

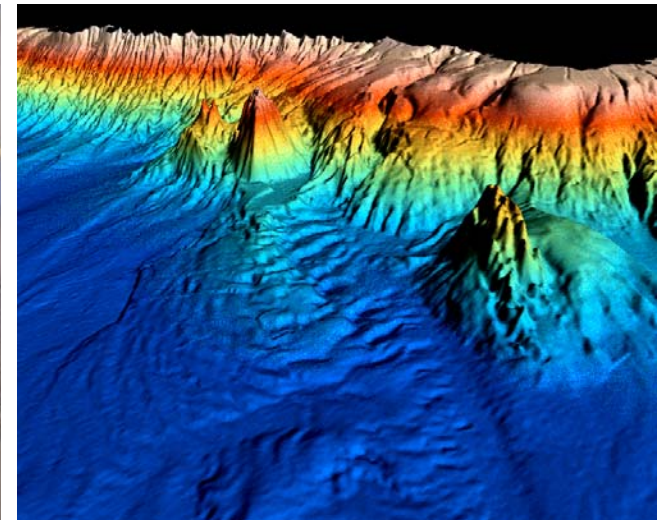
2nd INTERNATIONAL SYMPOSIUM ON SHALLOW FLOWS

December 10-12, 2008

The Hong Kong University of Science and Technology (HKUST)



Cyclic Steps: A Phenomenon of Supercritical Shallow Flow from the High Mountains to the Bottom of the Ocean



Gary Parker

**Department of Civil & Environmental Engineering and
Department of Geology,
University of Illinois Urbana USA**

This is a story about the genesis and flowering of an idea.

The following people played important roles in this process:

- John Ahern
- Jan Rik van den Berg
- Peter Brooks
- Bill Dietrich
- Juan Fedele
- Andrea Fildani
- Norihiro Izumi
- Tommy Gerber
- Svetlana Kostic
- David Mohrig
- Bill Normark
- Chris Paola
- Lincoln Pratson
- Tao Sun
- James Syvitski
- Kazuo Taki
- Kelin Whipple
- Benoit Spinewine
- Octavio Sequeiros
- Bruno Savoye



Bauhinia flower:
Emblem of Hong Kong

**It is also a celebration of the
SHALLOW WATER EQUATIONS,**

which result from the long-wave (slender flow) approximation applied to the layer-integrated Navier-Stokes and continuity equations applied to flows with an upper interface.

1D version:

t = time

U = flow velocity

η = bed elevation

e = coefficient of entrainment of ambient fluid (e.g. air)

x = distance

h = flow depth

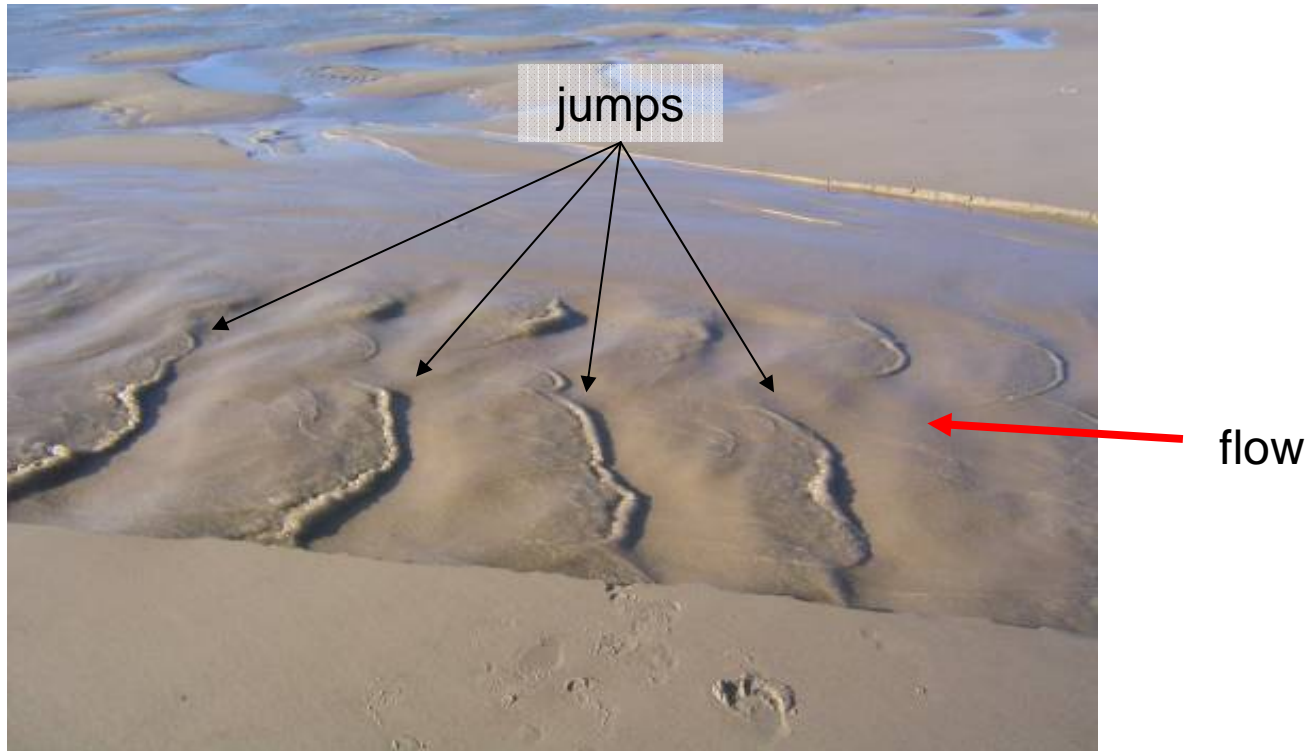
C_f = bed friction coefficient

$$\frac{\partial U}{\partial t} + \frac{\partial U^2 h}{\partial x} = -\frac{1}{2} g \frac{\partial h^2}{\partial x} - gh \frac{\partial \eta}{\partial x} - C_f U^2 \quad \text{momentum balance}$$

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x} (Uh) = eU \quad \text{flow mass balance}$$

I liked the paper so much that I recommended it for an award.

It didn't get the award.



Trains of cyclic steps in a coastal outflow channel on a beach in Calais, France.
Image courtesy H. Capart.

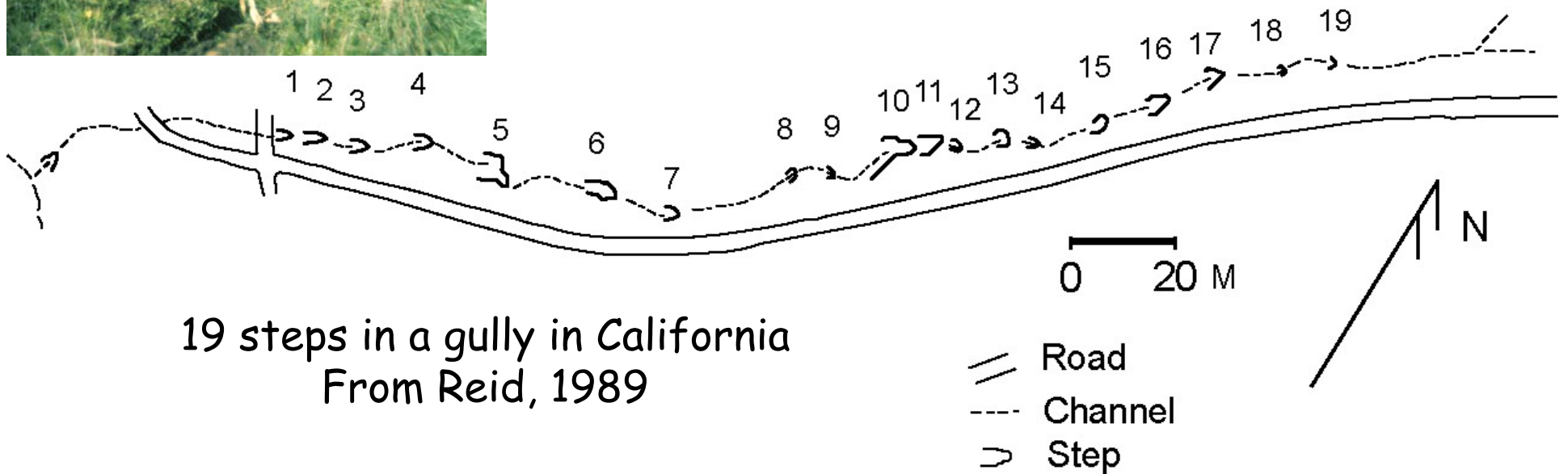
And then I forgot about it.

In 1992 I visited Bill Dietrich in the Department of Earth and Planetary Sciences, University of California Berkeley for several months.

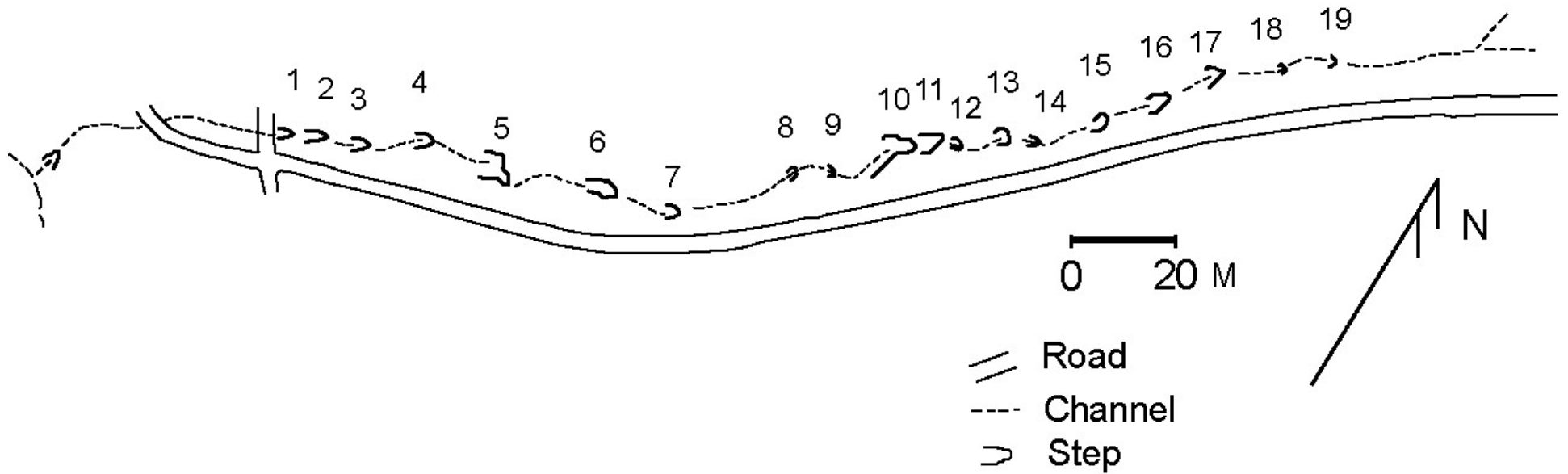




Bill took me to the Tennessee River Catchment, Marin County, and showed me something I had never seen before: DISCONTINUOUS GULLIES.



Why the series of upstream-migrating steps?



I stored it in memory, but no click.

In 1992 Chris Paola, Kelin Whipple and I began a research program for the tailings basin of the Hibbing Taconite Mine, Minnesota.

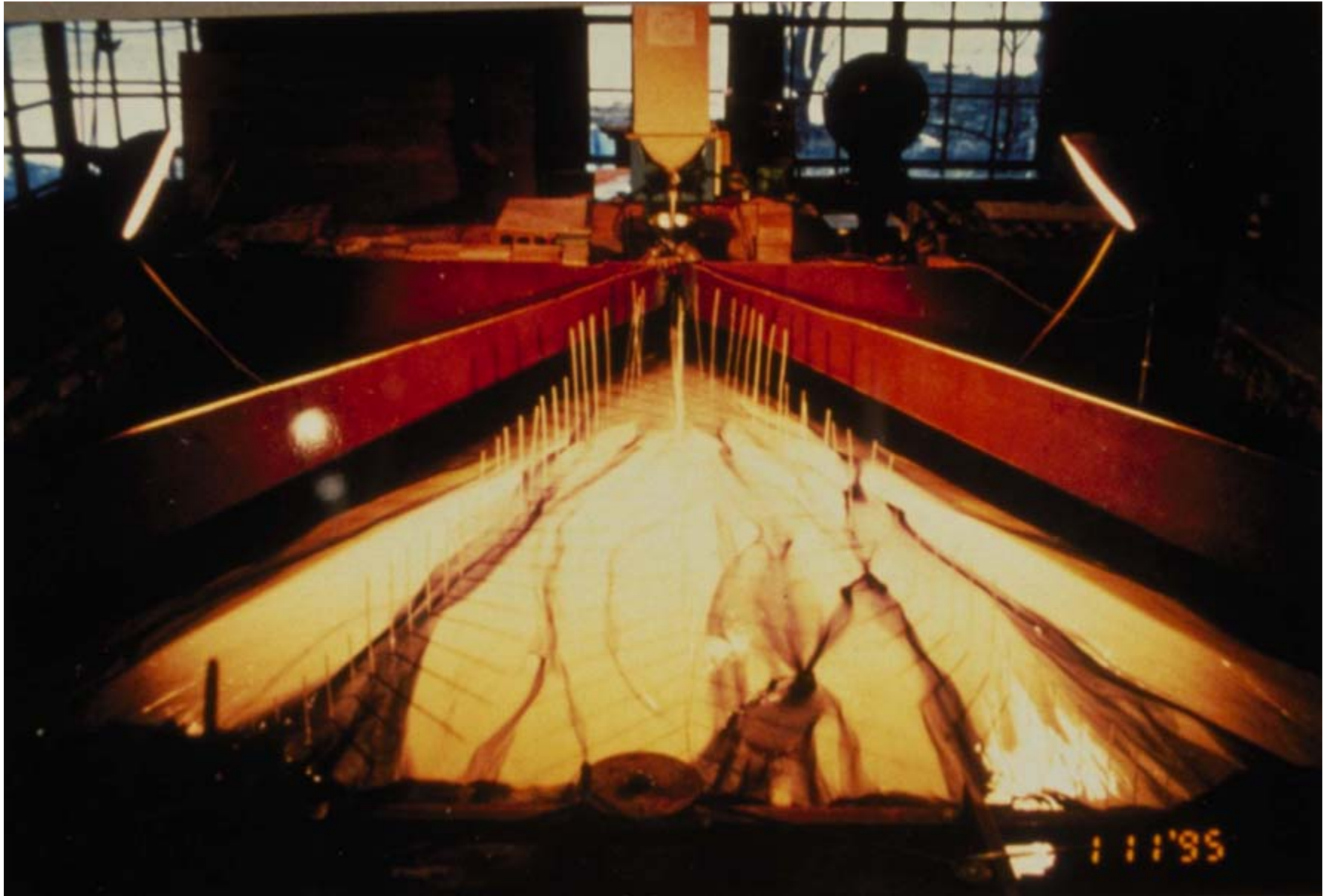


The tailings basin forms an alluvial fan

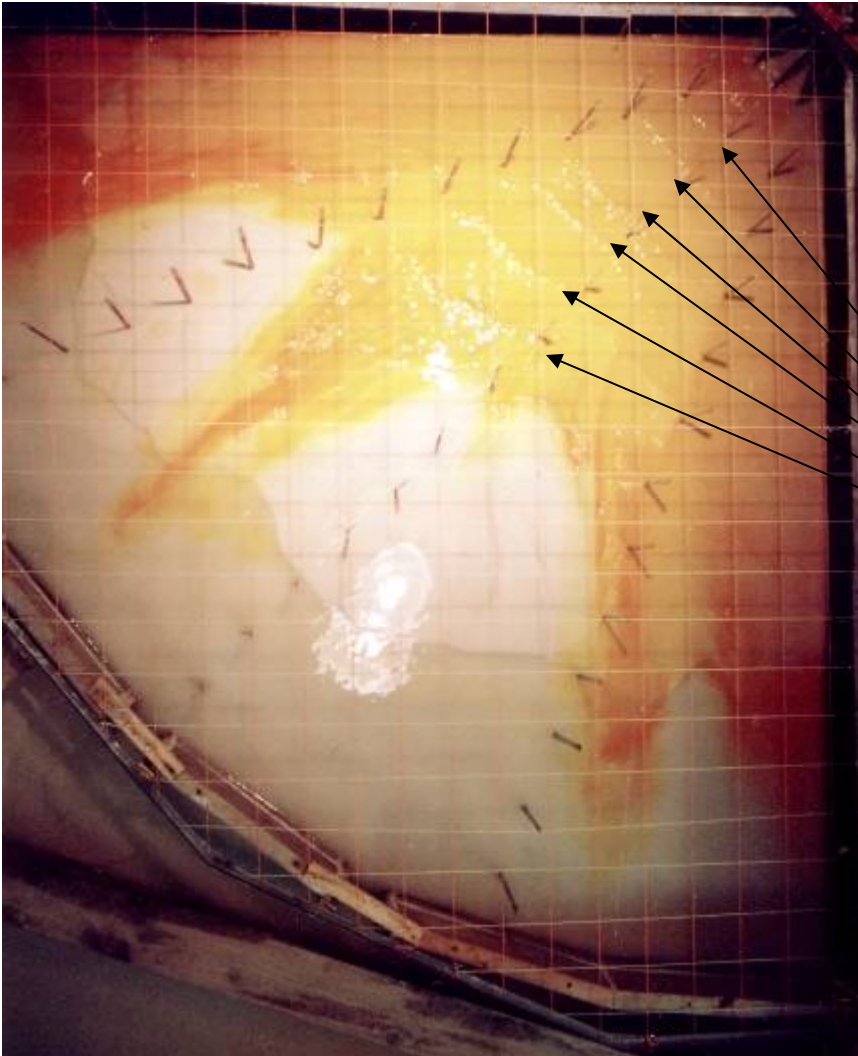
We got to see some big trucks.



We also got to do some experiments on fans, because the tailings basin of the mine is an alluvial fan.



One day John Ahern, an undergraduate student working for me on the project, asked me to come downstairs and explain something to him.

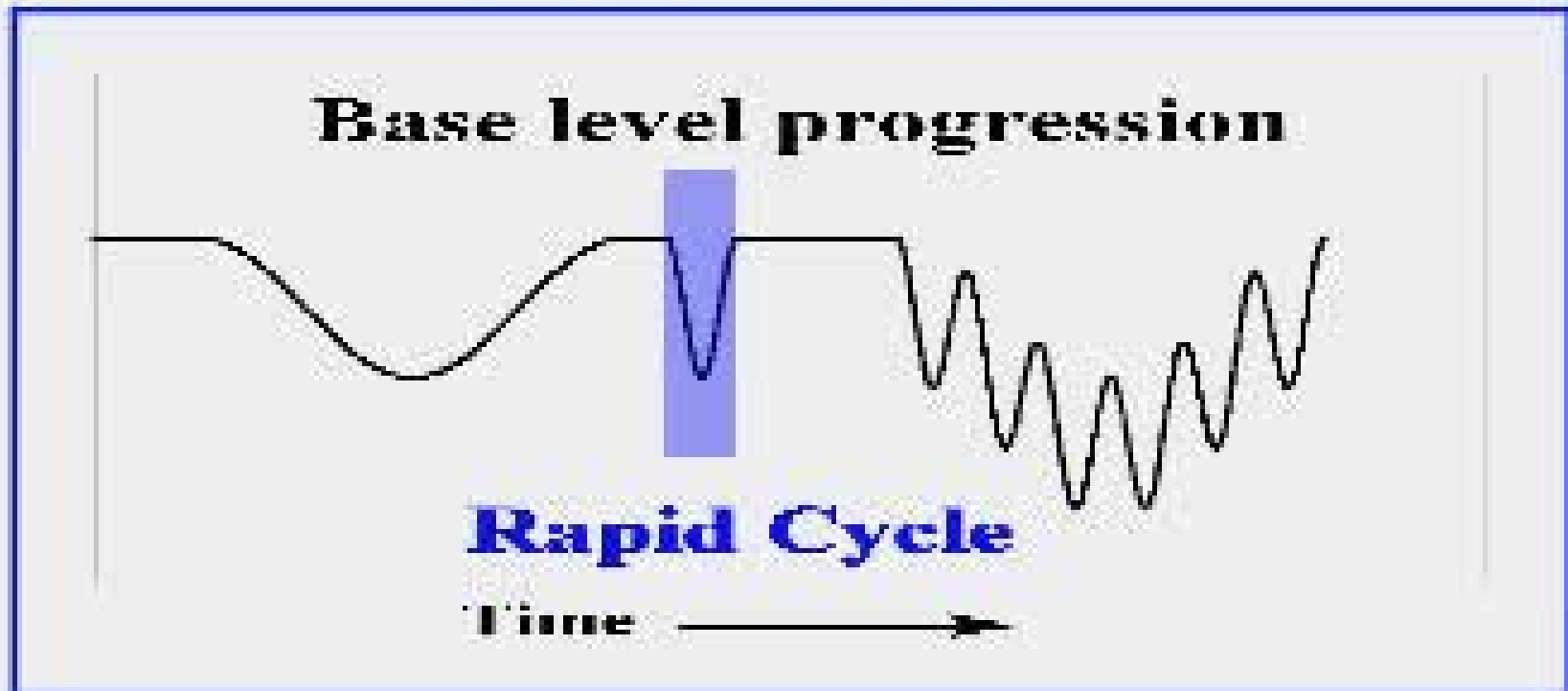


He asked:
What are these upstream-migrating steps bounded by hydraulic jumps?

I replied:
Beats the hell out of me.

You can see these steps in an experiment done (many years later) in the XES basin by Chris Paola's group.

