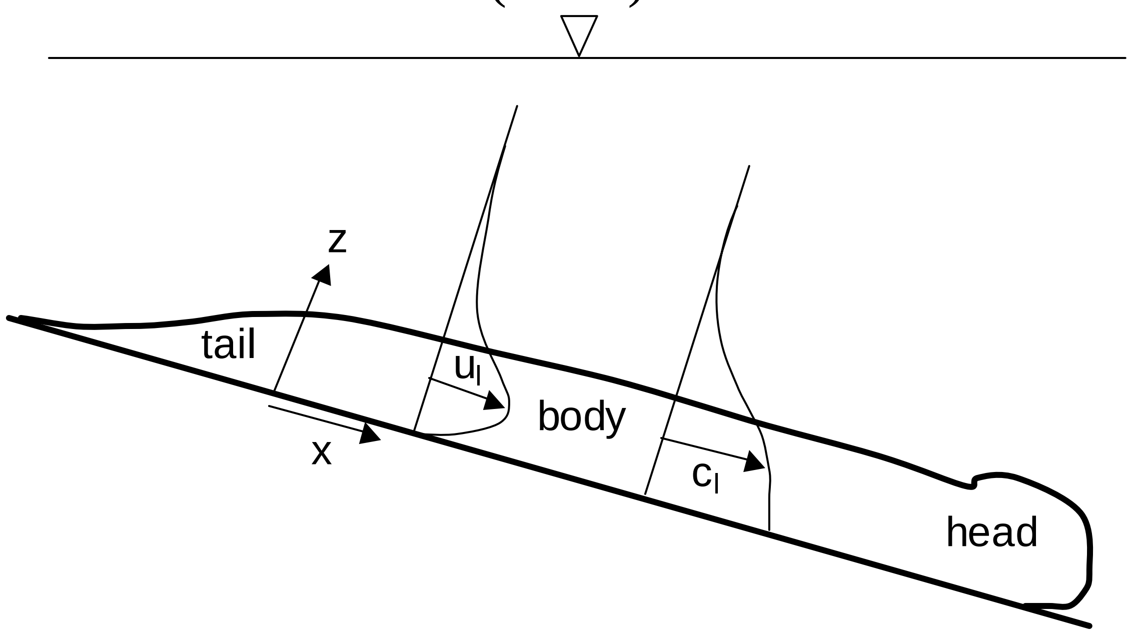
Short Course: Geomorphological Fluid Mechanics
Saint-Oyen, Italy, 2000

CHAPTER 3
TURBIDITY CURRENTS AND RESULTING MORPHOLOGY

Continental shelves and slopes
Submarine canyons
Channelized submarine fans
(slides)



Here we are concerned with quasi-continuous turbidity currents (sustained events with long bodies)

$u_l(x,z,t)$ = local vel. averaged over turbulence
$c_l(x,z,t)$ = local vol. sed. conc. averaged over turb.

Overview

Turbidity currents often contain
sand (size D) and *mud*

The presence of the sediment renders the dirty water slightly heavier than clear water:
Downslope gravitational driving force/ volume =

$$gS[\rho_s c_l + \rho(1 - c_l)] - \rho g S = \rho R g c_l S$$

where

g = gravitational accel. S = bed slope

ρ_s = sediment density ρ = water density

$$R = \frac{\rho_s}{\rho} - 1 \cong 1.65$$

For a dilute suspension $\boldsymbol{R}c_l << 1$
Thus downslope pull of gravity is vastly reduced compared to rivers

Turbidity current needs *turbulence* to hold sand in suspension

A turbidity current can hold fine mud in suspension for long distances even in the absence of turbulence due to the small fall velocity

Layer-averaged formulation for turbidity currents (analogous to St. Venant)

$$uh = \int_0^\infty u_l dz \qquad u^2 h = \int_0^\infty u_l^2 dz$$

$$uch = \int_0^\infty u_l c_l dz$$

Flow continuity:

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(uh) = e_w u$$

where

$$e_w = e_w(\mathbf{Ri_b}) \qquad \mathbf{Ri}_b = \frac{Rgch}{u^2}$$

($e_w \rightarrow 0$ as $\mathbf{Ri}_b \rightarrow \infty$)

