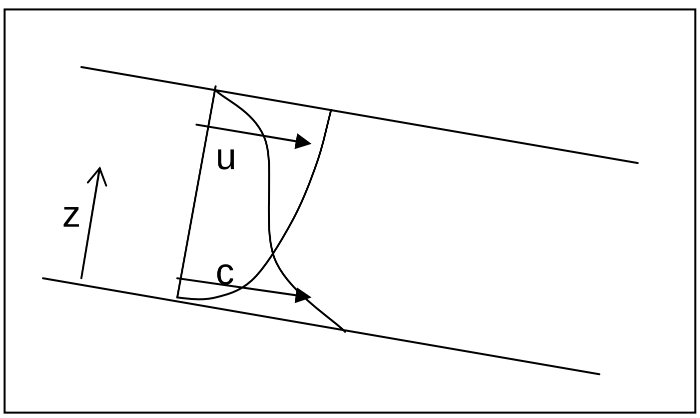
Note to St. Oyen students. This document consists of two parts. The first is a presentation of the algebraic turbulence closure scheme of J. Dungan Smith and colleagues for stratification effects due to suspended sediment in open channels. Two references are given at the end. The second is a list of references that pertain to Schmidt effects in suspensions.

STRATIFICATION OF SUSPENDED SEDIMENT IN OPEN CHANNEL FLOW

Gary Parker, May, 1999

Definitions

Consider steady, equilibrium open channel flow over an erodible bed in a wide channel with upward normal profiles streamwise velocity u = u(z) and volume suspended sediment concentration c = c(z), where z denotes a coordinate upward normal from bed. The flow has depth H and shear velocity u_*. (In the present formulation all the shear stress is assumed to be skin friction and none to be form drag.) The water has density ρ and kinematic viscosity ν.



The sediment is assumed to be uniform with size D, density ρ_s and fall velocity v_s. The submerged specific gravity of the sediment is given as

$$R = \frac{\rho_s}{\rho} - 1$$

and the explicit particle Reynolds number $\mathbf{Re}_p$ is given as

$$\mathbf{Re}_p = \frac{\sqrt{RgD}\,D}{\upsilon}$$

where g denotes gravitational acceleration. Particle fall velocity v_s is specified by a dimensionless relation of the form

$$\mathbf{R}_f = f(\mathbf{Re}_p)$$

where

$$\mathbf{R}_f = \frac{v_s}{\sqrt{RgD}}$$

and the functional relationship itself is given by Dietrich (1982).

Basic forms

The equation of momentum conservation for the flow takes the form

$$\nu_t \frac{du}{dz} = u_*^2 (1 - \frac{z}{H}) \tag{1}$$

where ν_t denotes a kinematic eddy viscosity. The corresponding form for conservation of suspended sediment is

$$v_s c + \nu_t \frac{dc}{dz} = 0 \tag{2}$$

Eddy viscosity ν_t is specified in the following form:

$$\nu_t = \kappa u_* H F_1(\zeta) F_2(\mathbf{Ri}) \tag{3}$$

where κ denotes the Karman constant, F_1 and F_2 are specified functions,

$$\zeta = \frac{z}{H}$$

and **Ri** denotes the gradient Richardson number, given by

$$\mathbf{Ri} = -Rg \frac{\frac{dc}{dz}}{\left(\frac{du}{dz}\right)^2} \tag{4}$$

The bottom boundary condition for velocity is determined by a match to the rough logarithmic law at normalized reference bed elevation ζ_r:

$$\frac{u|_{\zeta_r}}{u_*} = \frac{1}{\kappa} \ell n(30\zeta_r \frac{H}{k_s}) \tag{5}$$

where k_s denotes bed roughness height and κ denotes the Karman constant (0.4). The bottom boundary condition for concentration of suspended sediment is likewise given in terms of a specification of reference bed concentration c_r at ζ_r.

Dimensionless forms

Define dimensionless velocity and concentration as follows:

$$\mathit{u} = \frac{u}{u_*} \qquad \mathit{c} = \frac{c}{c_r}$$

Equations (1), (2), (4) and (5) can then be reduced to

$$\frac{d\mathit{u}}{d\varsigma} = \frac{(1-\varsigma)}{\kappa F_1(\varsigma)F_2(\mathbf{Ri})} \tag{6}$$

$$\frac{d\mathit{c}}{d\varsigma} = -\frac{1}{\kappa u_{*r}}\frac{1}{F_1(\varsigma)F_2(\mathbf{Ri})}\mathit{c} \tag{7}$$

$$\mathbf{Ri} = -\mathbf{Ri}_* \frac{\dfrac{d\mathit{c}}{d\varsigma}}{\left(\dfrac{d\mathit{u}}{d\varsigma}\right)^2} \tag{8}$$

$$\mathit{u}\big|_{\varsigma_r} = \frac{1}{\kappa}\ell n(\varsigma_r \frac{H}{k_s}) \tag{9}$$

where

$$u_{*r} = \frac{u_*}{v_s} \qquad \mathbf{Ri}_* = \frac{\mathit{R}gHc_r}{u_*^2} \tag{10a,b}$$

In addition, the near-bed condition for normalized concentration c becomes

$$\mathit{c}\big|_{\varsigma_r} = 1 \tag{11}$$

Forms for the functions F_1 and F_2

The standard form for the function F_1 is the parabolic one:

$$F_1 = \varsigma(1-\varsigma)$$

Smith and McLean (1977) offer the alternative form

$$F_1 = \begin{cases} \varsigma + 1.32892\varsigma^2 - 16.86321\varsigma^3 + 25.22663\varsigma^4 & \varsigma_r \le \varsigma < 0.3 \\ 0.160552 + 0.075605\varsigma - 0.1305618\varsigma^2 - 0.1055945\varsigma^3 & 0.3 \le \varsigma \le 1 \end{cases}$$