Run 02-1 Rapid Cycle 18 hours



**Not long after John Ahern showed me these steps,
something clicked.**

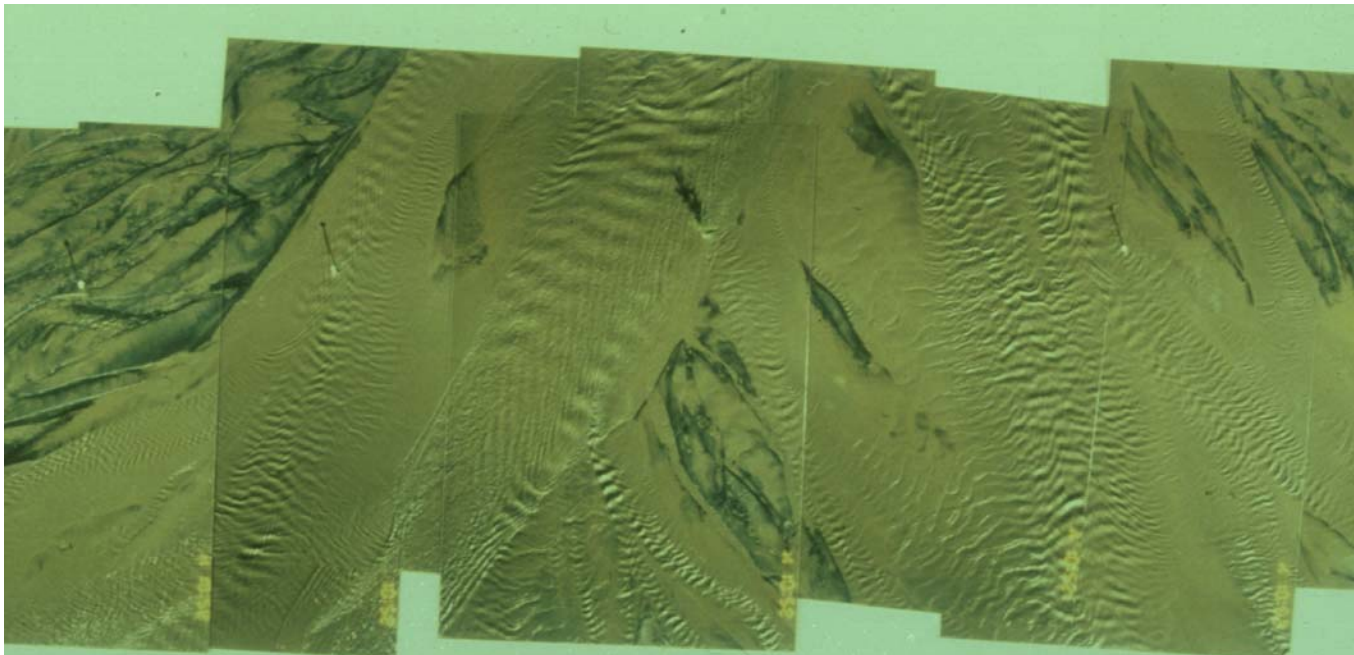
Froude Number

$$\mathbf{Fr} = \frac{U}{\sqrt{gh}}$$

U = flow velocity

g = gravitational acceleration

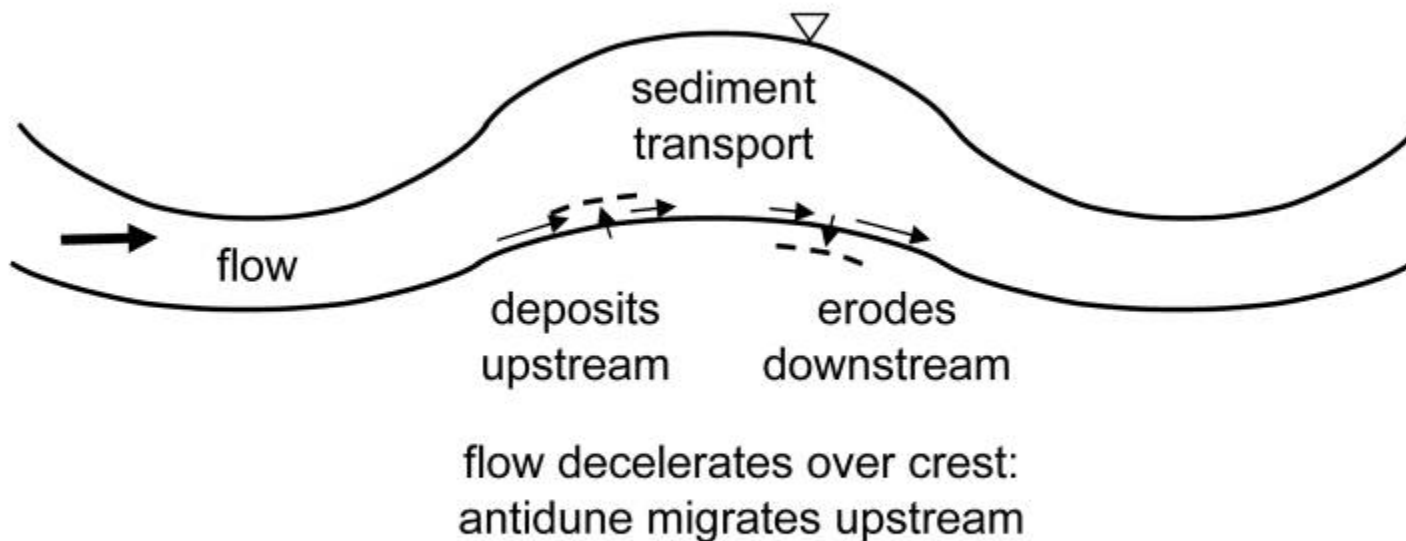
h = flow depth



Trains of antidunes in the tailings basin of the Hibbing Taconite Mine

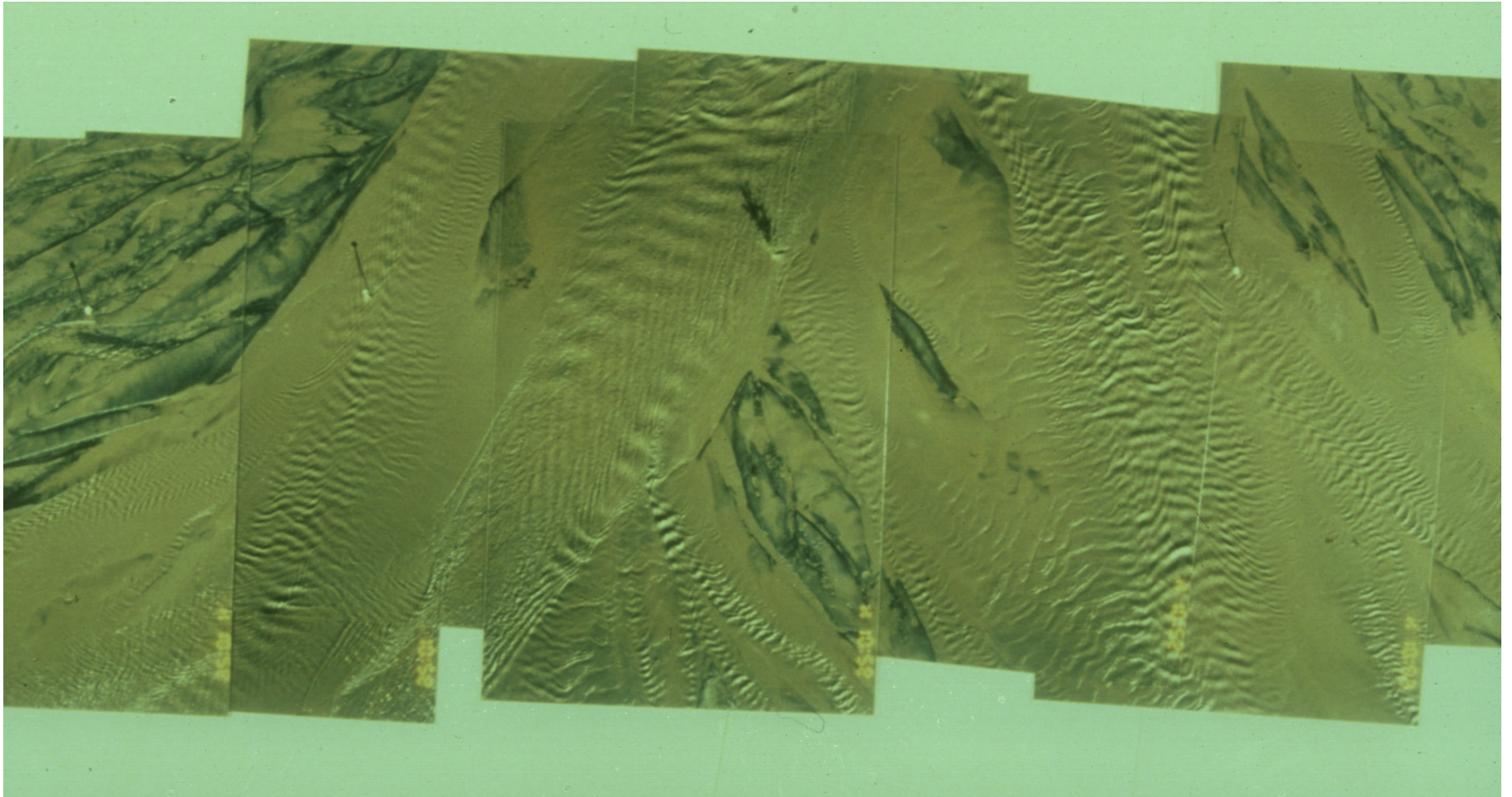
**The standard instability of Froude-supercritical flow,
 $Fr > 1$
is thought to lead to the antidune**

Antidunes are (usually) upstream-migrating rhythmic bedforms for which the water surface is in phase with the bed.



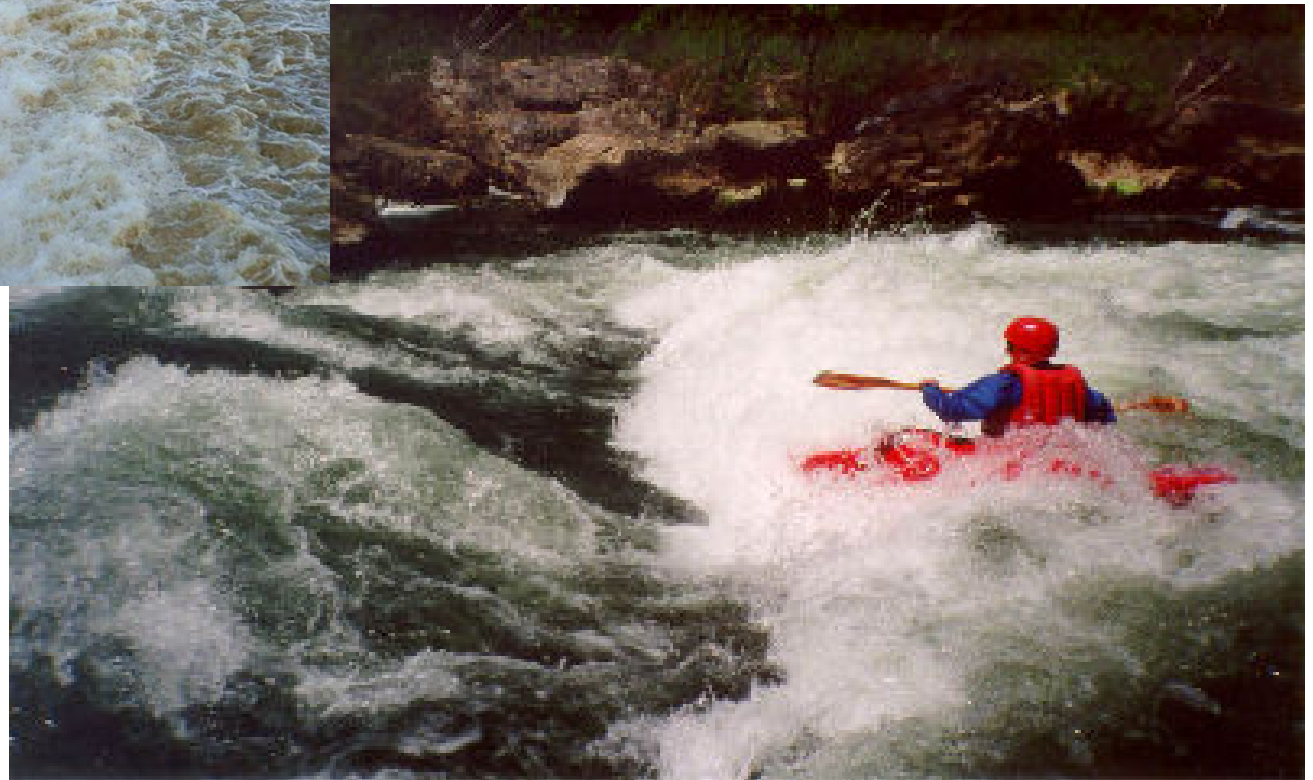
But antidunes are ephemeral: they form in trains, grow, collapse and reform elsewhere.

They do not form sustained, orderly, migrating trains.

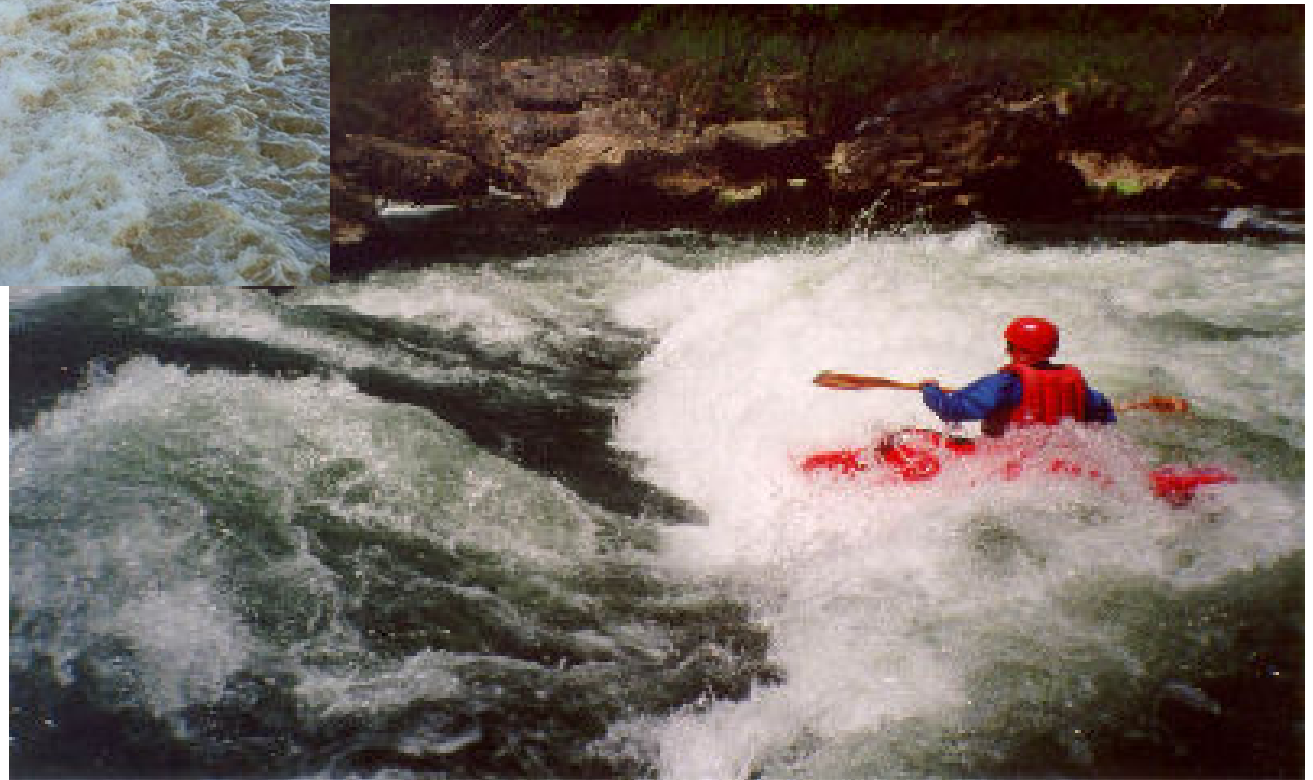




Suppose, however, that with a sufficiently high Froude number, supercritical flow over a plane, erodible bed would give way to a series of hydraulic jumps!



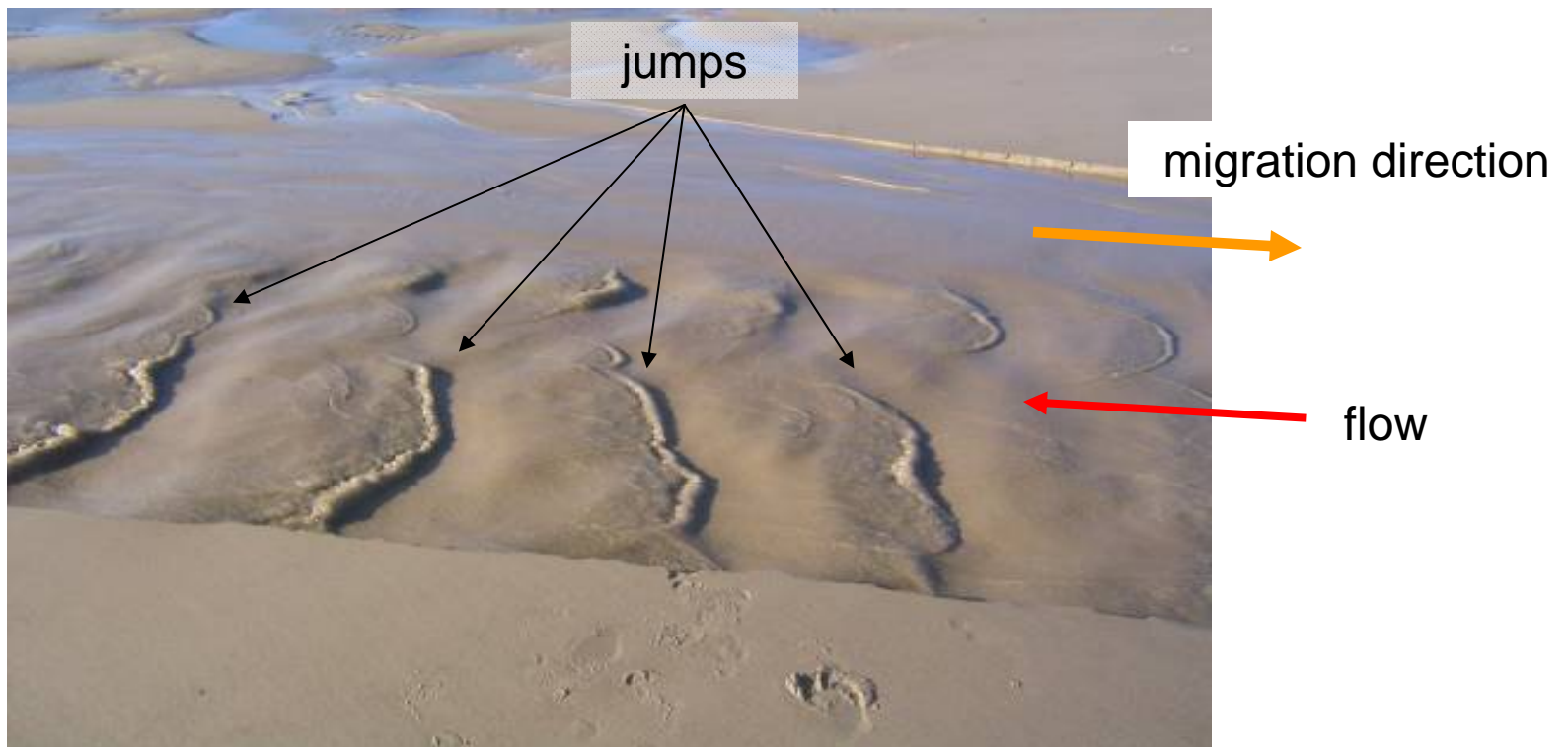
The jumps could stabilize a series of upstream-migrating steps into an orderly train.



Click

CYCLIC STEPS:

A long-wave bedform (cousin of the antidune, but much more stable), characteristic of flow that would otherwise be strongly Froude-supercritical.



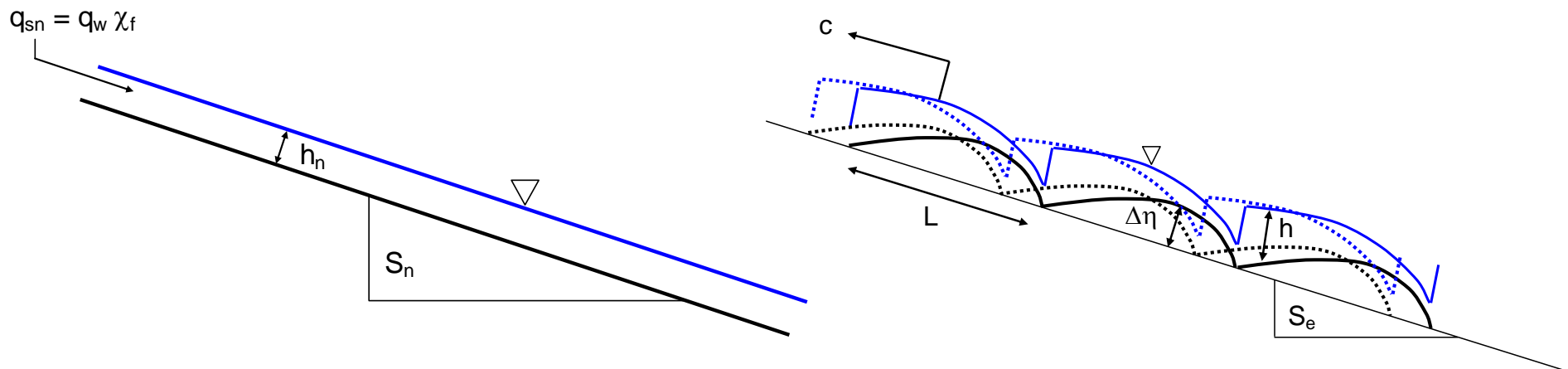
Trains of cyclic steps in a coastal outflow channel on a beach in Calais, France. Image courtesy H. Capart.

The idea:

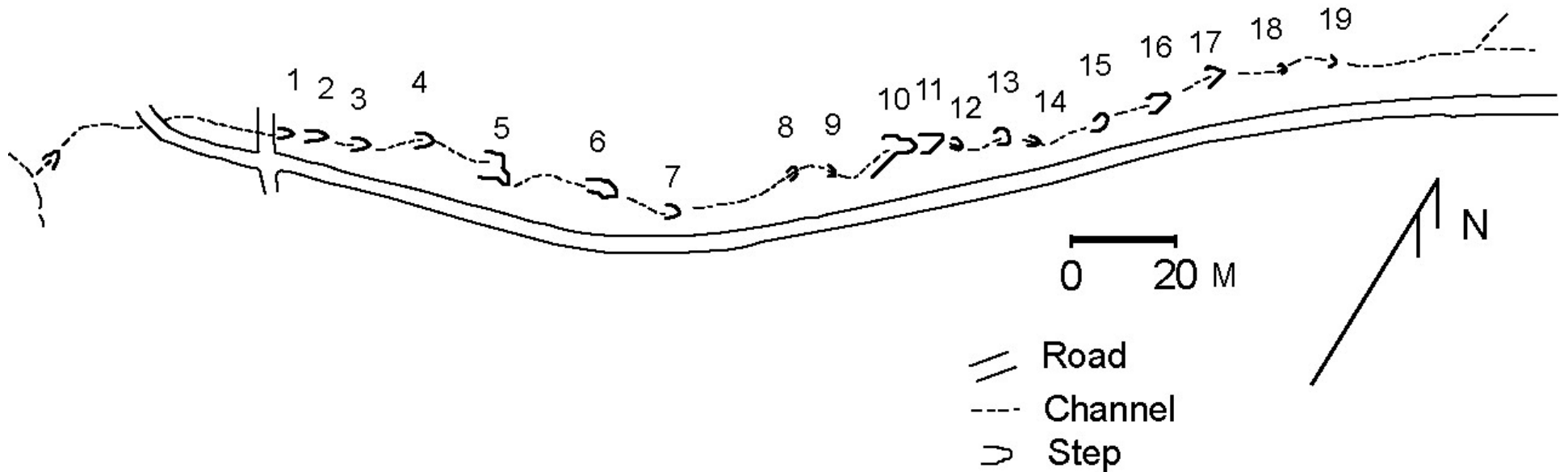
Steady, uniform (normal) Froude-supercritical flow ($Fr_n > 1$) over a freely-erodible bed of sand might be unstable,

and within an appropriate range might not devolve to ephemeral, short-wave ($L/h \sim 1$) antidunes, but instead would devolve to

orderly, sustained trains of long-wave ($L/h \ll 1$) cyclic steps, with regions of subcritical and supercritical flow bounded by hydraulic jumps



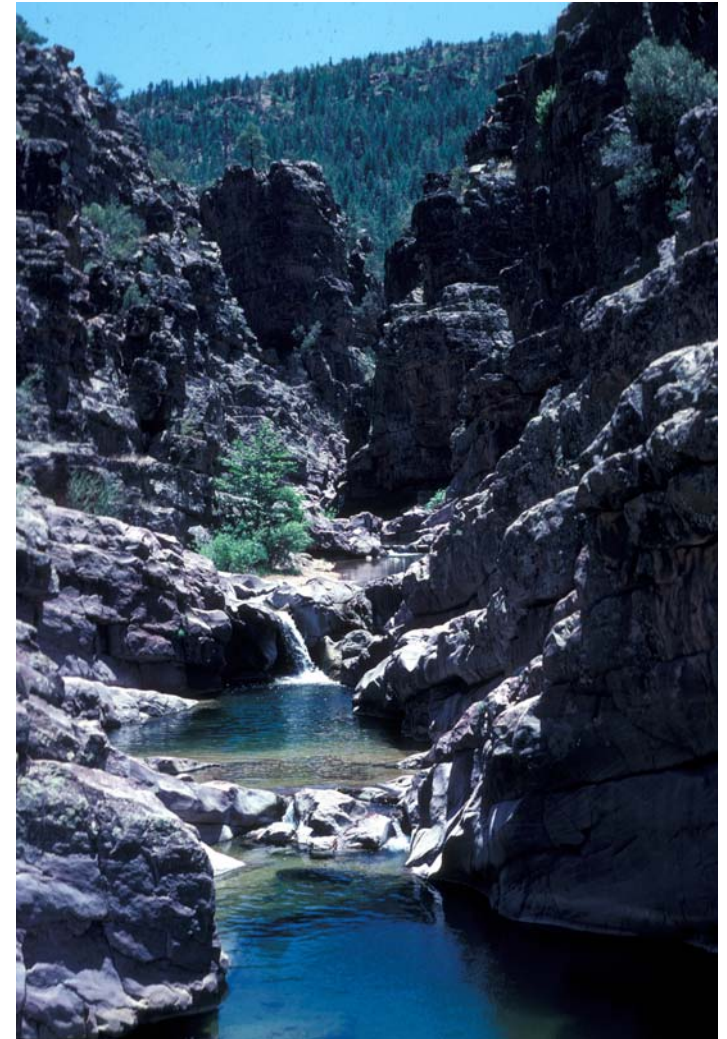
And then another click



Discontinuous gullies are typically carved into stiff clay that can be considered to be a kind of bedrock.

**Bedrock streams tend to be steep:
*A good place to expect supercritical flow during floods.***

Sure enough, such bedrock steps had been copiously documented by others.



Images courtesy Michael Neumann, Gough Island Weather Station and Ellen Wohl

Two flavors:
TRANSPORTATIONAL AND EROSIONAL
CYCLIC STEPS

The **transportational** case

- Bed covered in alluvium
- Froude-supercritical open-channel flow:
hydraulic jump
- Steps migrate upstream
- Equilibrium case: erosion
and deposition
balance over one
step

The **erosional** case

- Bed covered with erodible
bedrock or stiff clay
- Froude-supercritical open-
channel flow:
hydraulic jump
- Steps migrate upstream
- Equilibrium case: constant
degradation rate
averaged over one
wavelength

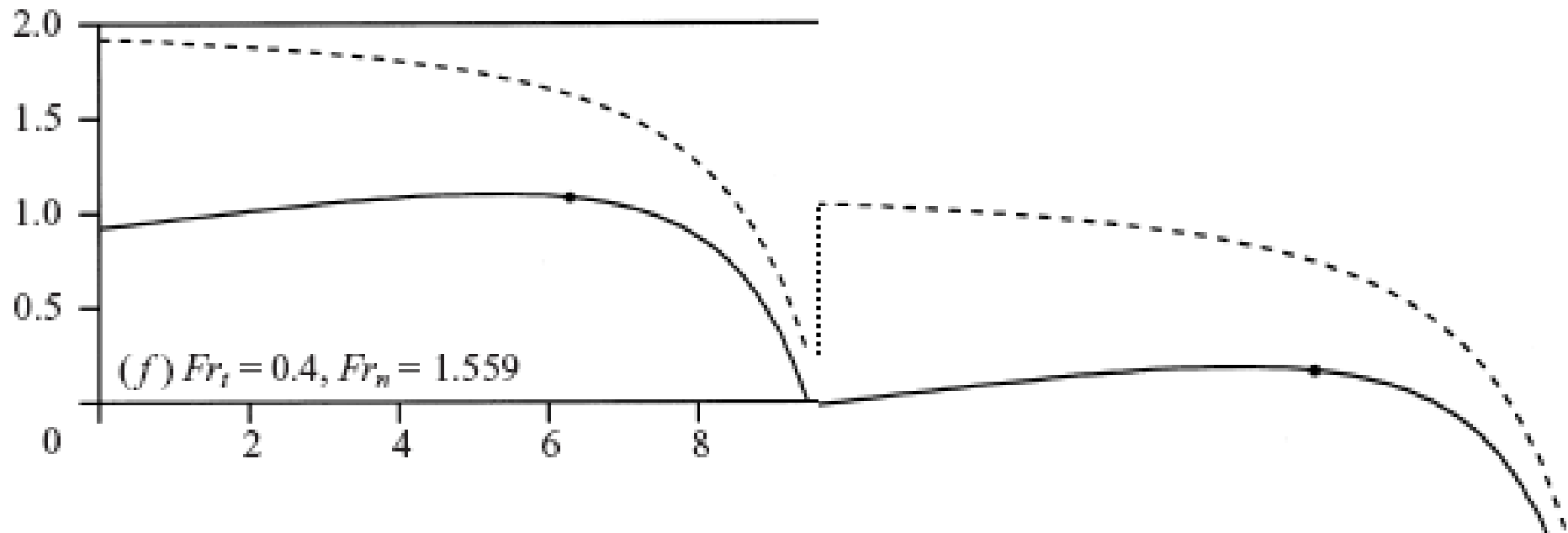
Our first work was theoretical and pertained to purely erosional cyclic steps.