A densimetric Froude number $\mathbf{Fr}_d$ can be defined such that

$$\mathbf{Fr}_d = (\mathbf{Ri}_b)^{-1/2} = \frac{u}{\sqrt{Rgch}}$$

Plays role analogous to **Fr** in open-channel flow

Layer-averaged formulation continued

Flow momentum balance:

$$\frac{\partial}{\partial t}(uh) + \frac{\partial}{\partial x}(u^2 h) =$$

$$-\frac{1}{2} R g \frac{\partial}{\partial x}(ch^2) + R g c h S - C_f u^2$$

Note how driving force ~ c,
resistance ~ u^2

Suspended sediment continuity:

$$\frac{\partial}{\partial t}(ch) + \frac{\partial}{\partial x}(uch) = v_s (E - r_o c)$$

where v_s = fall velocity and

$$r_o = \frac{c_{lb}}{c}$$

Entrainment E ~ τ_b^n ~ u^m, m = 2n,
deposition ~ c

Layer-averaged formulation continued

Exner equation of sediment continuity:

$$(1-\lambda_p)\frac{\partial \eta}{\partial t} = v_s(r_o c - E)$$

where
η = bed elevation, λ_p = bed porosity

Notes on closure

Friction coefficient C_f is often treated as a specified constant in the range 0.001 (large flows) to 0.01 (laboratory flows)

The entrainment function E is often given as a function of Shields stress τ^* or u_*/v_s, where

$$\tau^* = \frac{\tau_b}{\rho R g D} \qquad u_* = \sqrt{\frac{\tau_b}{\rho}} \qquad \tau_b = \rho C_f u^2$$

where τ_b denotes boundary shear stress

In general E is a function of u^m where $m > 2$

Ignition model for self-accelerating sandy turbidity currents

Only one size, sand

How can a turbidity current reach high enough velocities to excavate submarine canyons?

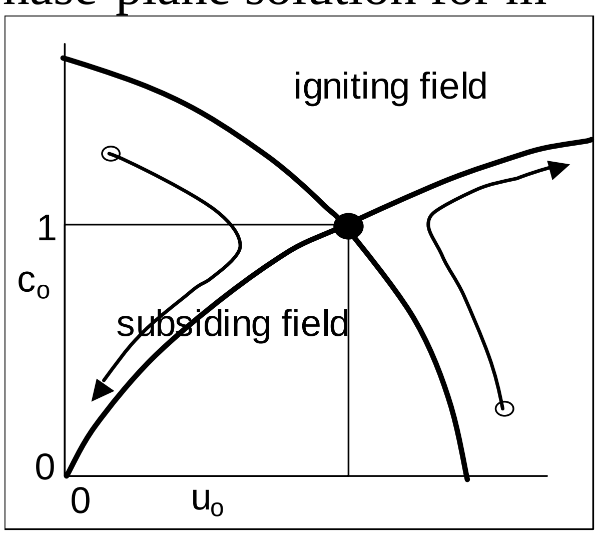
Heuristic model

$$\frac{\partial c}{\partial t} = u^m - c$$

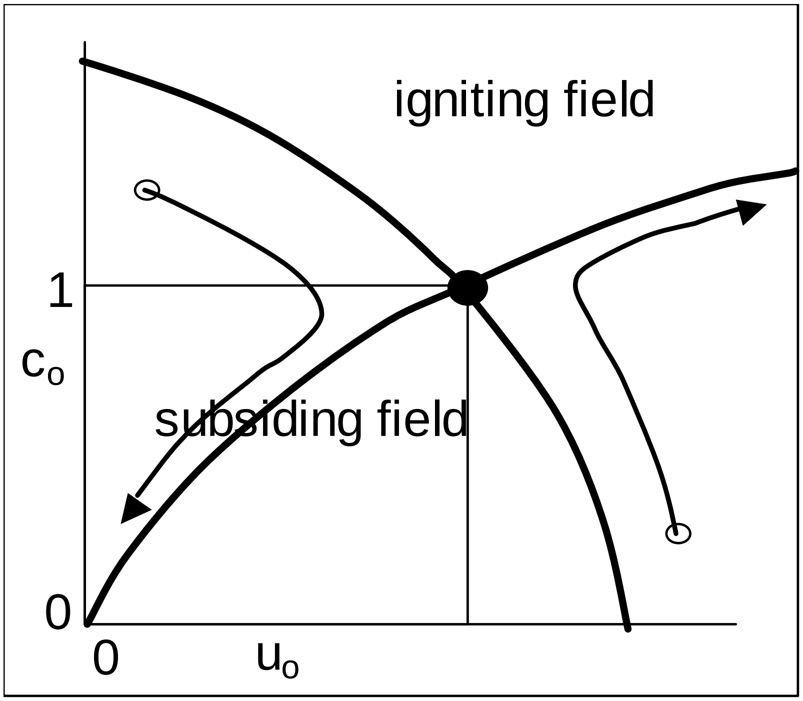
$$\frac{\partial u}{\partial t} = c - u^2$$

with $c(t = 0) = c_o$, $u(t = 0) = u_o$

Phase-plane solution for m > 2



Ignition model continued



Fixed point (equivalent of normal flow):