Gelfenbaum and Smith (1986) offer the alternative form

$$F_1 = \varsigma \exp(-\varsigma - 3.2\varsigma^2 + \frac{2}{3} 3.2\varsigma^2)$$

Smith and McLean (1977) provide the following form for F_2:

$$F_2 = 1 - 4.7\mathbf{Ri}$$

Gelfenbaum and Smith (1986) offer the alternative below:

$$F_2 = \frac{1}{1 + 10.0\,X} \qquad X = \frac{1.35\mathbf{Ri}}{1 + 1.35\mathbf{Ri}}$$

Form for near-bed concentration

In the analysis given here, the relation of Garcia and Parker (1991) relation is used. The value of ζ_r is set equal to 0.05 in this formulation. The specification for reference concentration is as follows.

$$c_r = \frac{AX_e^5}{1 + \frac{A}{0.3} X_e^5} \qquad X_e = u_{*r}\, \mathbf{Re}_p^{0.6}$$

where A = 1.3x10^{-7}, $u_{*r} = u_*/v_s$ and u_* should refer only to that part of the shear velocity pertaining to skin friction.

Solution

The solution used in the spreadsheet (***St Oyen students: not supplied here but available on request***) uses the parabolic form for F_1, the form for F_2 due to Gelfenbaum and Smith (1986) and the form for c_r due to Parker and Garcia (1991). The solution is implemented iteratively. In the zeroth-order solution F_2 is set equal to 1, corresponding to vanishing stratification (**Ri** = 0). This yields the solution

$$u = \frac{1}{\kappa} \ell n(30 \frac{H}{k_s} \varsigma) \qquad c = \left[\frac{(1-\varsigma)/\varsigma}{(1-\varsigma_r)\varsigma_r} \right]^{\frac{1}{\kappa u_{*r}}}$$

The above solution is then used to compute **Ri**, which is found after some manipulation to take the form

$$\mathbf{Ri} = \frac{\kappa F_1 F_2}{u_{*r}(1-\varsigma)^2} c$$

This in turn allows for an update of F_2, and thus the solution for u and c. The solution is iterated until convergence is obtained.

Other formulations

Villaret and Trowbridge (1991) have implemented a perturbation form of the above analysis for weakly stratified suspensions. A more advanced differential closure scheme for stratified flows that has only recently been applied to open channel suspensions is that of Mellor and Yamada (e.g. 1982)

References

Dietrich, W. E. 1982 Settling velocity of natural particles. *Water Resources Research*, 18(6), 1615-1626.

Garcia, M. and Parker, G. 1991 Entrainment of bed sediment into suspension. *J. Hydraul. Engrg.*, ASCE, 117(4), 414-435.

Gelfenbaum, G. and Smith, J. D. 1986 Experimental evaluation of a generalized suspended-sediment transport theory. In *Shelf and Sandstones*, Canadian Society of Petroleum Geologists Memoir II, Knight, R. J. and McLean, J. R., eds., 133 – 144.

Mellor, G. L. and Yamada, T. 1982 Development of a turbulence closure model for geophysical fluid problems. *Reviews of Geophysics and Space Physics*, 20(4), 851-875.

Smith, J. D. and McLean, S. R. 1977 Spatially averaged flow over a wavy surface. *J. Geophys. Res.*, 82(2), 1735-1746.

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SCHMIDT EFFECTS IN OPEN CHANNEL SUSPENSIONS

There are two other effects in open channel suspensions in addition to stratification effects. The first of these is a roughness adjustment due to bedload transport. That is, typically the effective roughness height k_s is larger when the bed is intensely mobile, increasing weakly with Shields stress τ^*. The second of these is an effect quantified in terms of the Schmidt number,

$$\mathbf{Sc} = \frac{\nu_t}{\nu_{st}}$$

where ν_t denotes the eddy diffusivity for momentum transfer and ν_{st} denotes the eddy diffusivity for suspended sediment. That is, even when stratification effects are negligible the Schmidt number tends to become greater than 1 as the sediment size becomes coarser. In particular, the speculated relationship (as yet poorly-defined) is of the form

$$\mathbf{Sc} = \mathbf{Sc}\left(\frac{u_*}{v_s}\right)$$

such that **Sc** $\rightarrow$ 1 as $u_*/v_s \rightarrow \infty$ and **Sc** increases monotonically as u_*/v_s becomes small (i.e. approaches unity, below which a sustained suspension cannot be maintained).

The Schmidt effect is associated with finite particle size, and the inertial lag between flow fluctuations and fluctuations in particle velocity. There are a variety of references on this subject which I do not have with me. One early paper that stands out is by L. Brush; another is by the meteorologist Businger. While I don't have these references, you may find them and a host of others in the paper by Lyn (1988).

Please understand that early researchers, not understanding either the inertial effect of stratification effects, tended to try to fit the data by adjusting the Rouse number Z_r, and in particular adjusting the Karman κ in order to fit the observed profiles, where

$$Z_r = \frac{v_s}{\kappa u_*}$$

You will find references to this type of work in Lyn (1988) and Coleman (1981); the researchers include Vanoni and Einstein. The adjustment of κ is not physically based, but it does provide an ad-hoc way of introducing inertial (and stratification) effects.

References

Coleman, N. H. 1981 *J. Hydraulic Res*, 19, 221-229

Lyn, D. A. 1988 *J. Fluid Mech.*, 193, 1-26

Vanoni, V. A. 1946 *Trans. Am. Soc. Civil Eng.*, 111, 67-133