J. Fluid Mech. (2000), vol. 419, pp. 203–238. Printed in the United Kingdom
© 2000 Cambridge University Press

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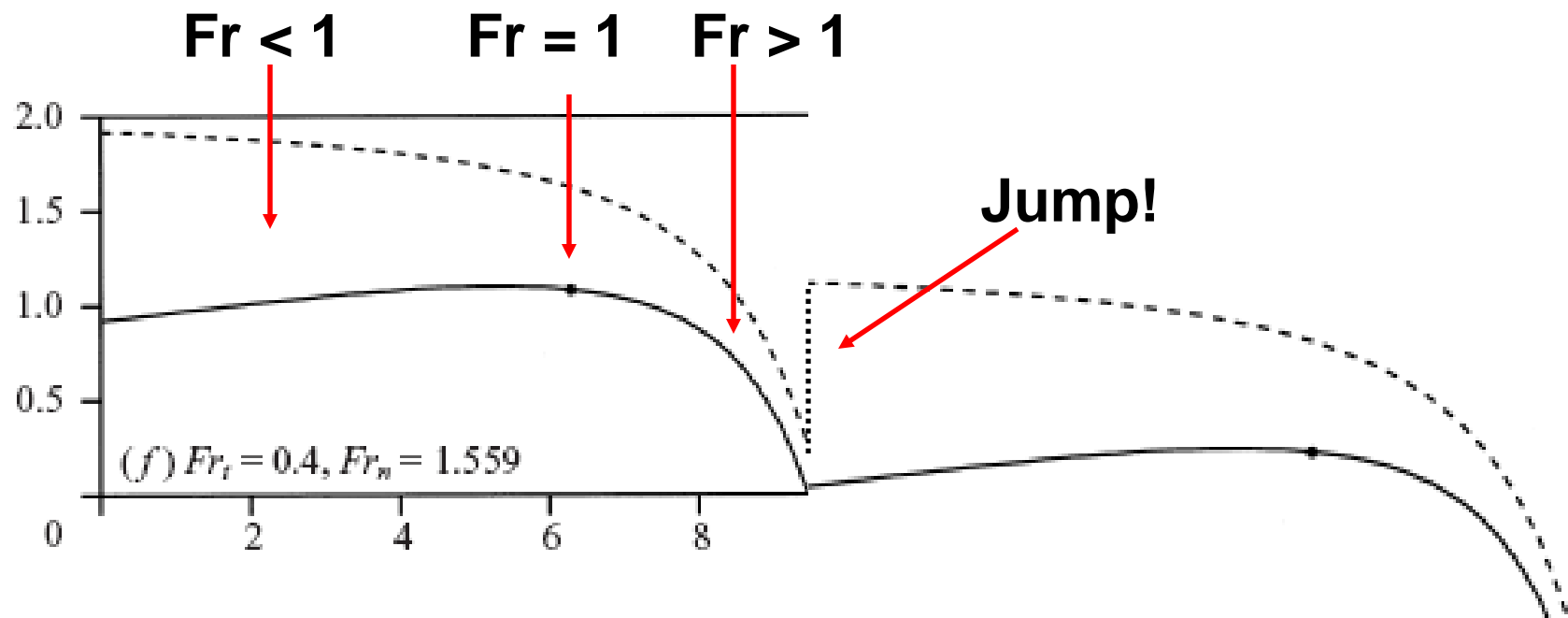
Purely erosional cyclic and solitary steps created by flow over a cohesive bed

By GARY PARKER¹ AND NORIHIRO IZUMI^{2†}



We sought (and found) a fully nonlinear periodic solution of permanent form with a constant upstream migration speed and constant incision rate (averaged over steps)

Sufficiently supercritical flow over a plane bed is subject to a *long-wave* instability that devolves into upstream-migrating supercritical and subcritical regions bounded by hydraulic jumps.



And we also modeled them experimentally.

Experiments of Brooks and Parker



They also appeared spontaneously in the drainage basin experiments of Hasbargen and Paola



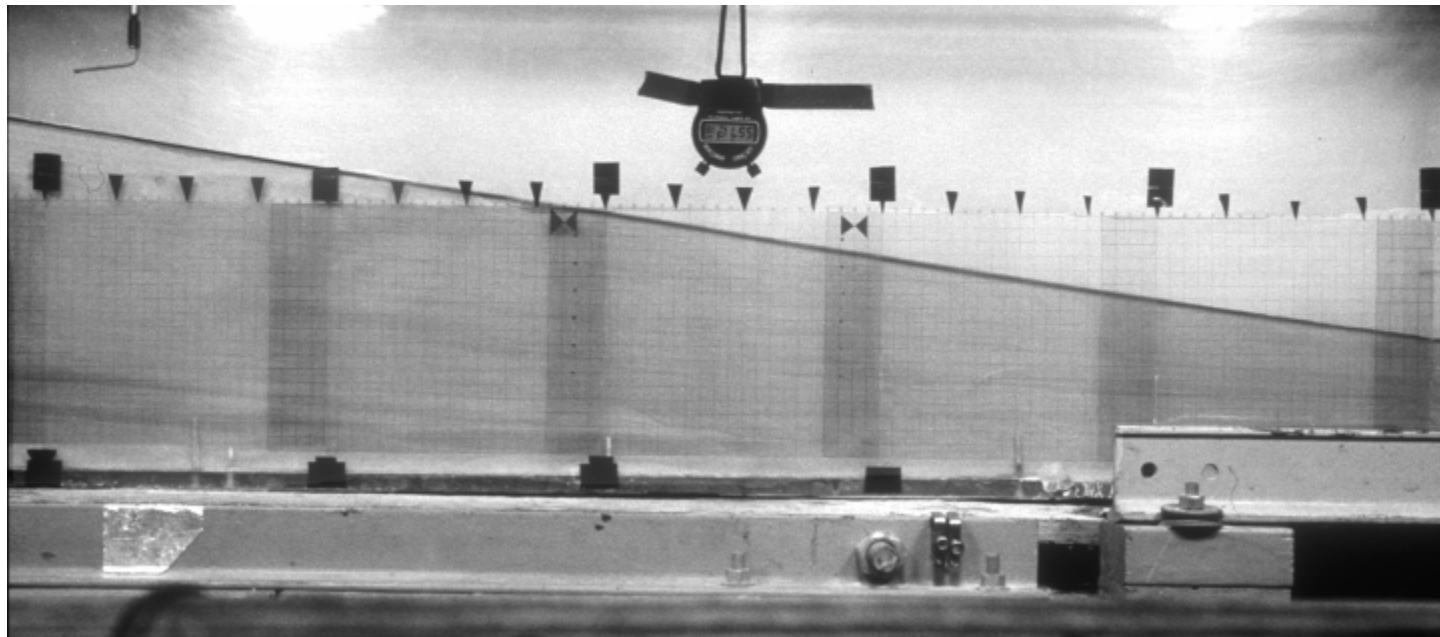
But what about the transportational case in alluvium?

Journal of Hydraulic Research

Transportational cyclic steps created by flow over an erodible bed.
Part 1. experiments

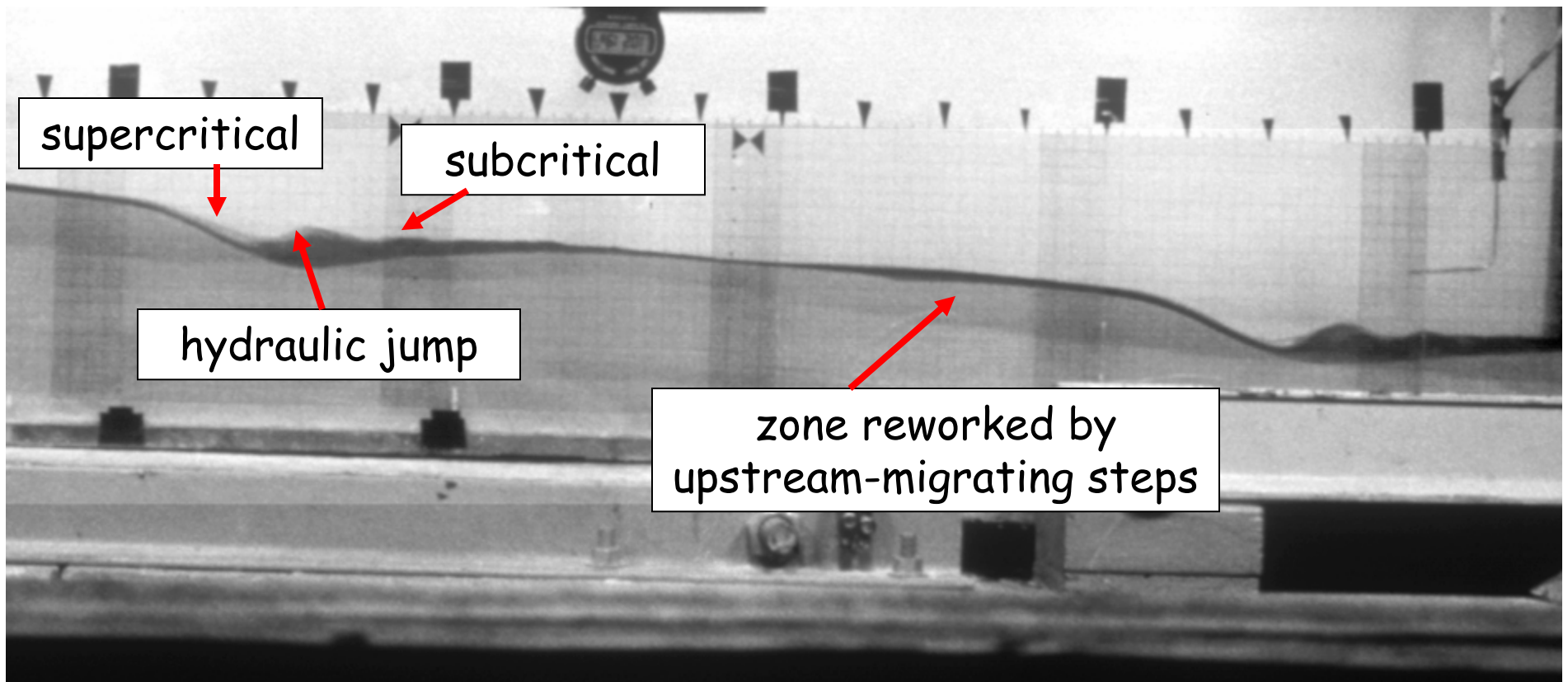
KAZUO TAKI, *Department of Life and Environmental Science, Chiba Institute of Technology, Tsudanuma 2-17-1, Narashino 275-8588, Japan*

GARY PARKER, *St. Anthony Falls Laboratory, University of Minnesota, Minneapolis, MN 55414, USA*



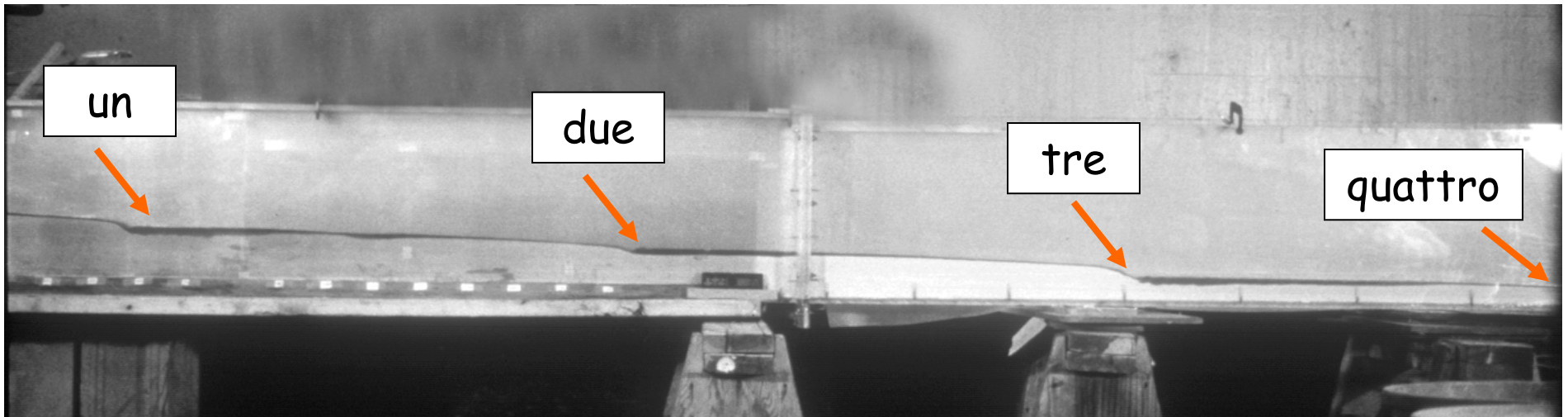
Flume is 10 cm wide, 2 or 4 m long.
Flow is only a few mm deep.
Steps have not yet formed.

Whenever the initial equilibrium flow was sufficiently Froude-supercritical, the flow devolved into cyclic steps.

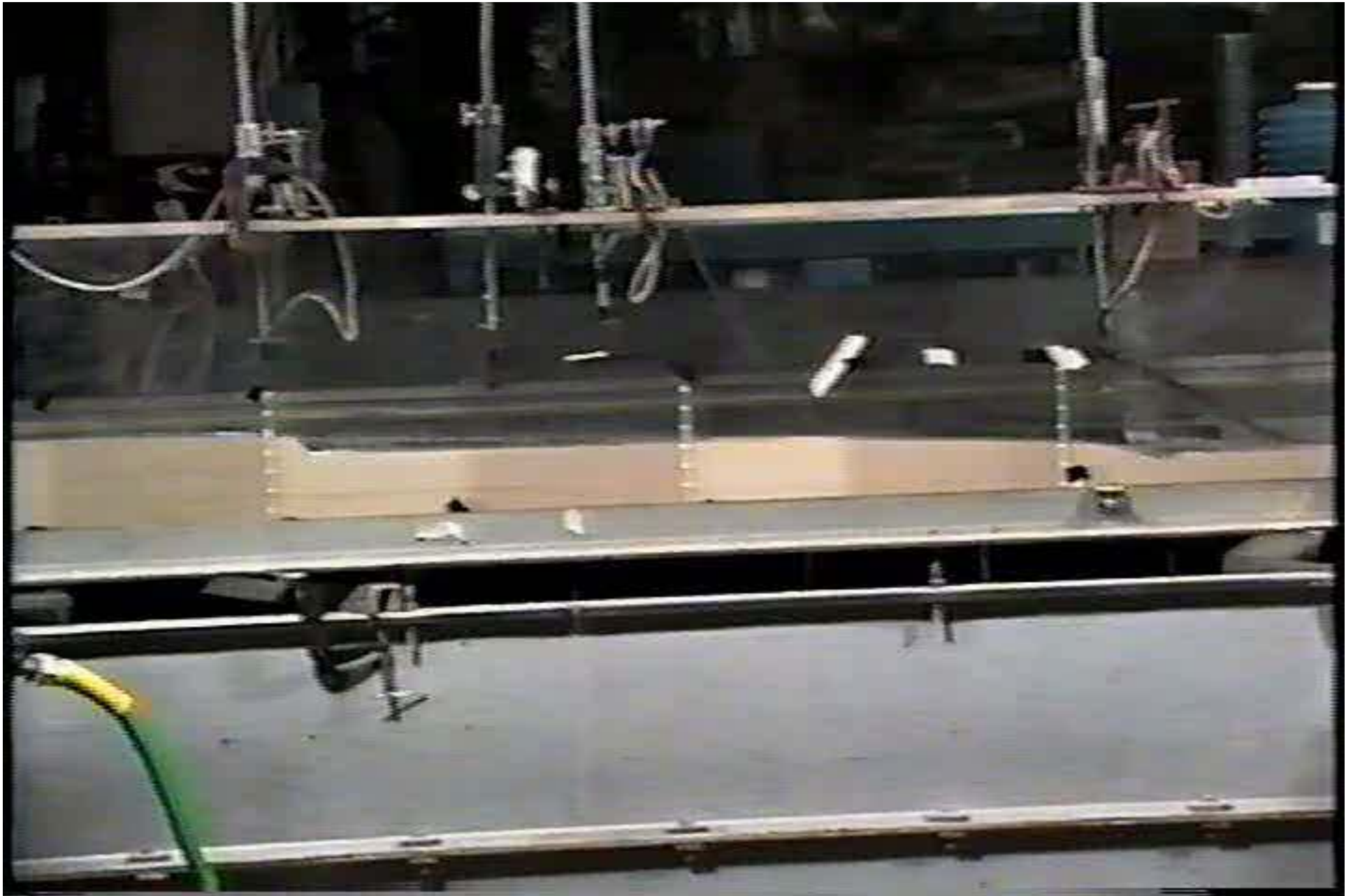


If the flume was short, we got one upstream migrating step that repeatedly reformed at the downstream end.

If the flume was long enough, we got multiple upstream-migrating steps



Let's look at a video clip



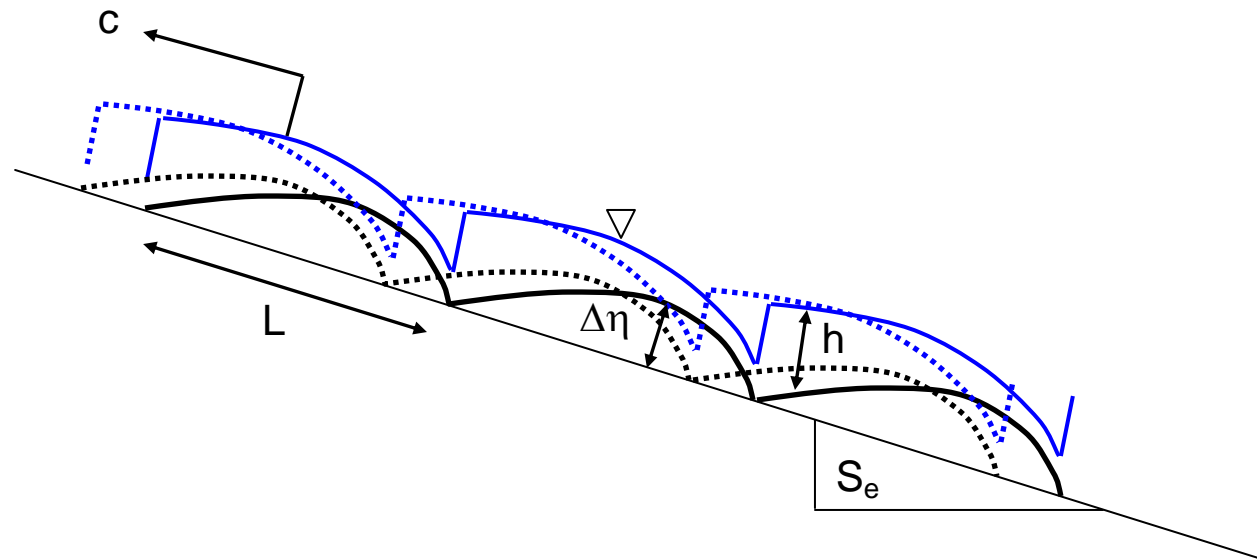
So maybe the governing equations admit a periodic finite-amplitude solution of permanent form, but this time with no net degradation or aggradation.

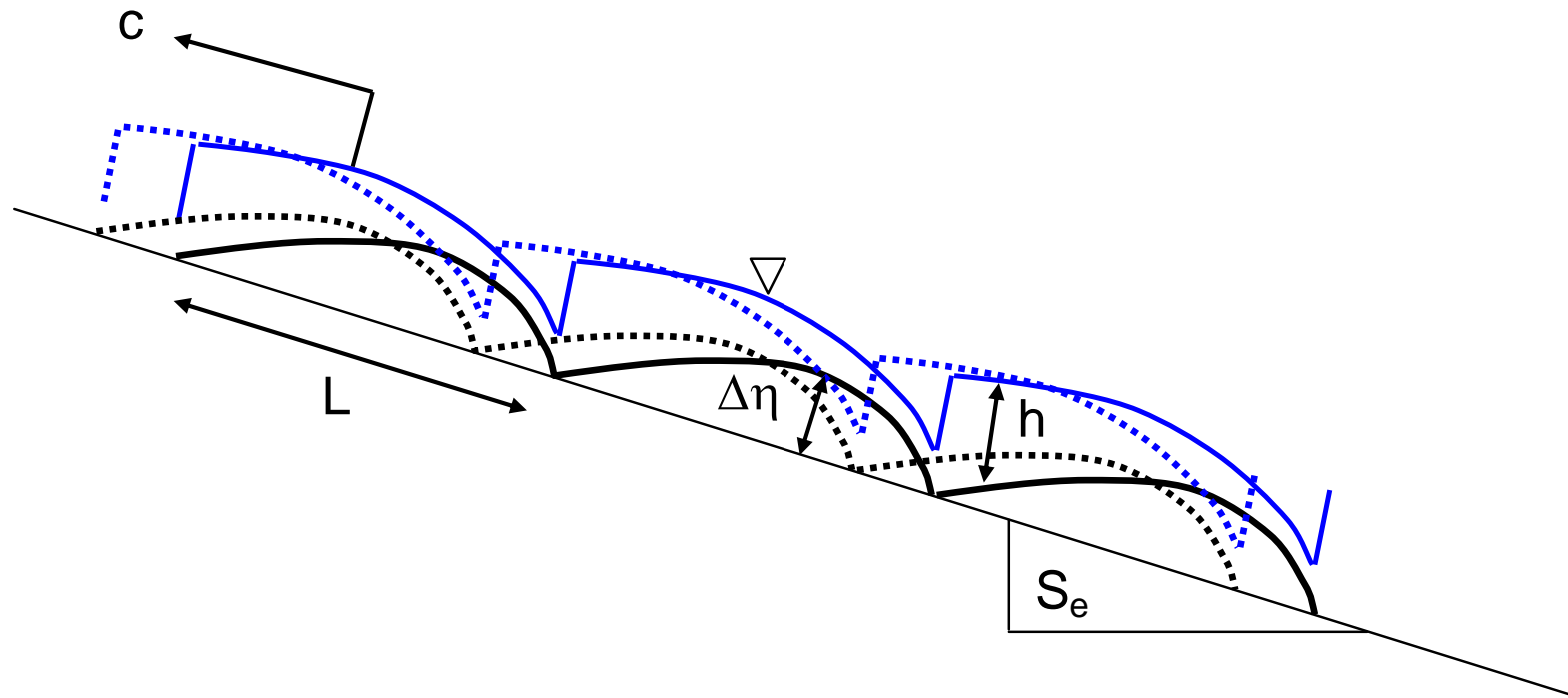
Journal of Hydraulic Research

Transportational cyclic steps created by flow over an erodible bed.
Part 2. Theory and numerical simulation

TAO SUN, *ExxonMobil Exploration Co., 233 Benmar, Houston, TX 77060, USA*

GARY PARKER, *St. Anthony Falls Laboratory, University of Minnesota, Minneapolis, MN 55414, USA*





Expected features

- Dominant transport mode: suspended sediment
- Spatially periodic, or cyclic
- Each step bounded by hydraulic jump
- Froude-subcritical → supercritical down step
- Deposition upstream, erosion downstream
- Migrates upstream with constant speed c
- Steps maintain permanent form as they migrate