(u, c) = (1, 1)

Fixed point is *stable* for m < 2:
$(u_o, c_o) \rightarrow (1, 1)$

Fixed point is *unstable* for m > 2:
$(u_o, c_o) \rightarrow (0, 0)$ or (∞, ∞)

Defines self-accelerating and
self-decelerating fields

Ignition model, continued

Full model: *steady, spatially developing* turbidity currents

$$\frac{dh}{dx} = \frac{-\mathbf{Ri}_b S + e_w\left(2 - \frac{1}{2}\mathbf{Ri}_b\right) + C_f + \frac{1}{2}\frac{v_s}{u} r_o \mathbf{Ri}_b\left(\frac{q_{se}}{q_s} - 1\right)}{1 - \mathbf{Ri}_b}$$

$$\frac{h}{u}\frac{du}{dx} = \frac{\mathbf{Ri}_b S - e_w\left(1 + \frac{1}{2}\mathbf{Ri}_b\right) - C_f - \frac{1}{2}\frac{v_s}{u} r_o \mathbf{Ri}_b\left(\frac{q_{se}}{q_s} - 1\right)}{1 - \mathbf{Ri}_b}$$

$$\frac{h}{q_s}\frac{dq_s}{dx} = \frac{v_s}{u} r_o \mathbf{Ri}_b\left(\frac{q_{se}}{q_s} - 1\right)$$

where

$$q_s = chu \qquad q_{se} = \frac{E}{r_o} hu$$

Ignition model, continued

The limiting case of a conservative dense bottom
flow is obtained as $v_s \rightarrow 0$
(Ellison and Turner, 1959)

Self-accelerating currents require
antecedent “fuel”, i.e.
bed sediment available for entrainment

High slopes favor igniting currents:
origin of submarine canyons

Low slopes favor subsiding currents:
origin of submarine fans

Need to include lateral dimension to explain
self-channelization in a depositional environment

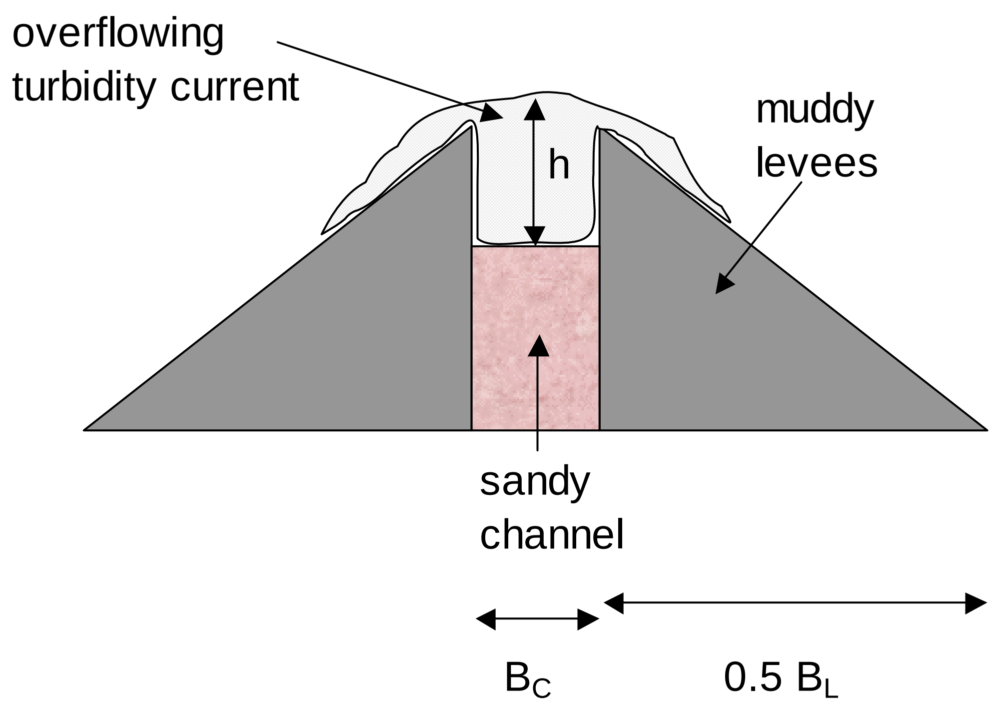
(Example of phase diagram from Scripps Submarine
Canyon, Fukushima et al., 1985)

Long profiles of channels on submarine fans

Many magnificent channelized submarine fans can be found on the bottom of the ocean, including: Amazon, Mississippi, Indus, Ganges, Submarine Fans

In many cases the channels are 100's to 1000's of km long.