Governing equations

Quasi-steady **shallow-water equations** (“d” means “has dimensions”)

$$\frac{dU^2h}{dx} = -\frac{1}{2}g\frac{dh^2}{dx} - gh\frac{d\eta}{dx} - C_fU^2 \quad \textit{momentum balance}$$

$$Uh = q_w \quad \textit{flow mass balance}$$

Quasi-steady equation of conservation of suspended sediment
(C = layer-averaged suspended sediment concentration)

$$\frac{d}{dx}(q_w\chi) = v_s(E - r_oC)$$

Exner equation of conservation of bed sediment (λ_p = porosity)

$$(1 - \lambda_p)\frac{\partial\eta}{\partial t} = v_s(r_oC - E)$$

Cyclic steps of permanent form

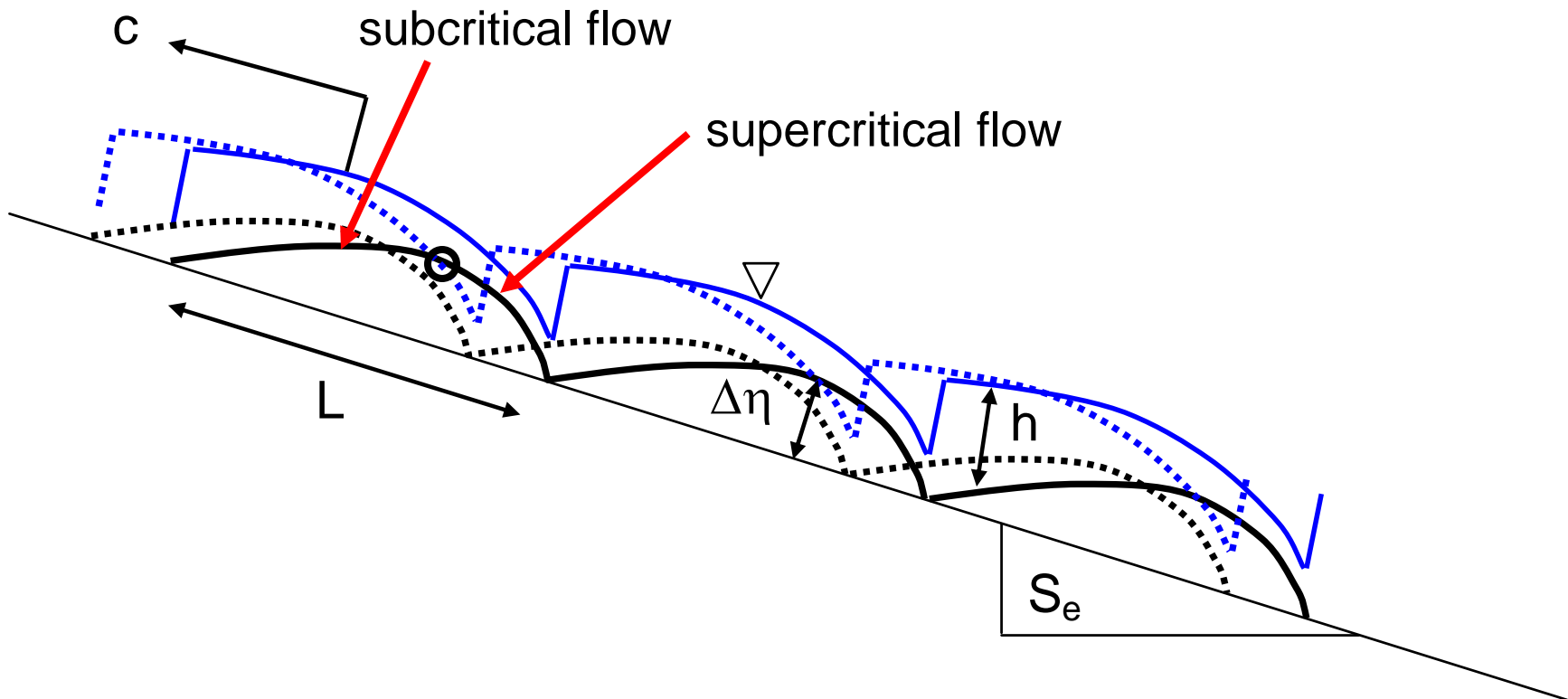
c = wave speed (positive upstream)
 L = wave length
 S_e = equilibrium average slope with steps

$$\eta = \eta_0 - S_e x - \eta_e(x - ct)$$

$$U = U(x - ct)$$

$$h = h(x - ct)$$

$$C = C(x - ct)$$



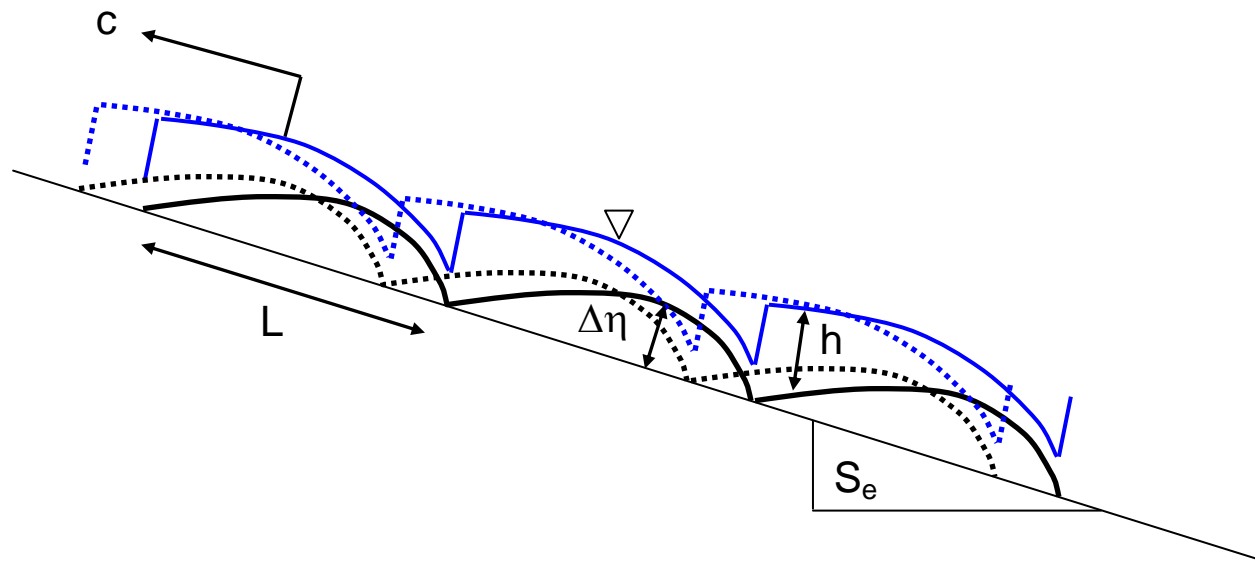
Boundary conditions

$$\tilde{x} = x - ct$$

Bed elevation

$$\eta_e|_{\tilde{x}=0} = 0$$

$$\eta_e|_{\tilde{x}=L} = 0$$



$$U|_{\tilde{x}_d=0} = U_t$$

Threshold flow

Flow velocity

$$U|_{\tilde{x}=L} = U_{\text{con}} \equiv \left(\frac{\sqrt{1 + 8\text{Fr}_t^2} - 1}{2} \right)^{-1} U_t$$

Conjugate depth of hydraulic jump

Boundary conditions contd.

Suspended sediment concentration

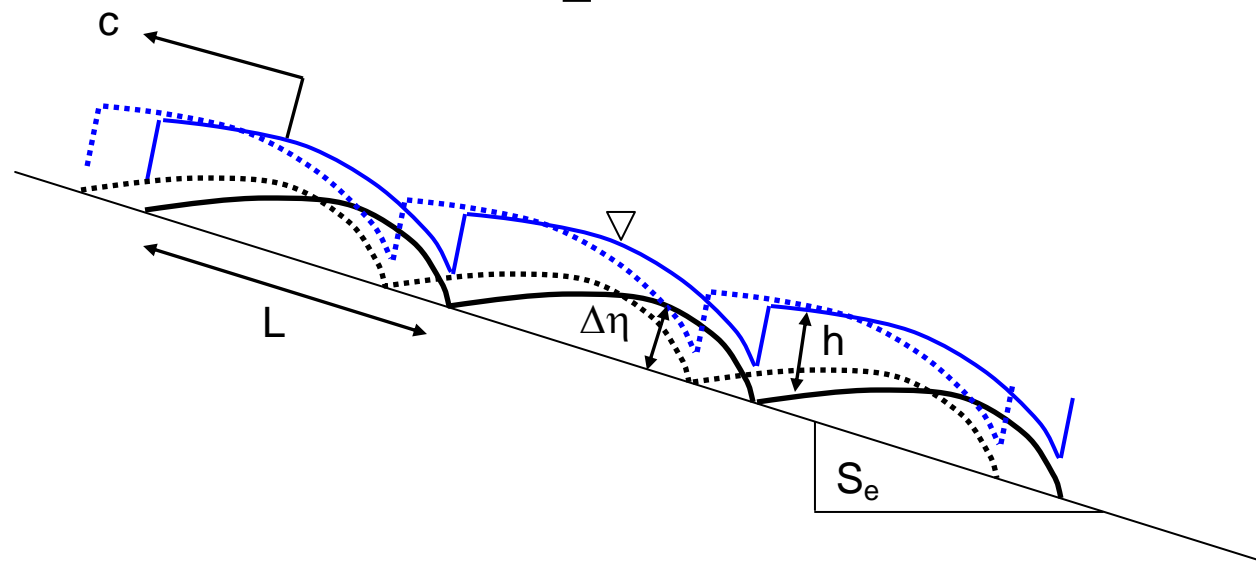
$$\frac{1}{T} q_w \int_0^{T_d} C dt = q_w C_{\text{feed}}$$

$$T = \frac{L}{c}$$

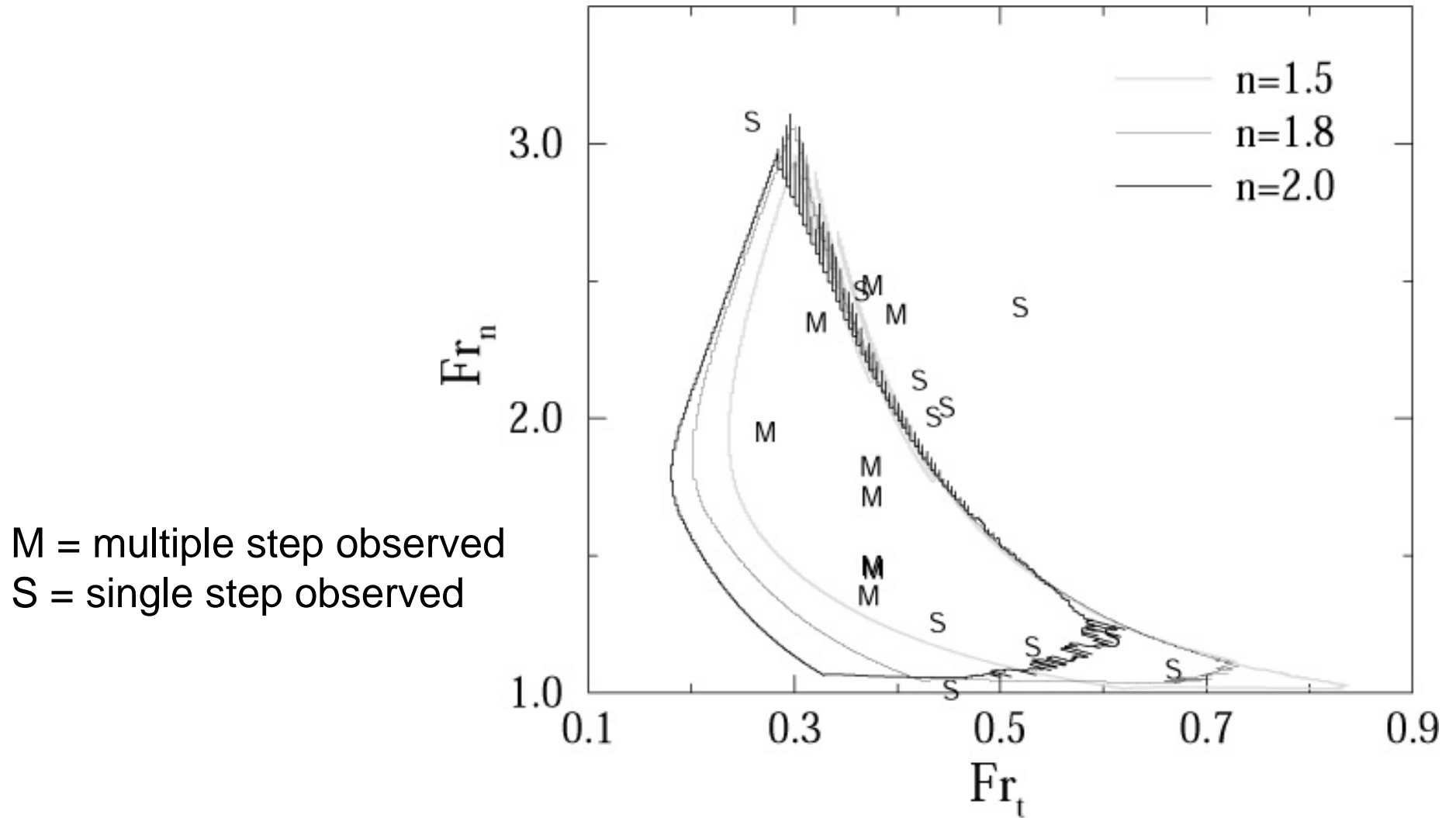
Suspended load averaged over one period = feed load

or thus

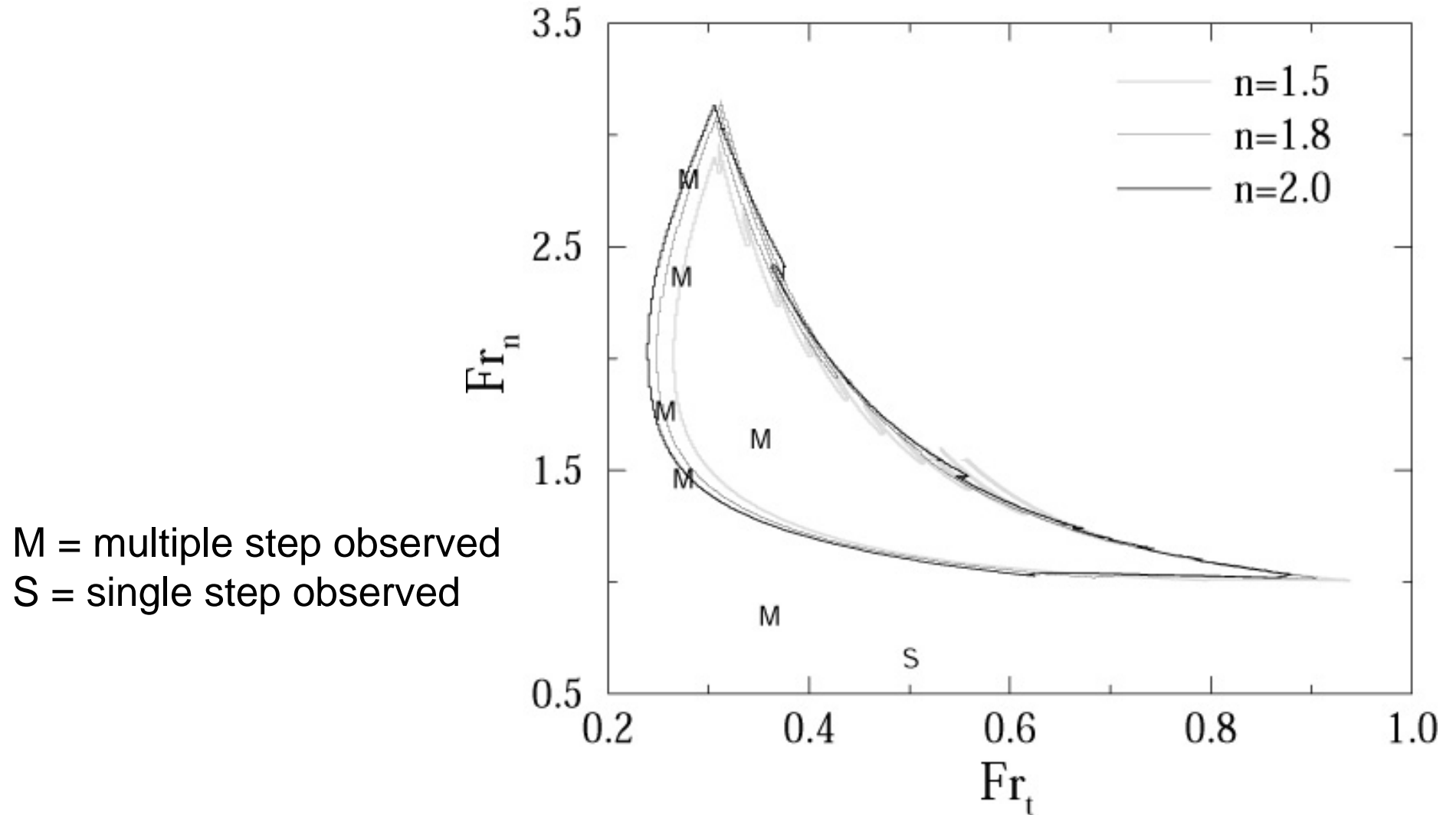
$$\frac{1}{L} \int_0^L C d\tilde{x} = \chi_{\text{feed}}$$



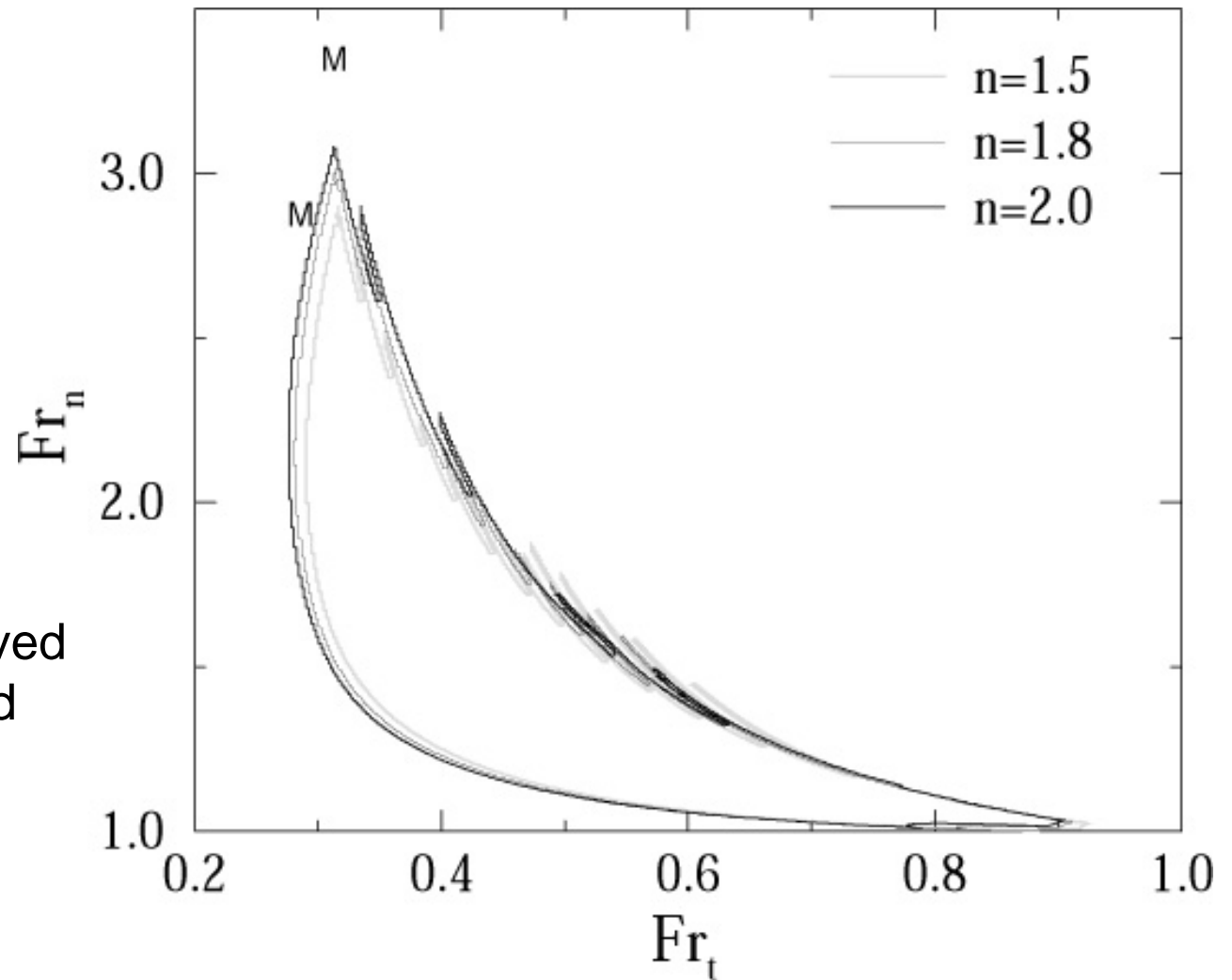
Predicted and observed region of formation of steps, 19 μm



Predicted and observed region of formation of steps, 45 μm

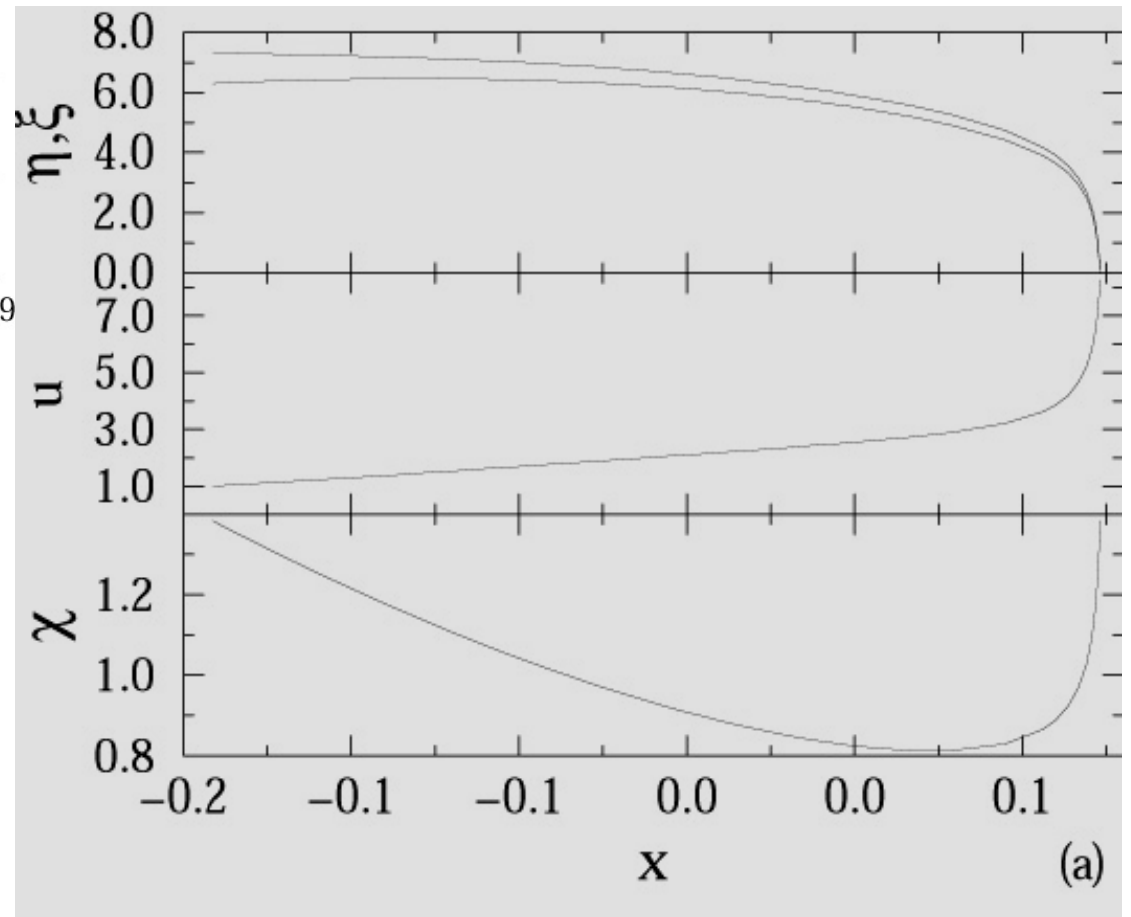
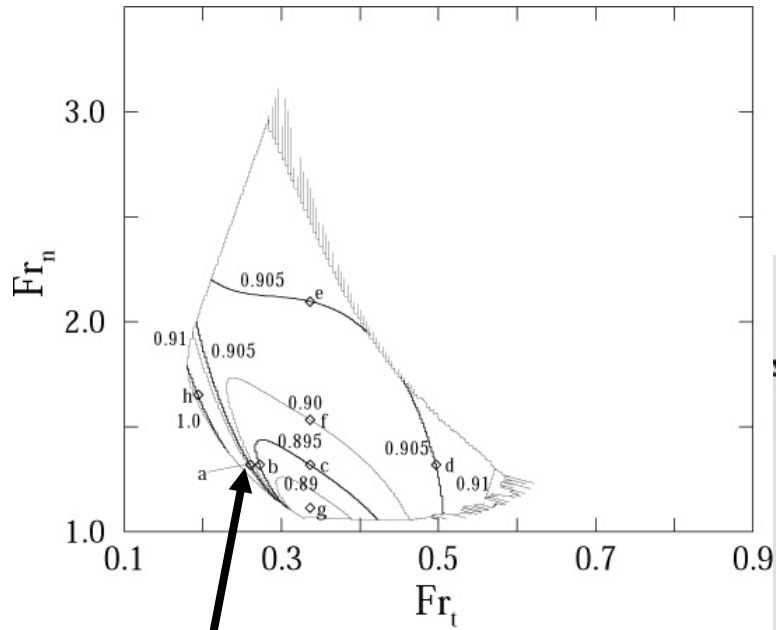