The channels typically consist of a ribbon of sand bounded by muddy levees.



(view slides)

Mud can help drive sand out 100's of km offshore

The current is driven by the excess weight of both the mud and the sand. The mud, however, needs minimal turbulence to be maintained in suspension

The mud thus acts like a passive contaminant in the channel. It spills out to form the levees, but does not deposit on the bed of the channel. It keeps the current going, so that the sand only drops out slowly.

Analogy to sand-bed rivers

Sand-bed rivers typically have negligible mud (fine silt or clay) on their beds.

Yet typically 90% to 95% of the sediment carried by a sand-bed river is mud.

The mud deposits on the levees and floodplain.

Difference between mud/sand turbidity currents and sand-bed rivers:

Mud helps drive the turbidity current, whereas mud does not help drive the river flow

Assumptions

- The flow is subcritical in the densimetric Froude sense, so that water entrainment from above can be neglected.
- The turbidity currents in question are long and sustained, and can be treated as continuous quasi-steady events.
- Bedload can be neglected.
- Sand has size D; total sand load can be computed with the Engelund-Hansen formula.
- The friction coefficient C_f of the flow is constant.
- The flow can be approximated with the normal flow assumption.
- Sand deposits in the channel, mud deposits on the levees due to a barely overflowing current.

Key assumption

Channels avulse, form, elongate, aggrade and avulse again

There is assumed to be an intermediate state during which the morphologically averaged aggradation rate $\bar{v}_a$ is roughly constant in time and over long distances in space

Assumptions continued

Hypothesis of constant morphologically averaged aggradation.

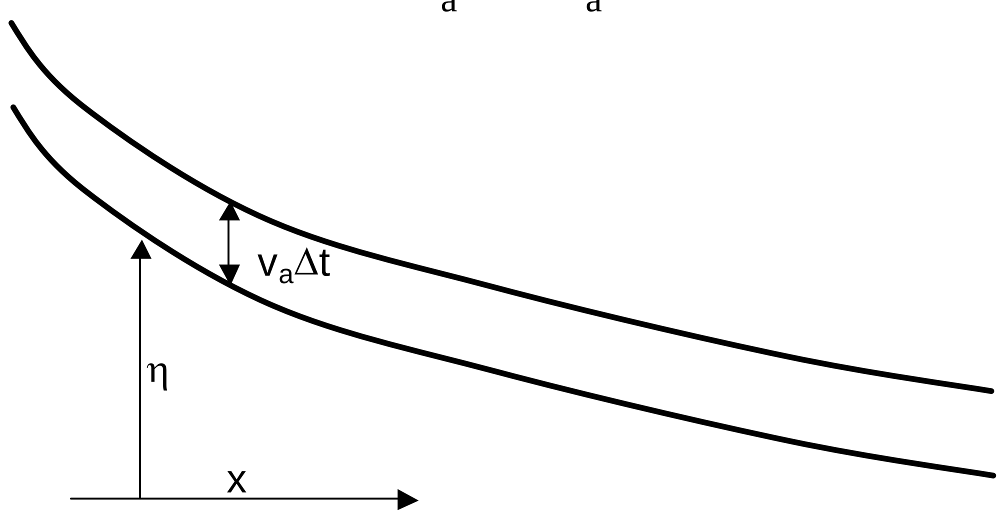
The aggradation rate during a flow event is v_a;
I denotes the intermittency of flow events:

$$I = \frac{T_c}{T_i + T_c}$$

where T_c denotes the time duration of a current event and T_i denotes the time in between events.