Predicted and observed region of formation of steps, 120 μm

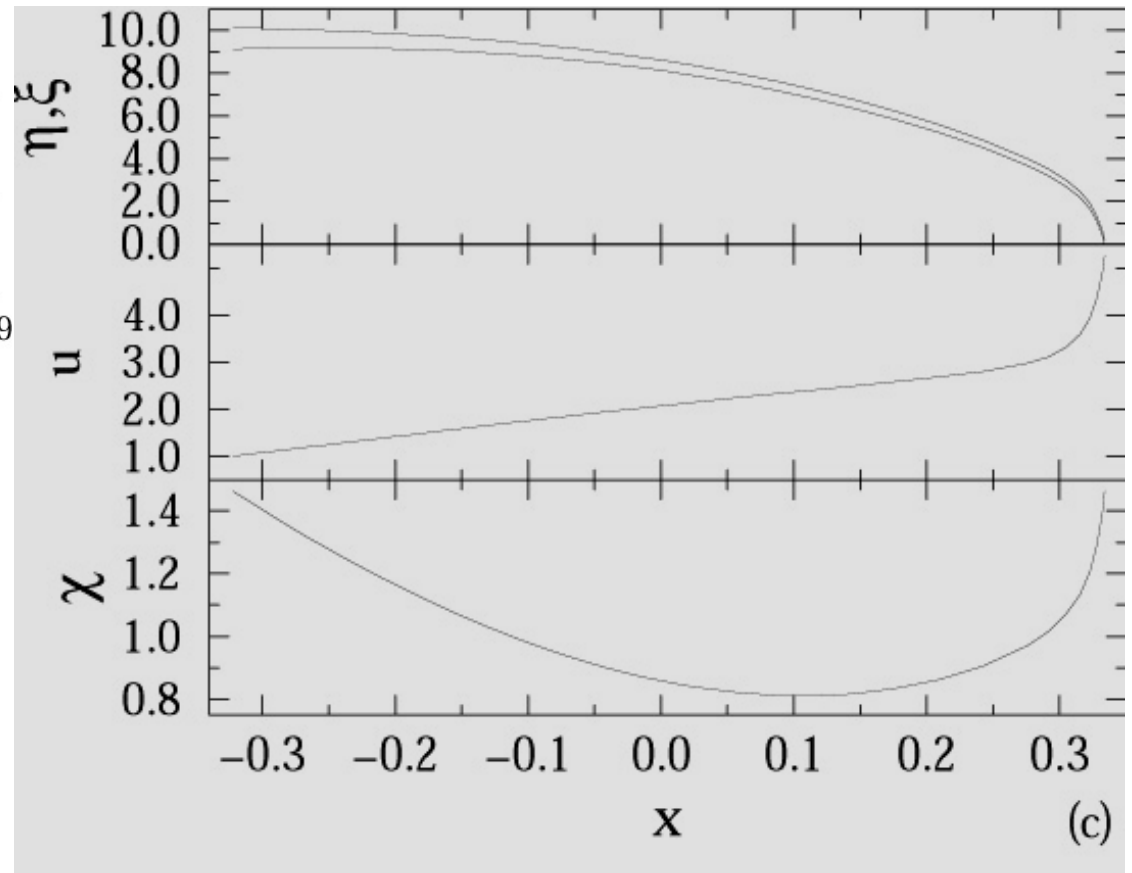
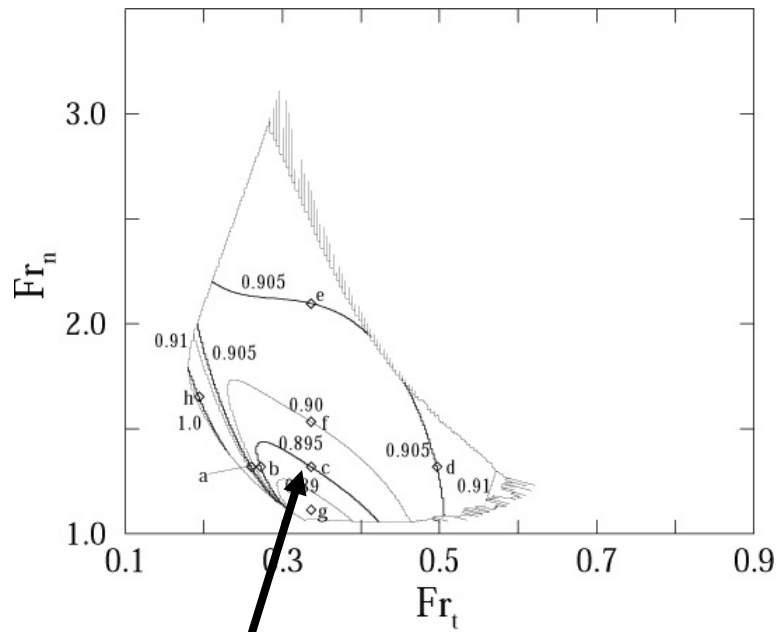


M = multiple step observed
S = single step observed

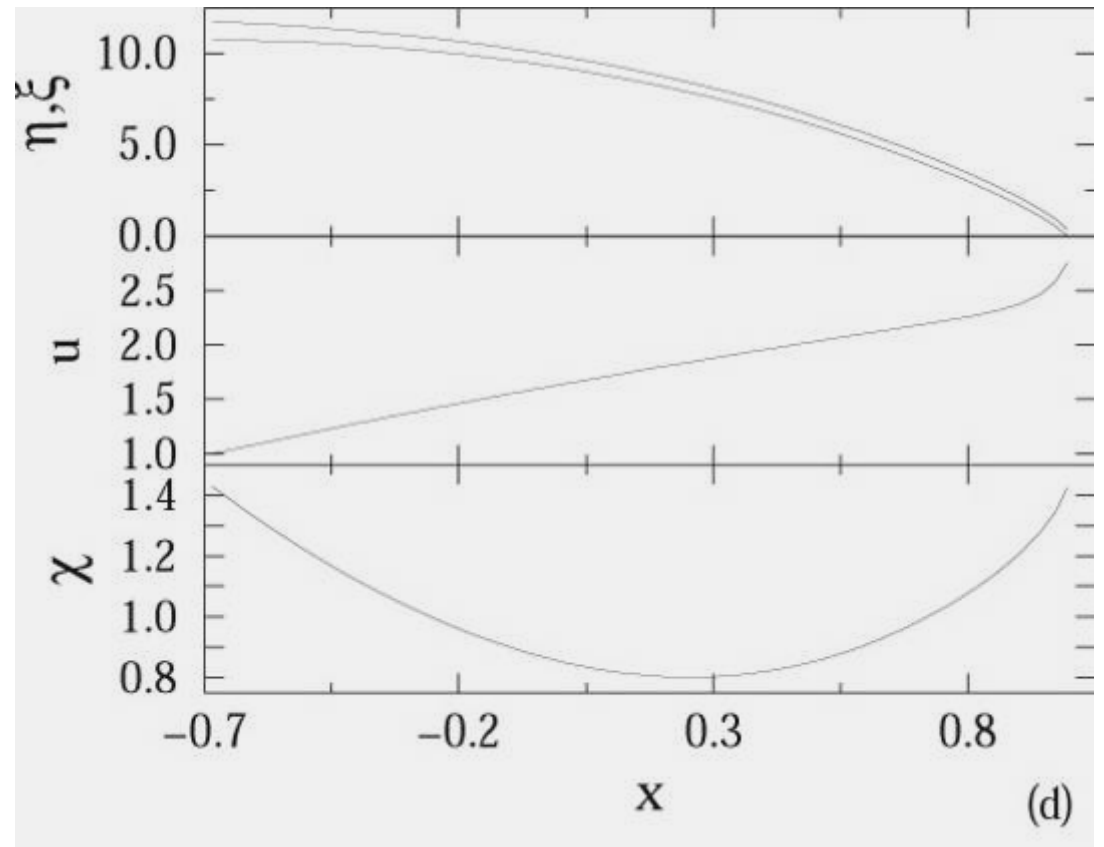
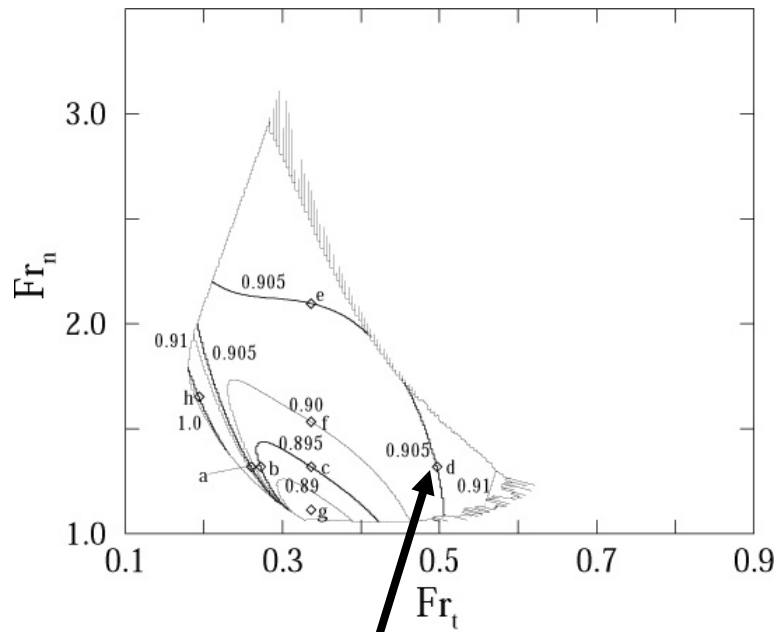
Predictions, 19 μm , point "a"



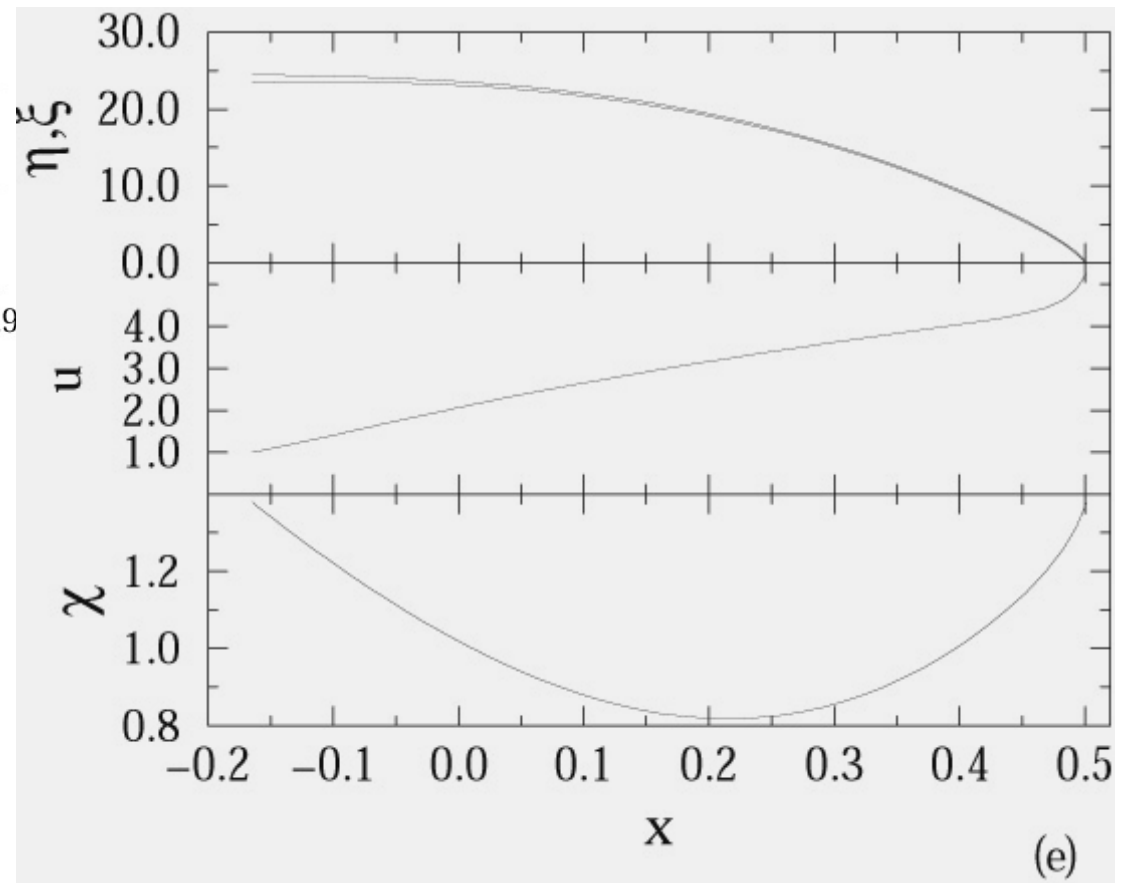
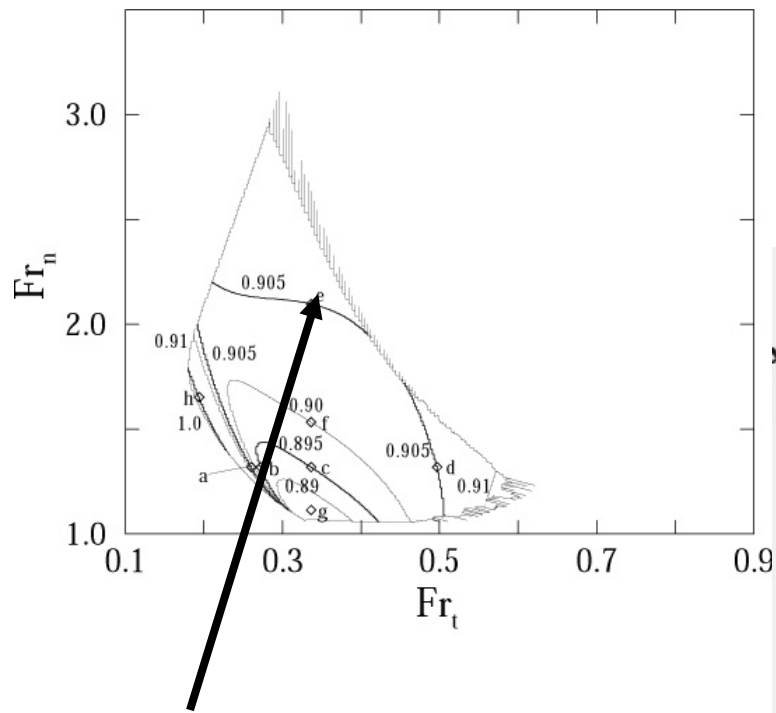
Predictions, 19 μm , point "c"



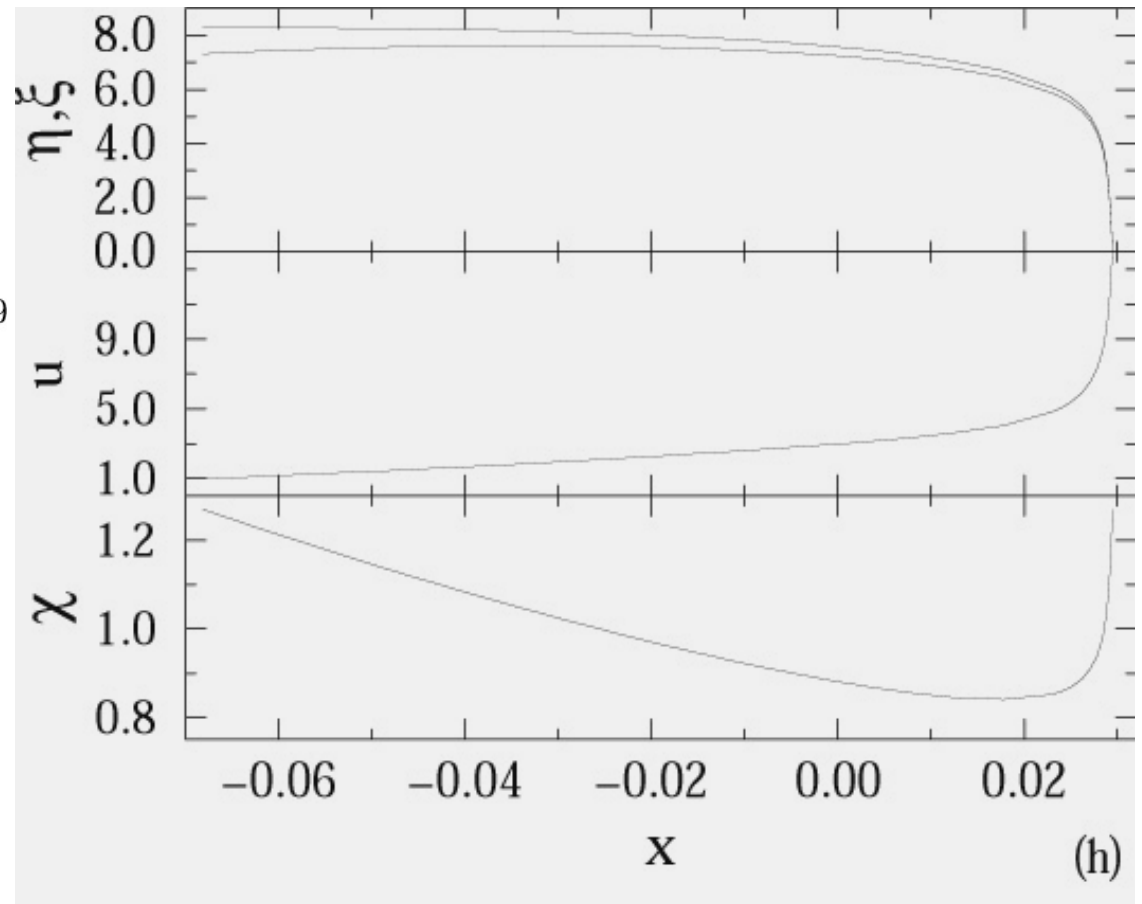
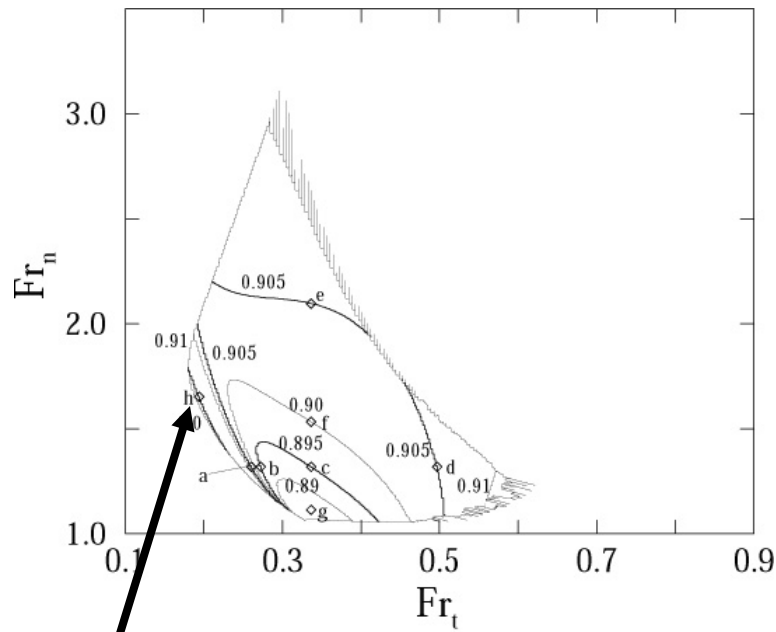
Predictions, 19 μm , point "d"



Predictions, 19 μm , point "e"

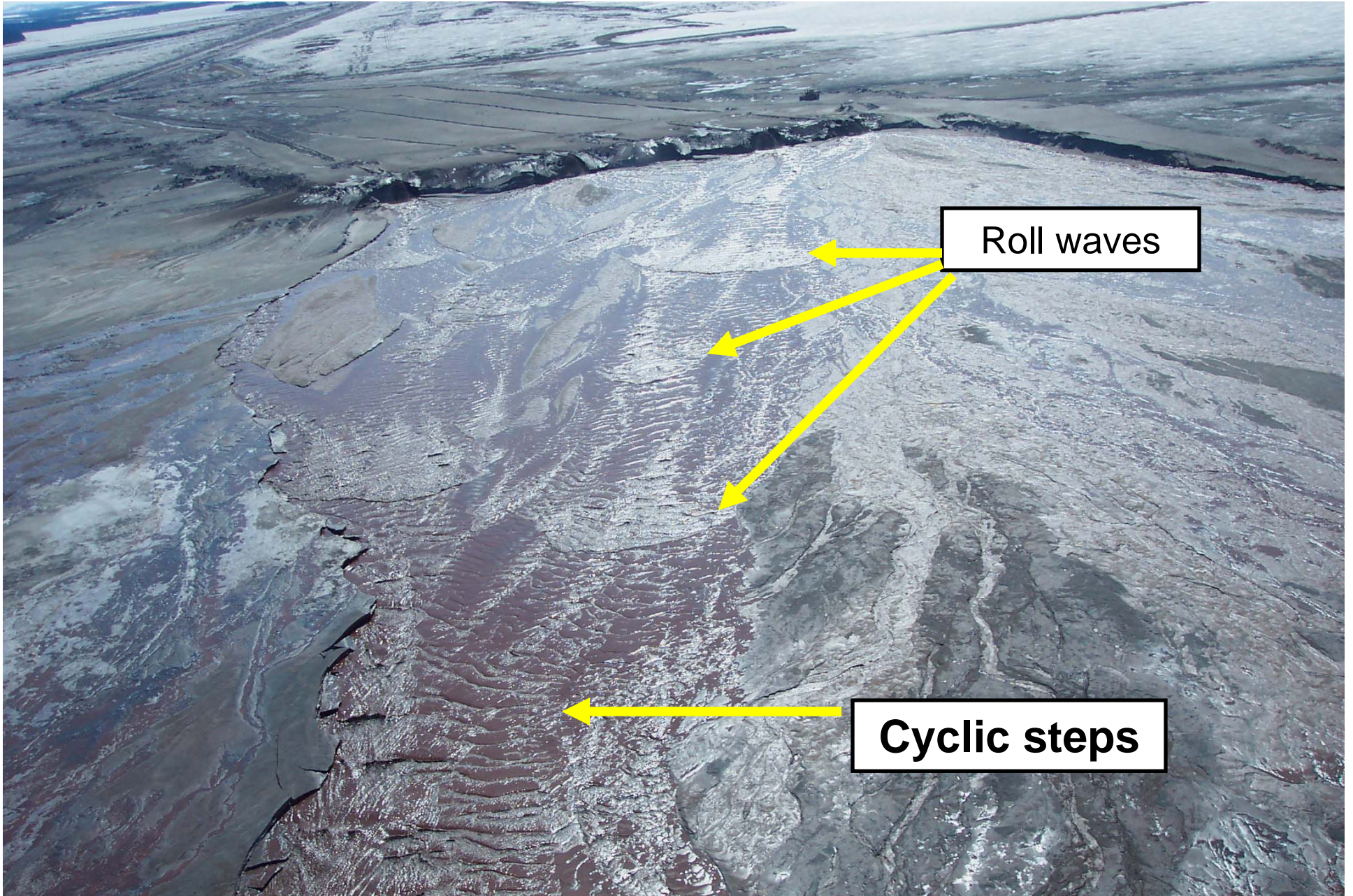


Predictions, 19 μm , point "h"



**Cyclic steps in the field:
Lake Wabush – tailings basin of an iron mine in Labrador**





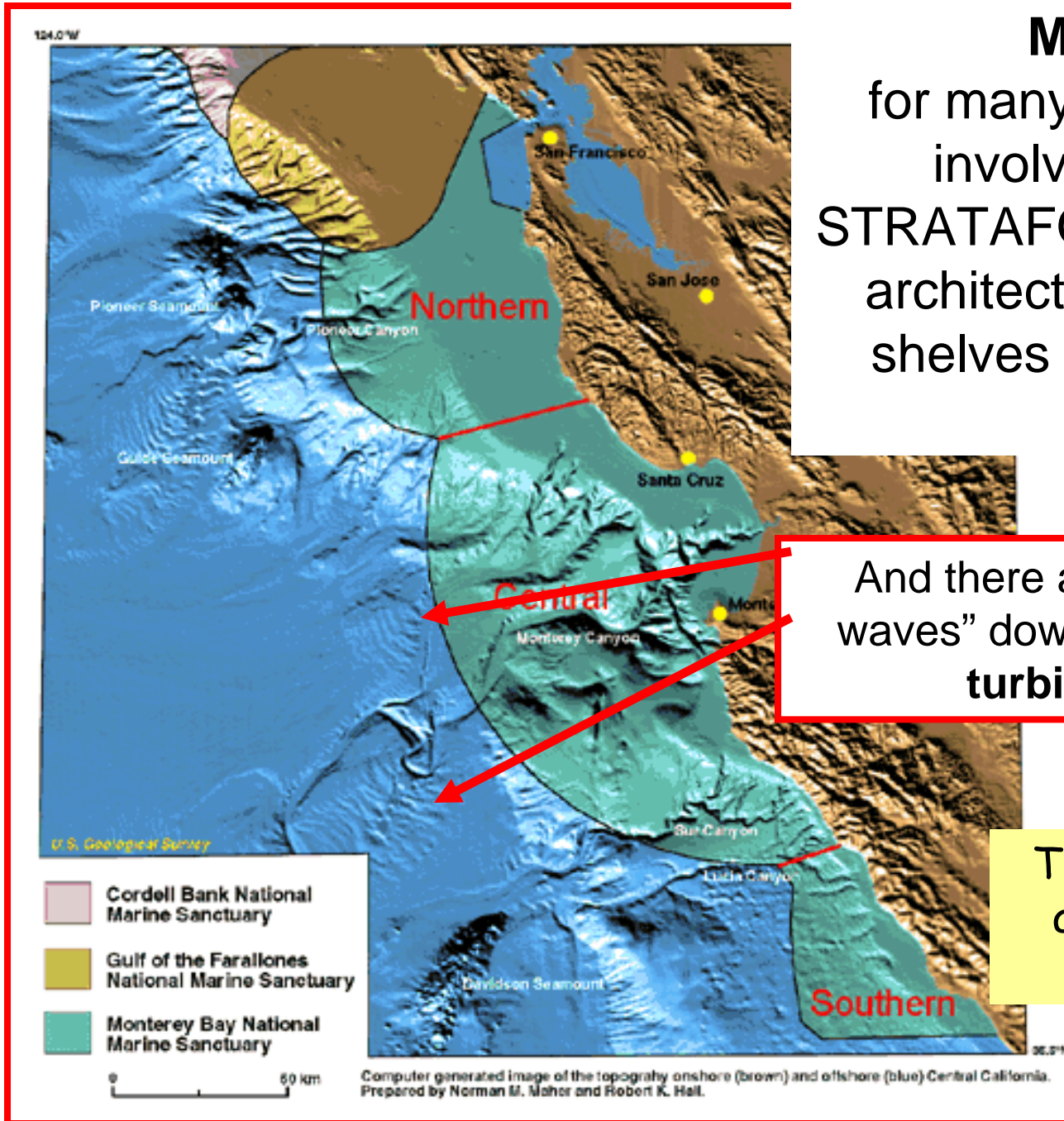
Roll waves

Cyclic steps

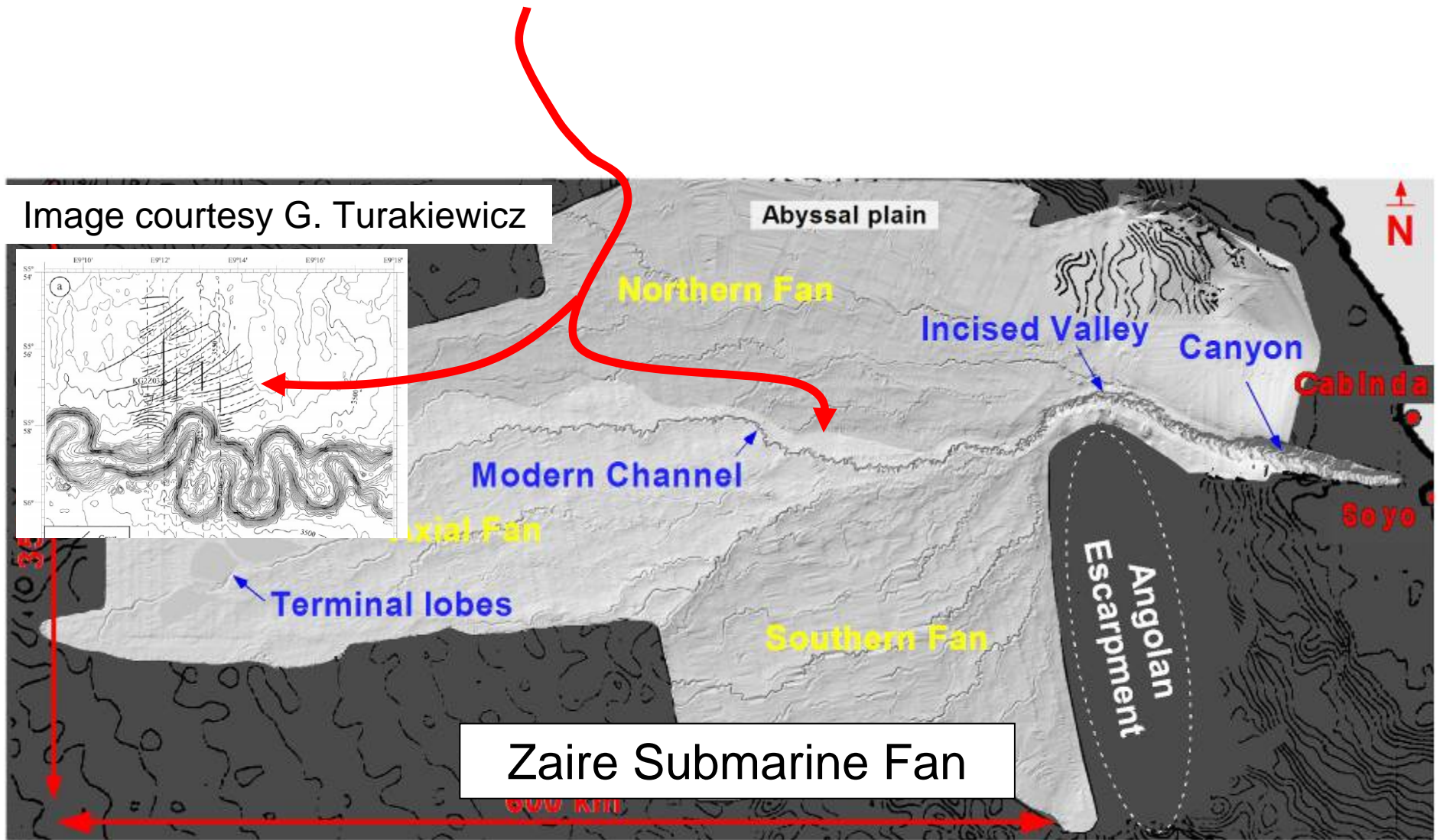
Meanwhile.
for many years I had been involved in the ONR STRATAFORM Project on the architecture of continental shelves and slopes in the ocean.

And there are lots of “sediment waves” down there, produced by **turbidity currents.**

These sediment waves are common at slope breaks

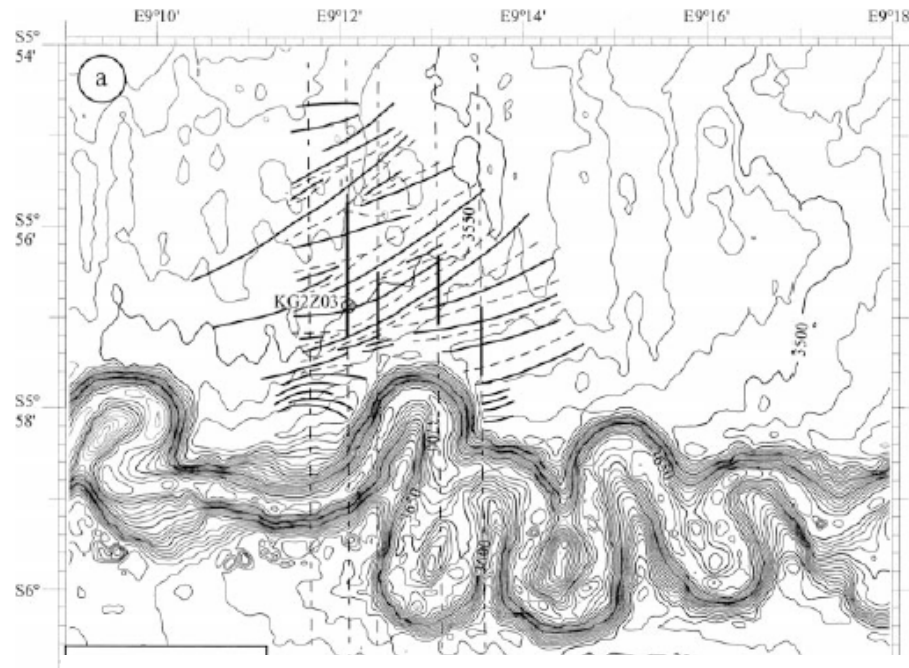


“Sediment waves” are also common on the back side of levees of channels on submarine fans.

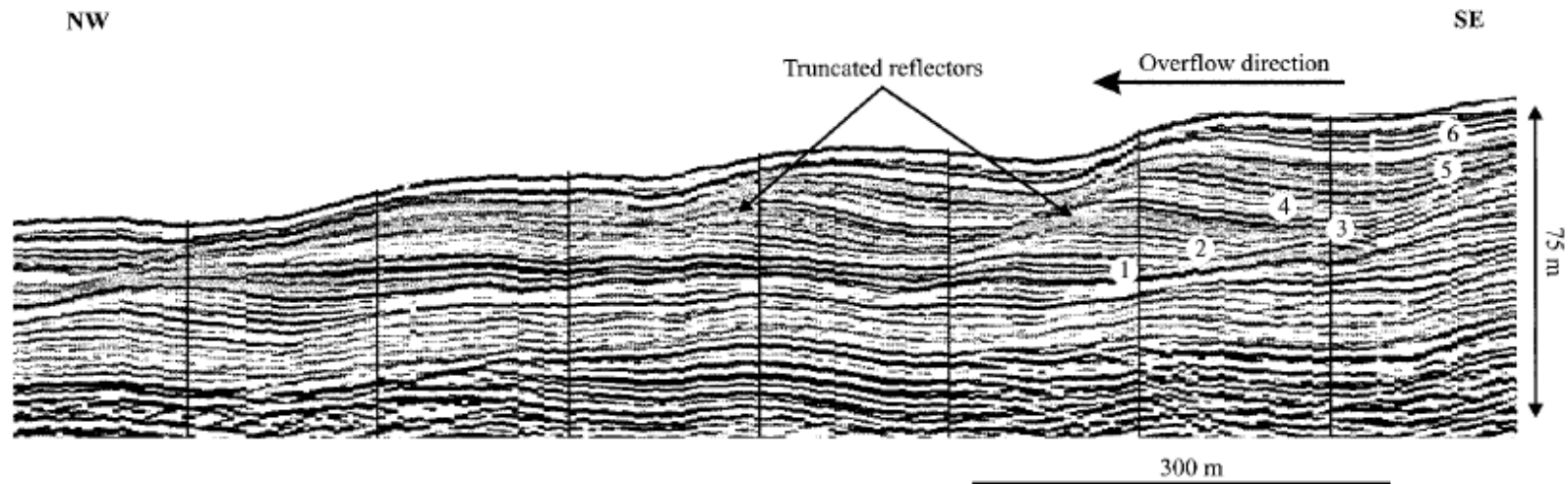


EM12 Bathymetry of the Pleistocene to Present Zaire Deep-Sea Fan.

The “sediment waves” invariably migrate upstream in orderly trains.



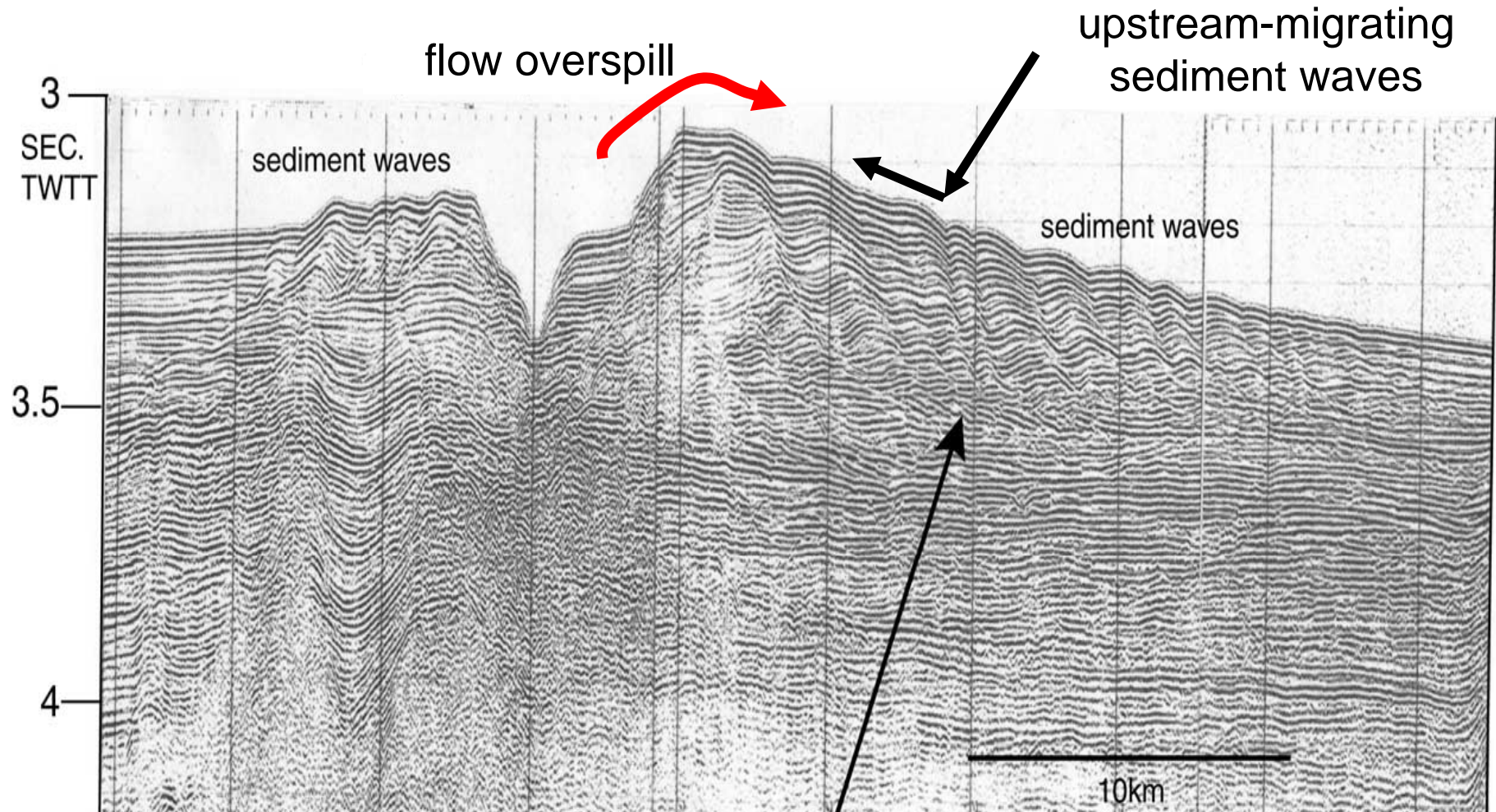
From Savoye et al.



The turbidity currents that create these waves constitute submarine analogs of rivers.

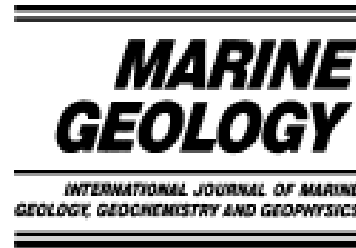


Sediment waves have been commonly interpreted as “antidunes.”



Levee on the Toyama Submarine Channel.
Main flow is into the page.

But the trains are so orderly and so persistent that I wondered whether they might be cyclic steps instead.



Distinguishing sediment waves from slope failure deposits:
field examples, including the 'Humboldt slide',
and modelling results

Homa J. Lee^{a,*}, James P.M. Syvitski^b, Gary Parker^c, Daniel Orange^d,
Jacques Locat^e, Eric W.H. Hutton^b, Jasim Imran^f

and then forgot about them.

Meanwhile others (Kubo and Syvitski, Fedele and Garcia)
were pursuing the antidune analogy.

But I still had an itch.

Rivers with supercritical flow ($\mathbf{Fr} > 1$) are often so steep that they have trouble maintaining an alluvial cover.

$$\frac{dU^2h}{dx} = -\frac{1}{2}g\frac{dh^2}{dx} - ghS - C_fU^2 \quad , \quad \mathbf{Fr} = \frac{U}{\sqrt{gh}}$$

In the case of dilute-concentration ($C \ll 1$) turbidity currents, however, only gravity operating on suspended sediment pulls the flow downslope. The corresponding Froude number is the densimetric Froude number \mathbf{Fr}_d .

$$\frac{dU^2h}{dx} = -\frac{1}{2}Rg\frac{dCh^2}{dx} - RCghS - C_fU^2 \quad , \quad \mathbf{Fr}_d = \frac{U}{\sqrt{RCgh}}$$

$$R = \frac{\rho_s}{\rho} - 1 \cong 1.65 \quad , \quad C \ll 1$$

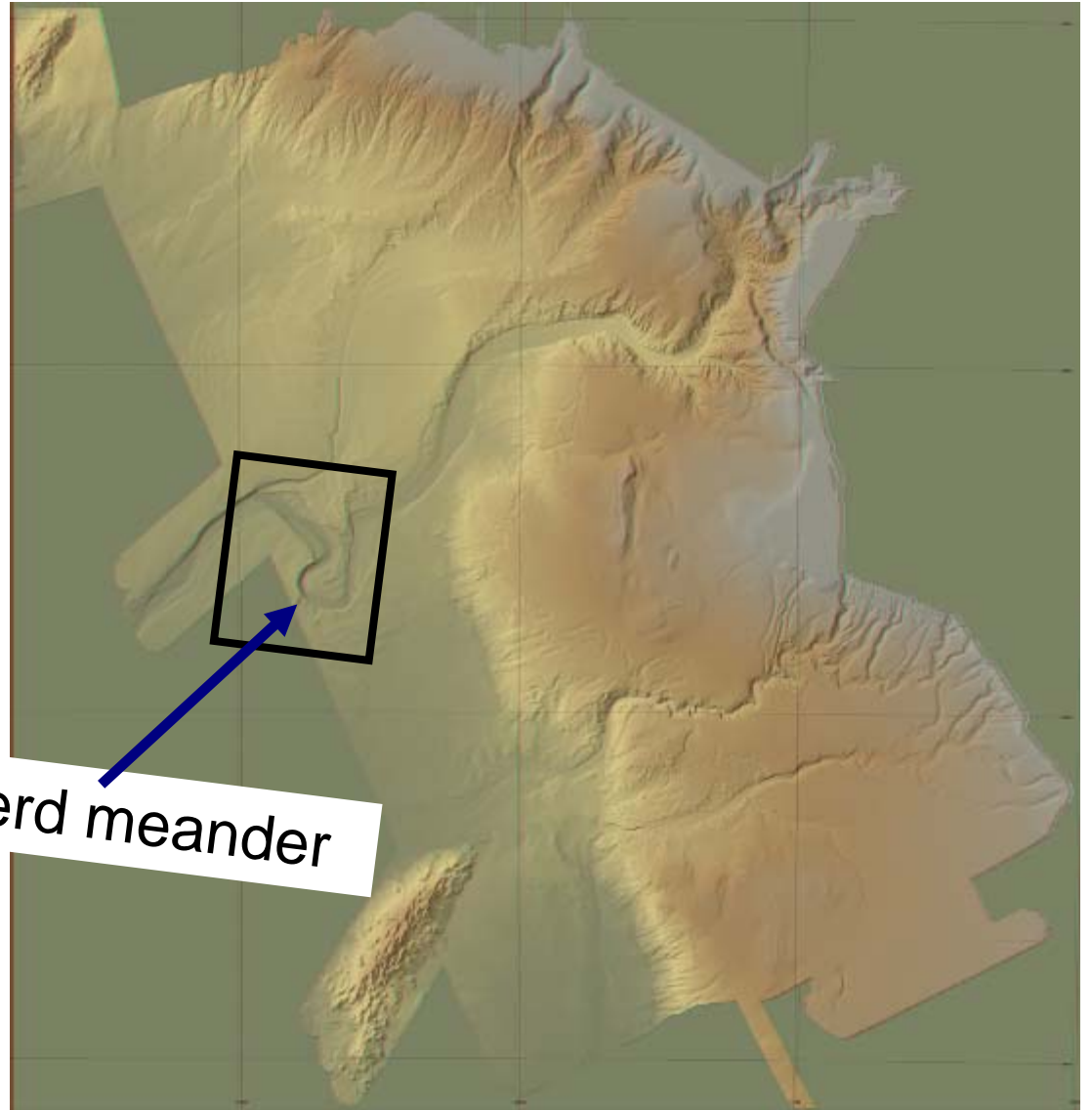
Since volume concentration $C \ll 1$, turbidity currents with the same U and h as a river are more likely to be supercritical.

So I asked my colleague and expert in submarine morphology, Bill Normark, if he had ever seen erosional steps in the submarine environment.

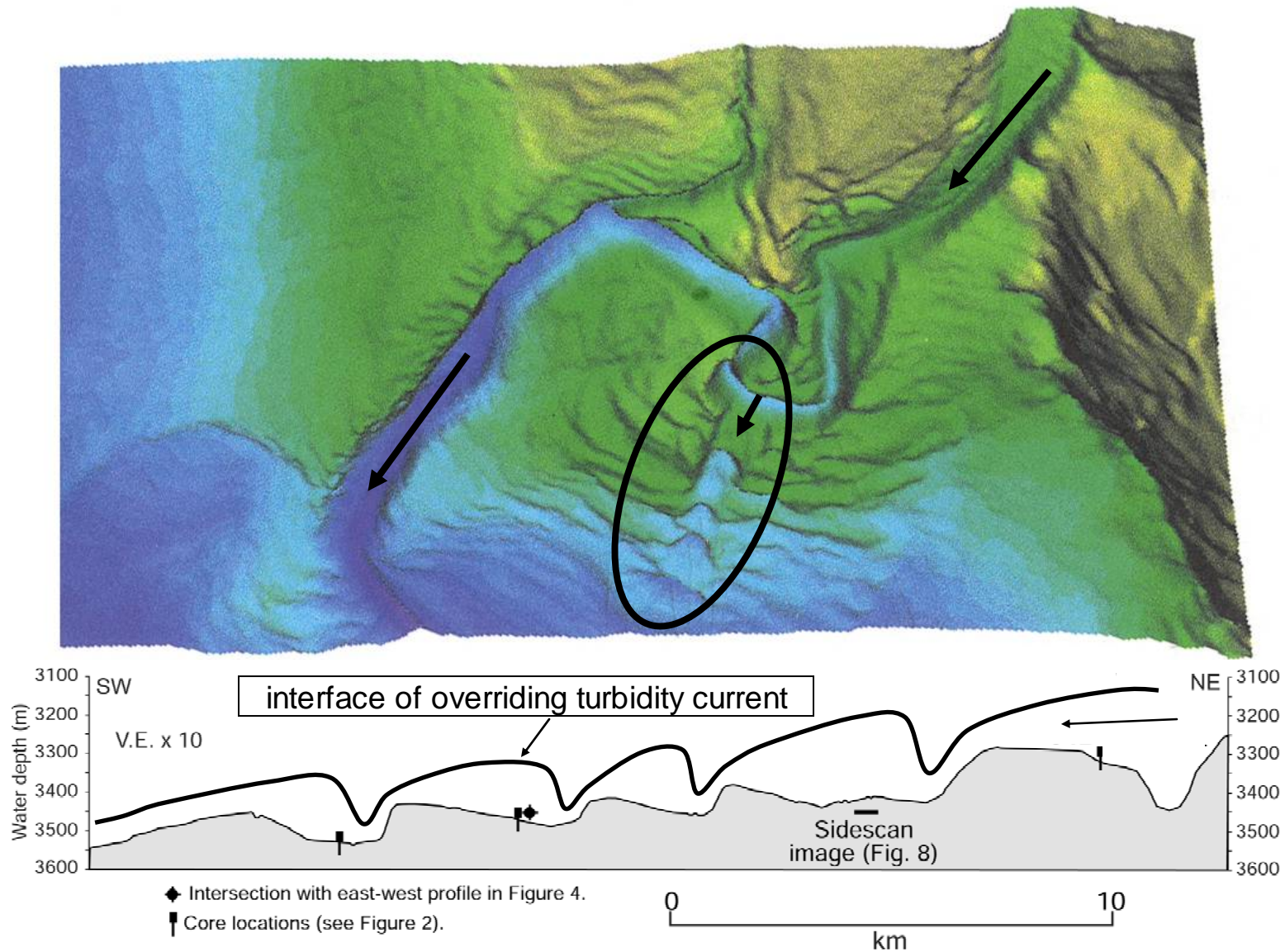
Within a few months he was back to me.

Monterey
Submarine Canyon

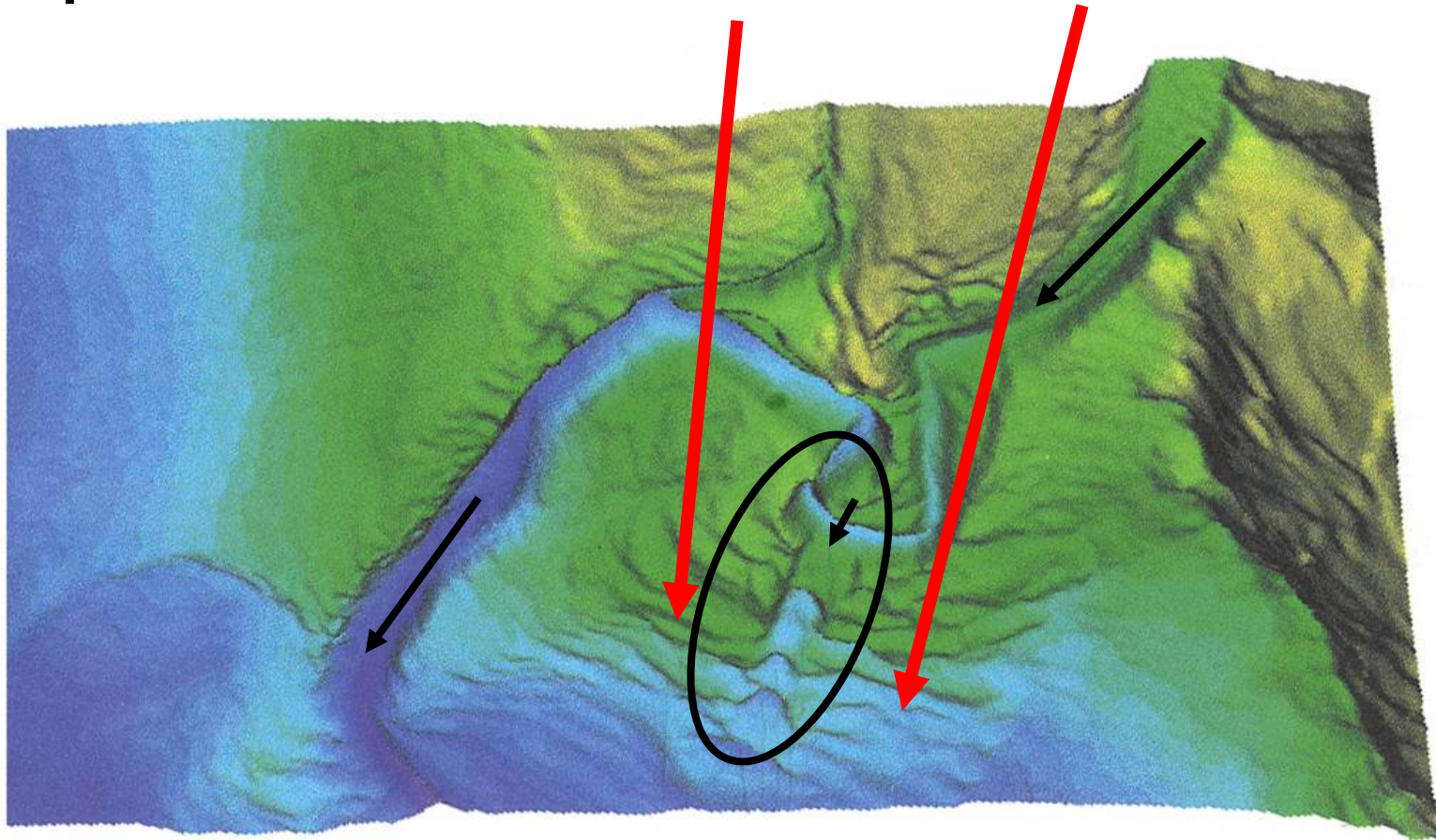
Sheperd meander



Subaqueous erosional cyclic steps incised into the outer levee of the Sheperd meander?