Averaged over this intermittency, the channel aggradation rate $\overline{v}_a$ is assumed to be constant and given by the relation

$$\overline{v}_a = I v_a$$



Governing equations

Flow momentum balance

$$\frac{\partial}{\partial t}(uh) + \frac{\partial}{\partial x}(u^2 h) =$$

$$-\frac{1}{2} Rg \frac{\partial}{\partial x}(ch^2) + RgchS - C_f u^2$$

approximates to

$$Rg(c_s + c_m)hS = C_f u^2$$

where c_s and c_m denote the concentrations of sand and mud, respectively

Flow mass balance

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(uh) = e_w u$$

approximates to

$$Q_f = uhB_C$$

where Q_f is constant flow discharge

Governing equations continued

Conservation of suspended sand

$$\frac{\partial}{\partial t}(c_s h) + \frac{\partial}{\partial x}(u c_s h) = v_s (E - r_o c_s)$$

Exner equation of continuity of bed sand

$$(1 - \lambda_{ps})\frac{\partial \eta}{\partial t} = v_s (r_o c - E)$$

where λ_{ps} is the porosity of sand

But

$$\frac{\partial \eta}{\partial t} = v_a$$

And thus where $Q_s = u c_s h B_C$ is the volume sand load,

$$\frac{dQ_s}{dx} = -(1 - \lambda_{ps}) v_a B_C$$

Governing equations continued

Conservation of suspended mud

The muddy levees must grow at the same rate as the sand bed.

Thus where $Q_m = uc_m hB_C$ denotes the volume mud load,

$$\frac{dQ_m}{dx} = -(1-\lambda_{pm})v_a \chi B_C$$

where λ_{pm} denotes the porosity of the mud deposit and

$$\chi = \frac{B_L}{B_C}$$

is the ratio of levee width to channel width, assumed constant here.

Assumptions concerning sand and mud transport, and channel width

Since the mud does not settle out in the channel, the concentration of mud down the channel is taken to be constant

$$c_m = c_{mo}$$

The rate of transport sand is assumed to obey the Engelund-Hansen total bed material load equation

$$C_f q_s^* = 0.05(\tau^*)^{5/2}$$

where

$$q_s^* = \frac{Q_s}{B_C \sqrt{RgD}\, D} \qquad \tau^* = \frac{C_f u^2}{RgD}$$

An empirical relation is assumed for channel width in the lack of better information:

$$B_C = \alpha Q_f^m$$

where α is a constant and $m \approx 0.5$ for rivers

Statement of the problem

8 equations for 8 parameters
u, S, h, B_C, c_s, c_m, Q_s, Q_m

$$Rg(c_s + c_m)hS = C_f u^2$$
$$Q_f = uhB_C$$
$$Q_m = uhc_m B_C$$
$$Q_s = uhc_s B_C$$
$$\frac{dQ_s}{dx} = -(1-\lambda_{ps})v_a B_C$$
$$\frac{dQ_m}{dx} = -(1-\lambda_{pm})v_a \chi B_C$$
$$C_f \frac{Q_s}{B_C\sqrt{RgD}\,D} = 0.05\left(\frac{C_f u^2}{RgD}\right)^{5/2}$$
$$B_C = \alpha Q_f^m$$
$$c_m = c_{mo}$$

The constant flow discharge Q_f must be specified, along with the channel width B_{Co} at x = 0 and the input rates of sand and mud:

$$Q_s\big|_{x=0} = Q_{so} \qquad Q_m\big|_{x=0} = Q_{mo}$$

Solution

Dimensionless parameters governing the solution

$$\varphi = \frac{Q_{so}}{Q_{so} + Q_{mo}}$$

$$\Pi\alpha = \frac{v_a B_{Co} L(1-\lambda_{ps})}{Q_{so}}$$

$$\mathbf{Ri}_o = \frac{Rg(c_{so} + c_{mo})h_o}{u_o^2}$$

where L is the length of the channel and the subscript "o" denotes values at x = 0

φ = fraction of incoming sediment that is sand

$\Pi\alpha$ = order-one Paola basin filling number

$\mathbf{Ri}_o$ = bulk Richardson number at x = 0 (> 1 for subcritical flow)

Dimensionless formulation

$$\hat{x} = \frac{x}{L} \quad \hat{B} = \frac{B_C}{B_{Co}} \quad \hat{Q}_f = \frac{Q_f}{Q_{fo}} \quad \hat{S} = \frac{S}{S_o}$$

$$\hat{Q}_s = \frac{Q_s}{Q_{so}} \qquad \hat{Q}_m = \frac{Q_m}{Q_{mo}} \qquad \hat{u} = \frac{u}{u_o}$$