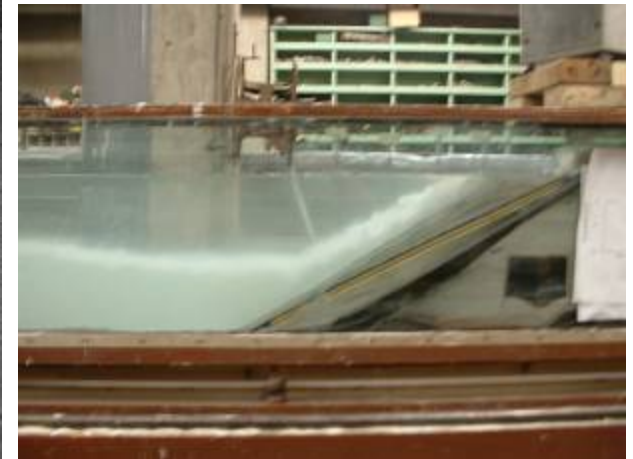
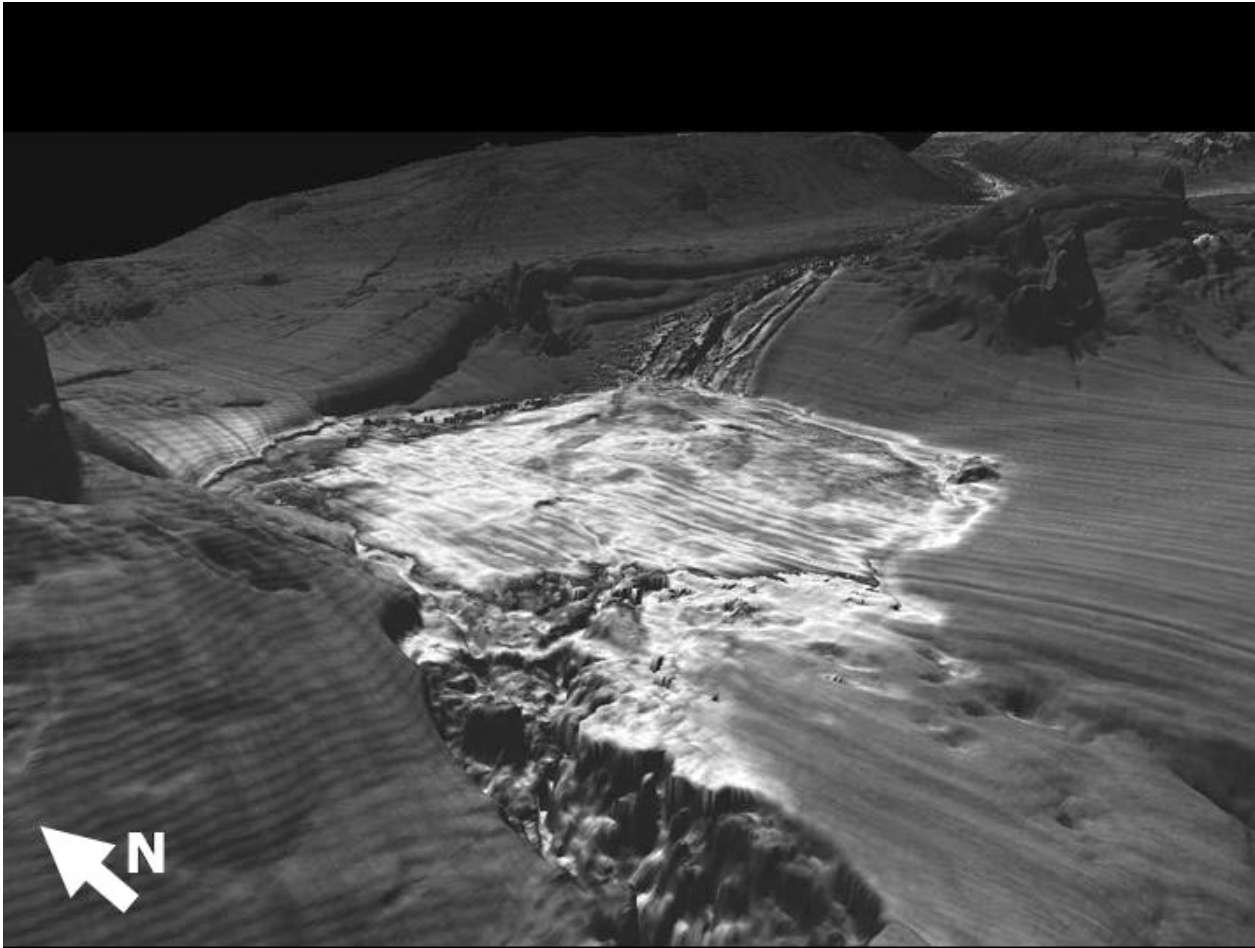


But my colleague David Mohrig pointed out to me that the erosional steps may simply reflect the imprint of antecedent sediment waves on the levee.

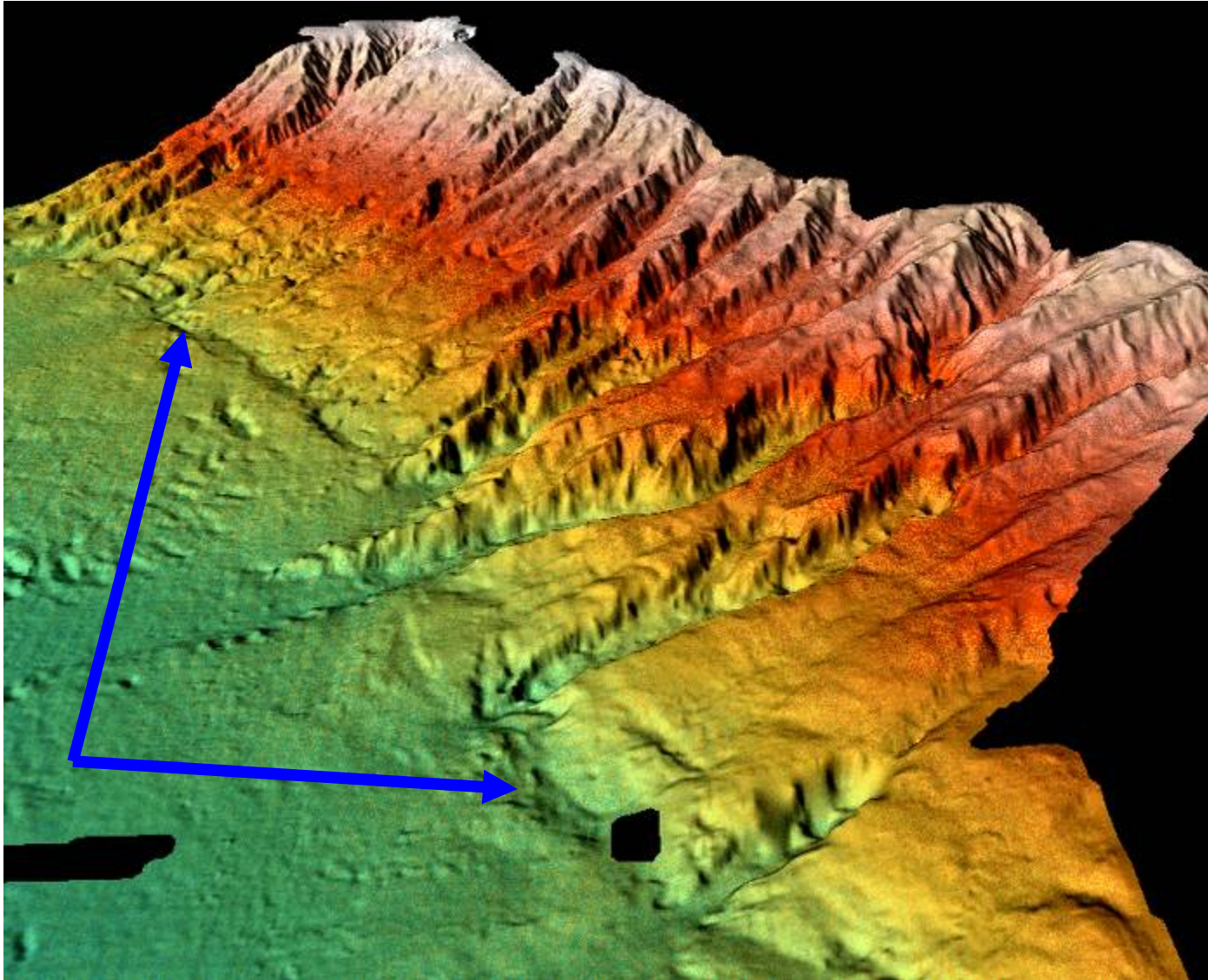


In the event, Bill Normark, Andrea Fildani and I decided to write a descriptive paper of the train of scours that suggested that they might be erosional submarine cyclic steps.

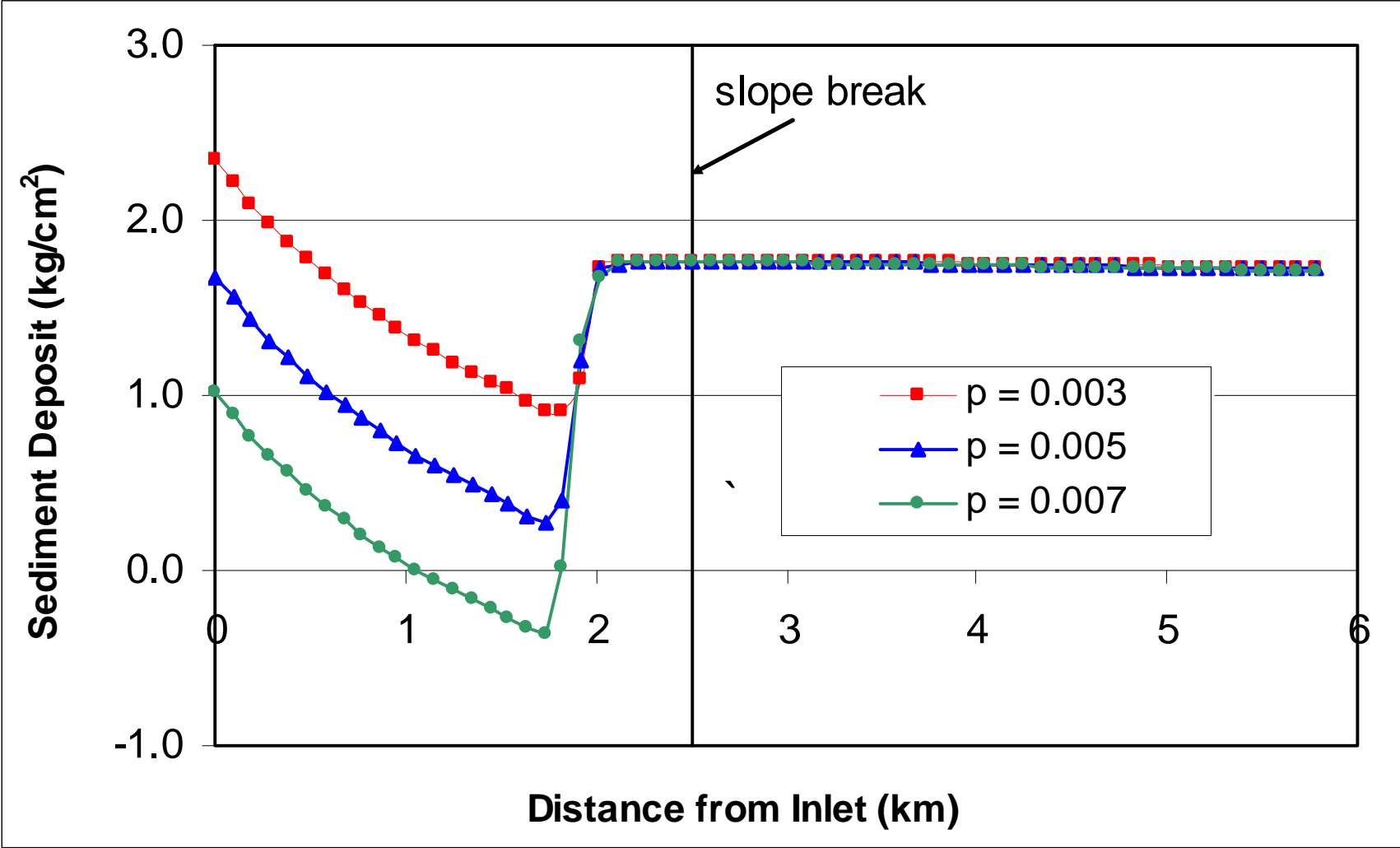
Meanwhile, my postdoctoral researcher in California, Svetlana Kostic, was working away at the depositional response of a turbidity current to a slope break.



We had reason to expect a backward-facing step at a slope break,, such as at the base of the continental slope off New Jersey,



And we got one from the numerical model.



Flow

But in October, 2004,

I got an e-mail from Svetlana Kostic saying that she was either

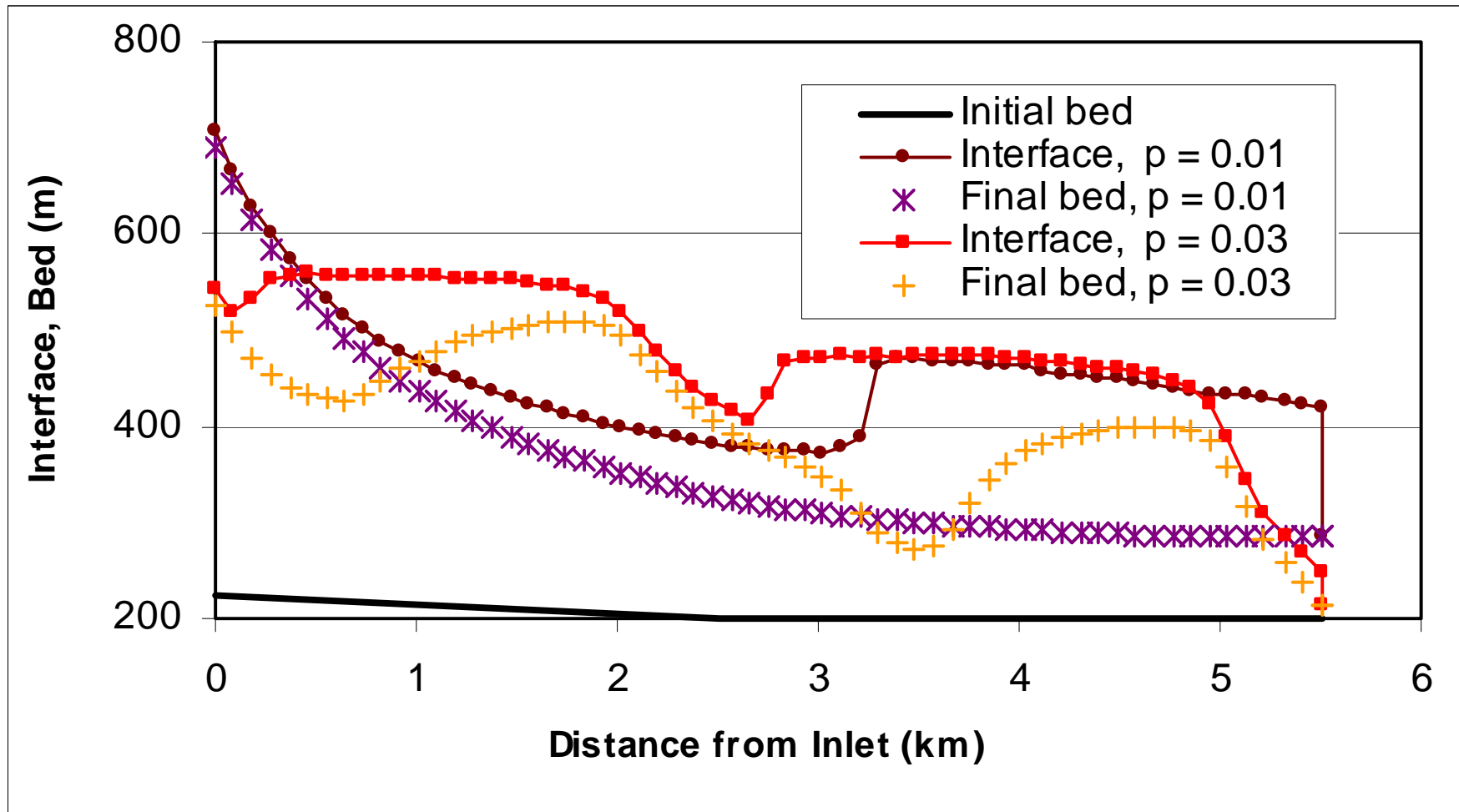
a) having trouble, or

b) getting interesting results using the same model (4-eqn model) of turbidity currents that Juan Fedeles and Marcelo Garcia had used to study antidunes created by density flows.

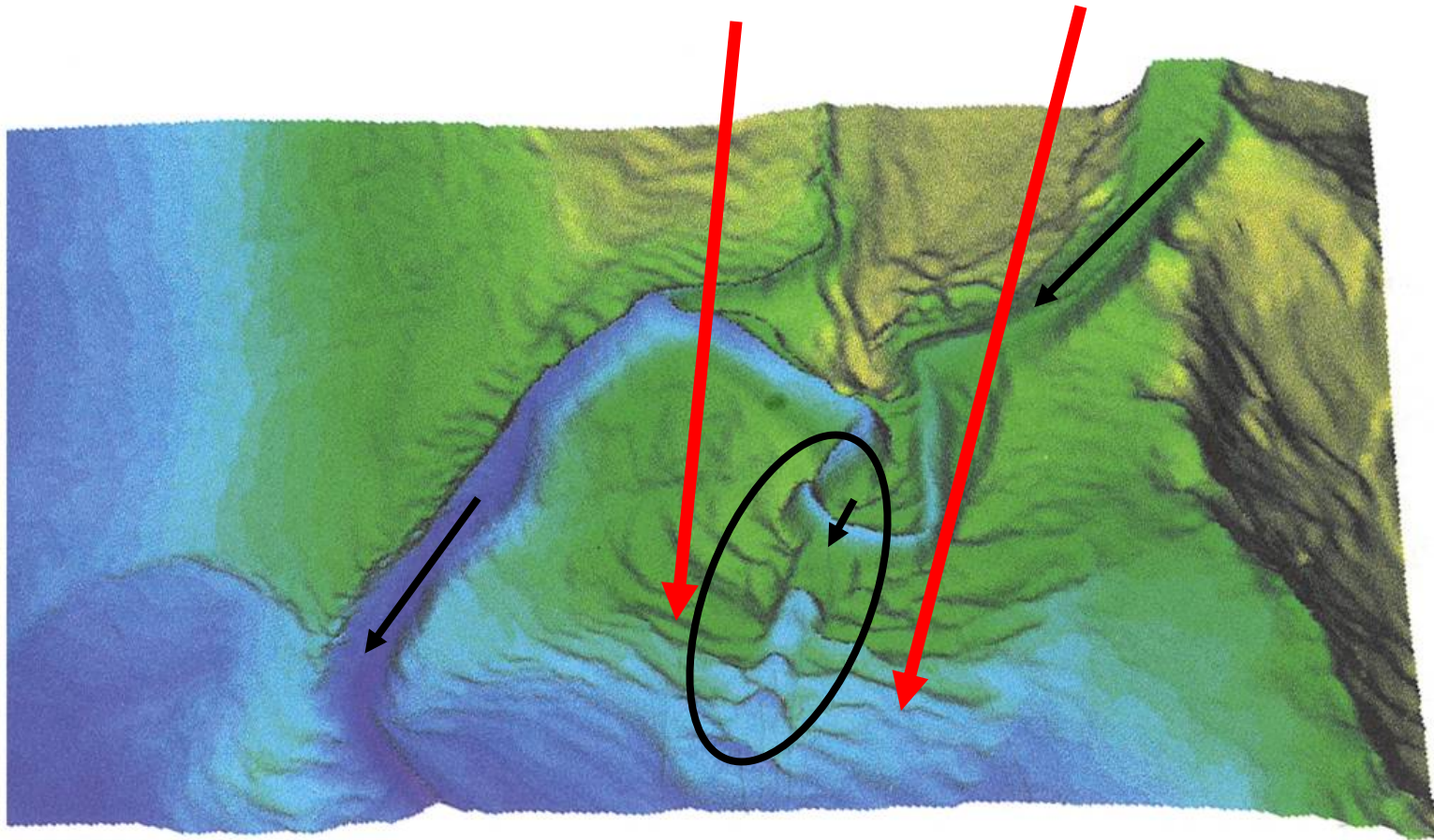
I was really busy.

It took me two months to read the e-mail.

When I finally read it, there they were:
Net *depositional* cyclic steps in the submarine
environment triggered by a slope break!



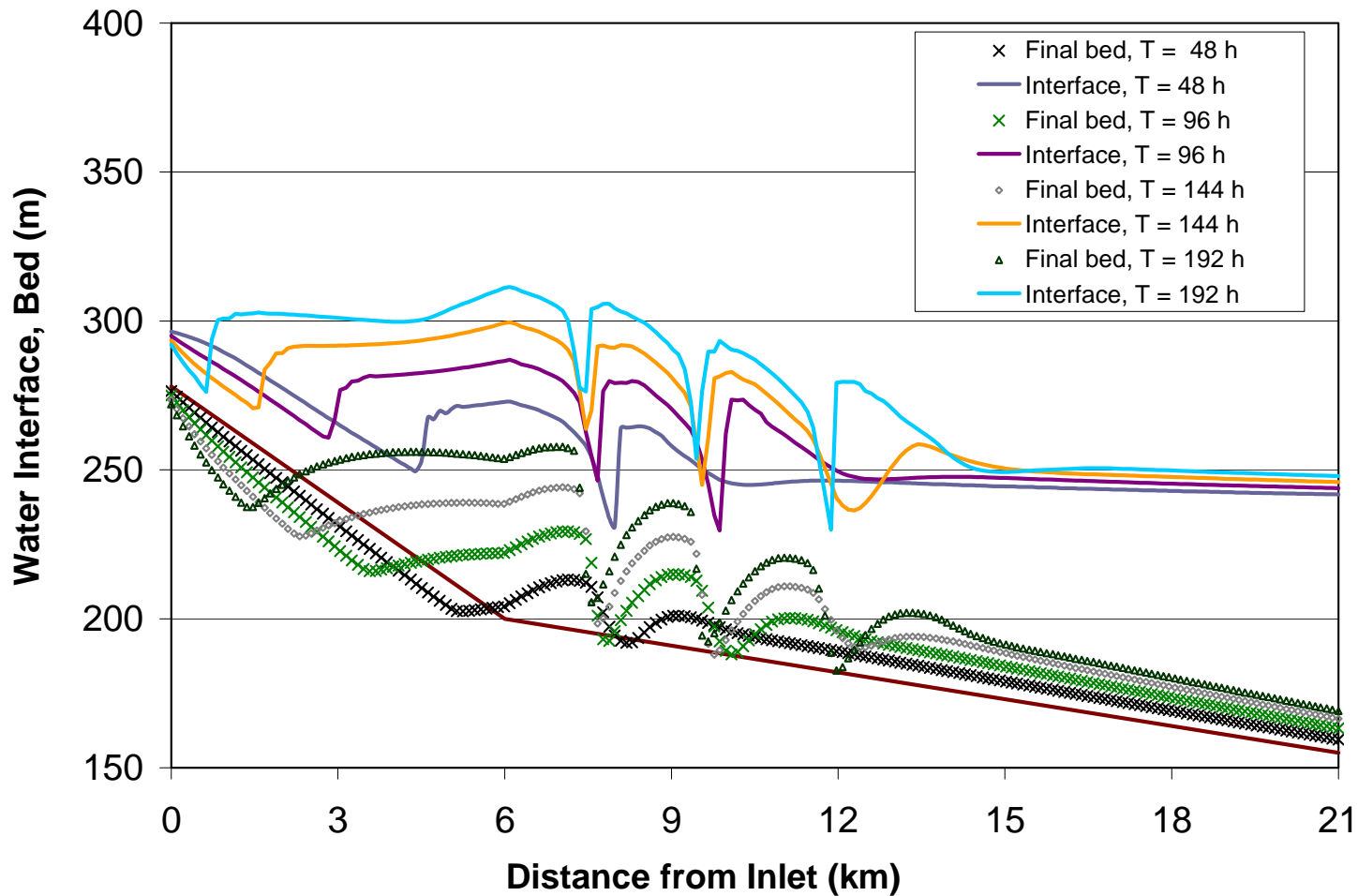
So in a sense David Mohrig was right: the scour holes and sediment waves are closely related.