Dimensionless governing relations

Conserve momentum: at $\hat{x} = 0$: $\mathbf{Ri}_o S_o = C_f$

at any $\hat{x}$: $\frac{\varphi \hat{Q}_s + (1-\varphi)\hat{Q}_m}{\hat{B}} \hat{S} = \hat{u}^3$

Const mud concentration: $\hat{Q}_m = \hat{Q}_f$

Sand transport: $\hat{Q}_s = \hat{u}^5 \hat{B}$

Width: $\hat{B} = \hat{Q}_f^m$

Conserve sand: $\frac{d\hat{Q}_s}{d\hat{x}} = -\boldsymbol{\Pi\alpha}\, \hat{B}$

Conserve mud: $\frac{d\hat{Q}_m}{d\hat{x}} = -\boldsymbol{\Pi\alpha}\, r_p \frac{\varphi}{1-\varphi} \chi \hat{B}$

$$r_p = (1-\lambda_{pm}) / (1-\lambda_{ps})$$

More formulation and solution

B.c.'s: at $\hat{x} = 0$

$$\hat{Q}_f = \hat{Q}_s = \hat{Q}_m = \hat{B} = \hat{S} = \hat{u} = 1$$

$$\text{at } \hat{x} = 1 \quad \hat{Q}_s = 0 \quad \hat{Q}_m \geq 0$$

Thus

$$\hat{Q}_f = \hat{Q}_m = [1-(1-m)\,\Pi\alpha\ \gamma\hat{x}]^{[1/(1-m)]}$$

$$\hat{B} = [1-(1-m)\,\Pi\alpha\ \gamma\hat{x}]^{[m/(1-m)]}$$

$$\hat{Q}_s = 1 - \frac{1}{\gamma}\{1-[1-(1-m)\,\Pi\alpha\ \gamma\hat{x}]^{[1/(1-m)]}\}$$

and

$$\hat{S} = \frac{\hat{B}^{2/5}\hat{Q}_s^{3/5}}{\varphi\hat{Q}_s + (1-\varphi)\hat{Q}_m}$$

where

$$\gamma = r_p \frac{\varphi}{1-\varphi}\chi$$

$$\Pi\alpha = \frac{1}{(1-m)\gamma}[1-(1-\gamma)^{(1-m)}]$$

Long profile of channel

By definition, $S = -\frac{d\eta}{dx}$

Now defining a dimensionless variable $\hat{\eta}$ such that

$$\hat{\eta} = \frac{\eta}{S_o L}$$

and applying the normalization $\hat{\eta}\big|_{\hat{x}=1} = 0$

it is found that

$$\hat{\eta} = \int_{\hat{x}}^{1} \hat{S} d\hat{x}$$

or equivalently

$$\tilde{\eta} = \frac{\int_{\hat{x}}^{1} \hat{S} d\hat{x}}{\int_{0}^{1} \hat{S} d\hat{x}}$$

where $\tilde{\eta} = \frac{\int_{\hat{x}}^{1} \hat{S} d\hat{x}}{\int_{0}^{1} \hat{S} d\hat{x}}$

Sample calculations

Input parameters.

Parameter	Value	Units	Basis
χ	10		Cross-sections, Pirmez (1994)
m	0.5		Analogy to rivers
λ_{ps}	0.4		Common value for clean sand
λ_{pm}	0.6		Common value for silty mud
φ	0.1		Educated guess
L	350	Km	Figure 5; Pirmez (1994)
h_o	90	M	Pirmez (1994)
S_o	0.059		Figure 5; Pirmez (1994)
B_{Co}	1590	M	Pirmez (1994)
D	200	mμ	
R	1.65		Most common value for natural sediment
$\mathbf{Ri}_o$	2		Educated guess, but see text
T_c	0.25	Day	
T_i	50	years	

Output Parameters

Parameter	Value	Units	Notes
γ	0.74		< 1 as required by theory
$\Pi\alpha$	1.33		Order-one
C_f	0.012		
C_{sto}	0.0139		Dilute suspension, as postulated
C_{so}	0.0014		
C_{mo}	0.0125		
u_o	3.18	m/s	Compare with estimates of Pirmez (1994) based on bend superelevation
Q_{fo}	4.54×10^5	m^3/s	
Q_{so}	6300	m^3/s	
v_a	2.50×10^{-6}	m/s	
Δ	54.1	mm	Event deposit thickness
I	1.37×10^{-5}		
$\overline{v}_a$	1.08	mm/year	

Sample calculations are shown with slides and an LCD projector