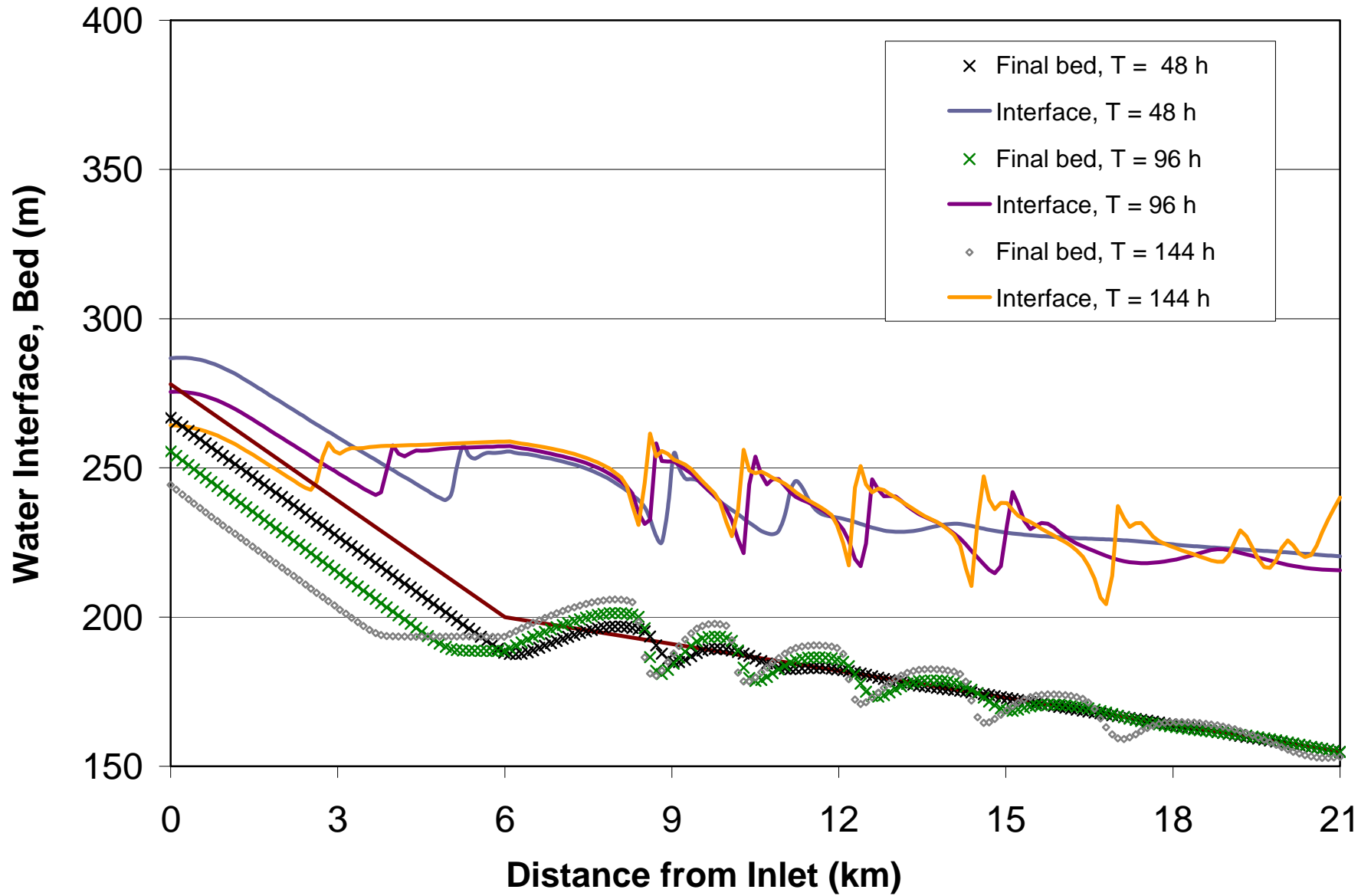
They are both cyclic steps, one net depositional and one net erosional.

And the numerical model, run for reasonable inputs for a sheet turbidity current spilling over the levee of the Monterey channel, yields both types!

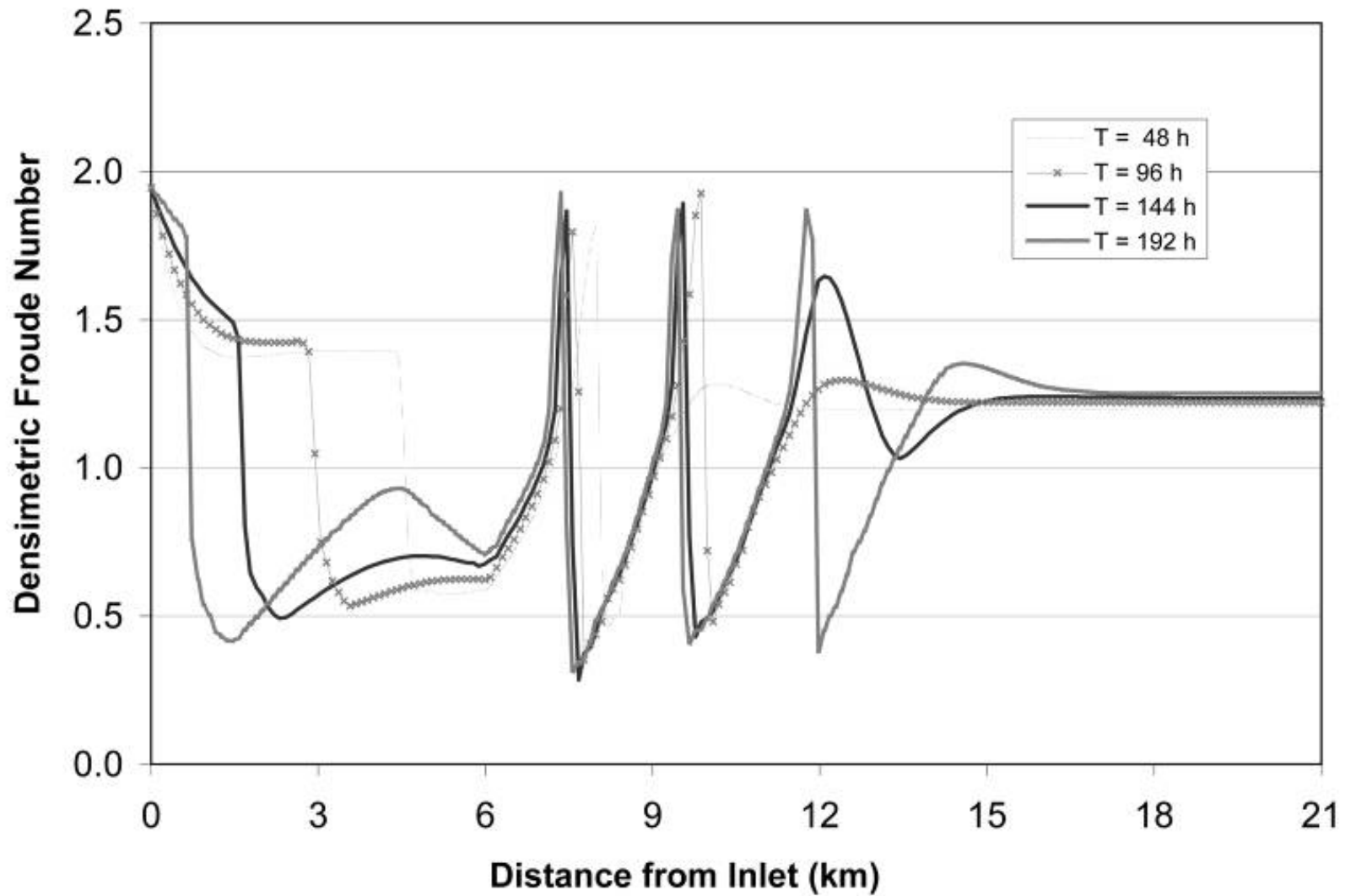
Net depositional,



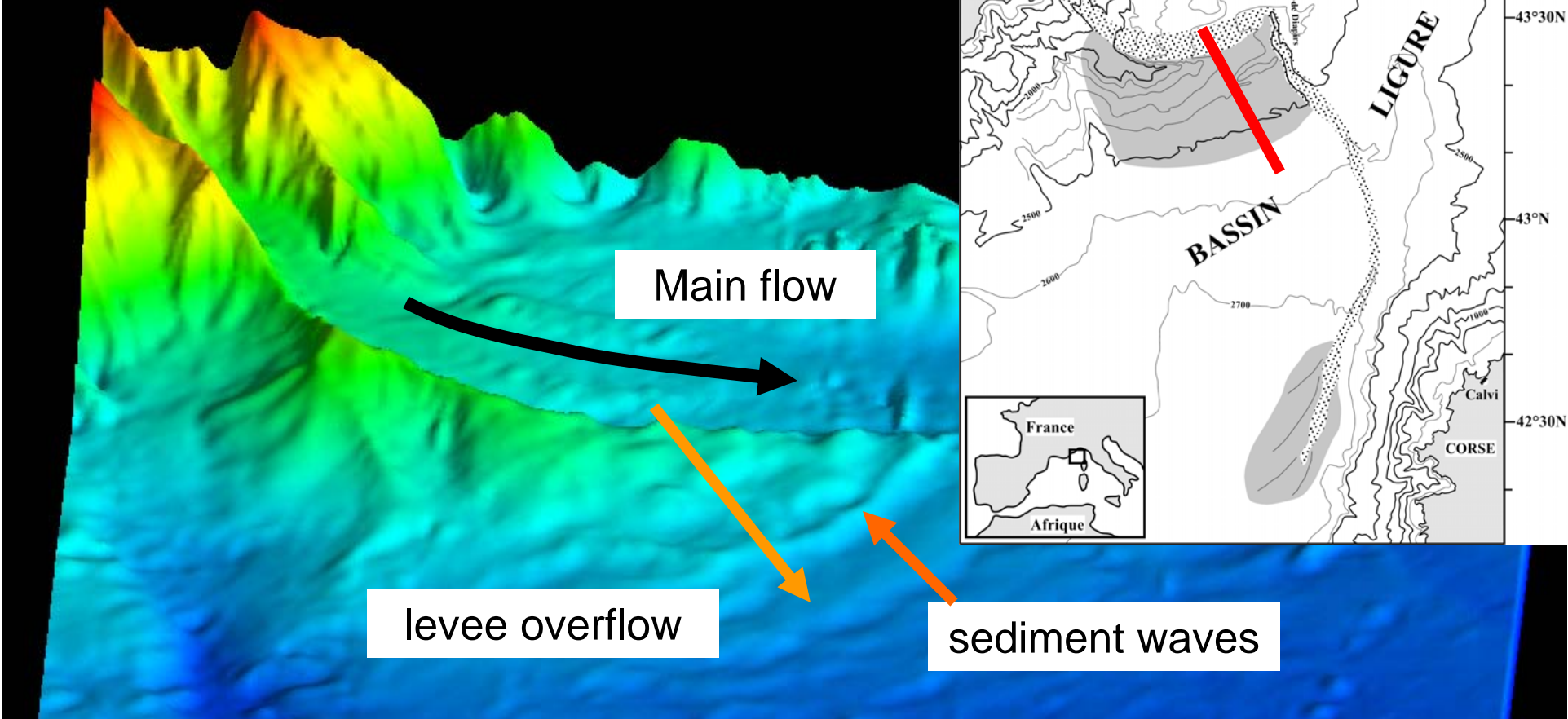
and net erosional.



And yes, the steps migrate upstream and are bounded by hydraulic jumps!

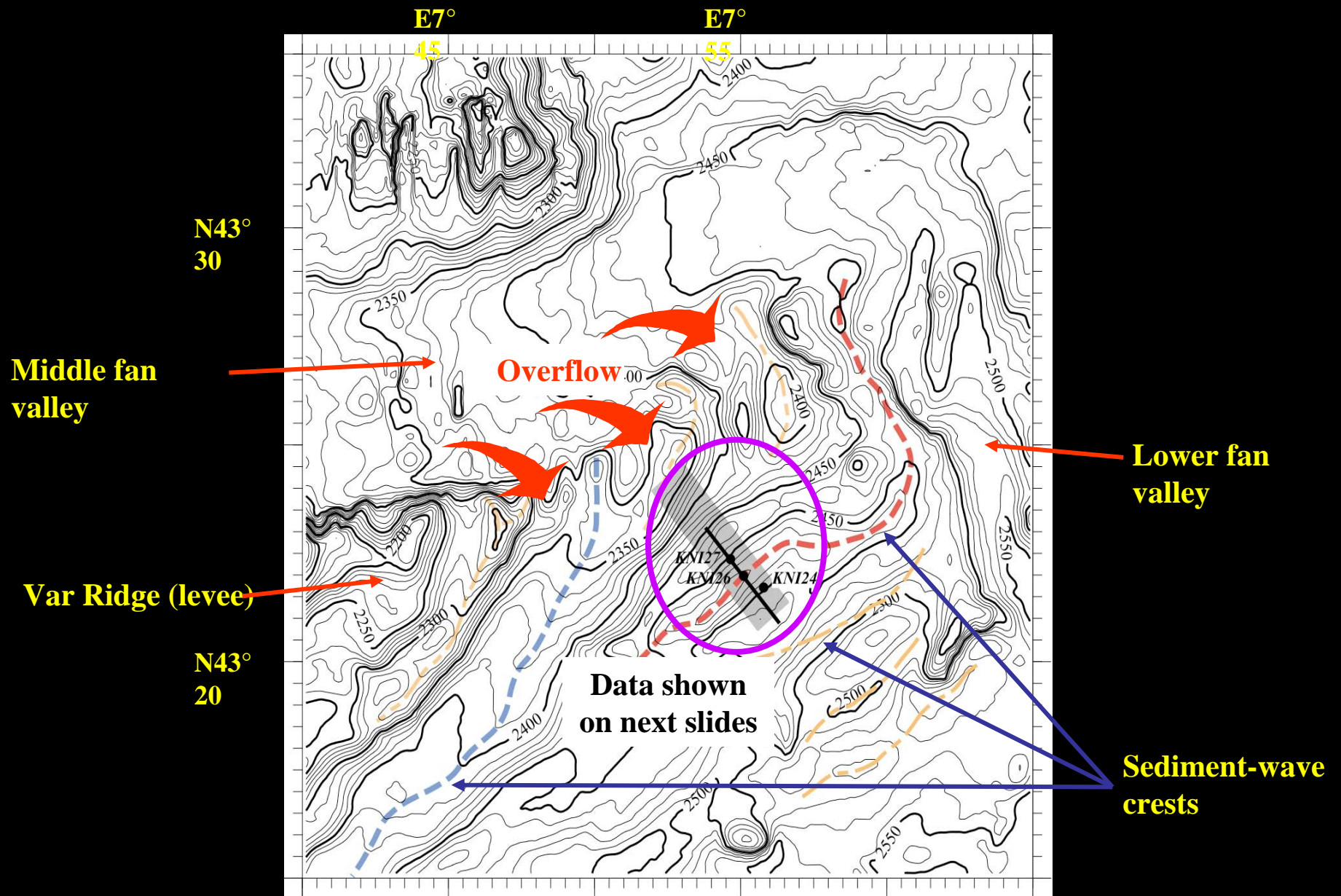


The Var Sedimentary Ridge

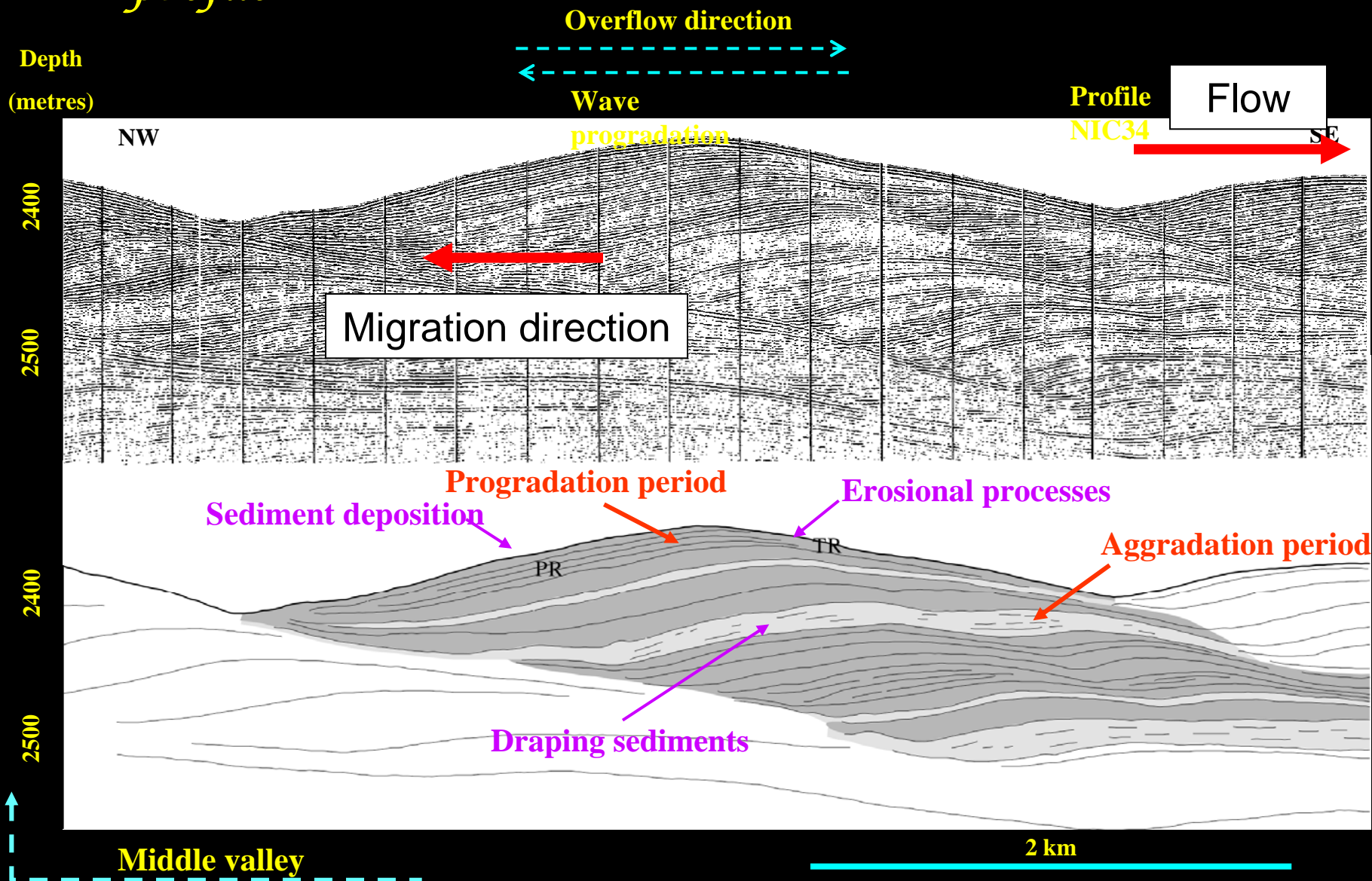


The Var Sedimentary Ridge: a huge (~ 300 m high) submarine levee created as turbidity currents bank around a curve.

Var turbidite system: distal part of the levee

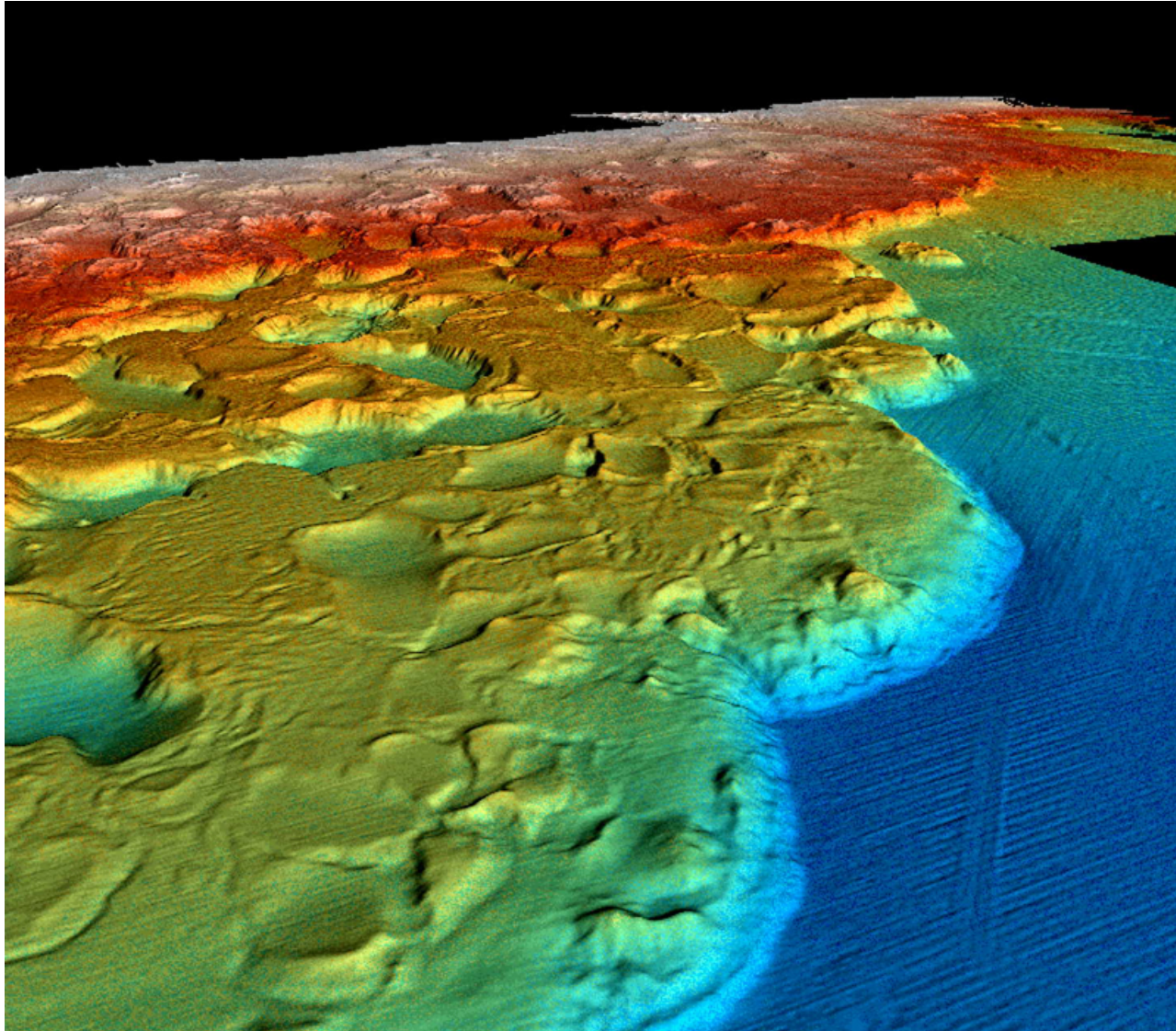


Sediment wave: high-resolution seismic-reflection profile



wavelength ~ 3 km, wave height ~ 50 m

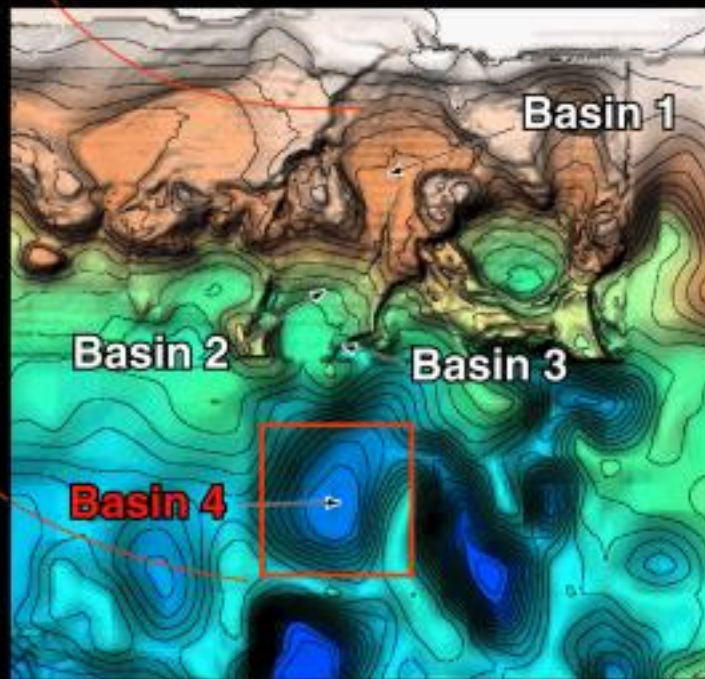
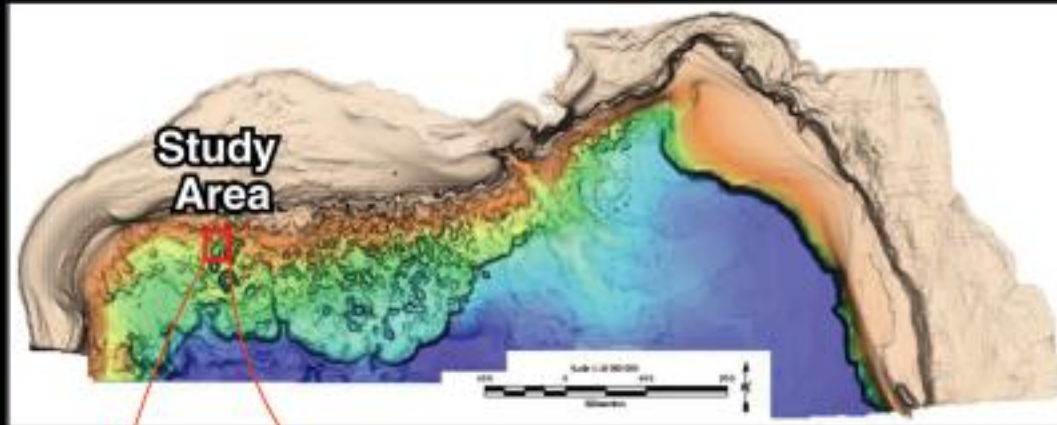
In 2007, we were performing conditions to determine whether it was possible for turbidity currents to form deep-water deltas in submarine minibasins.



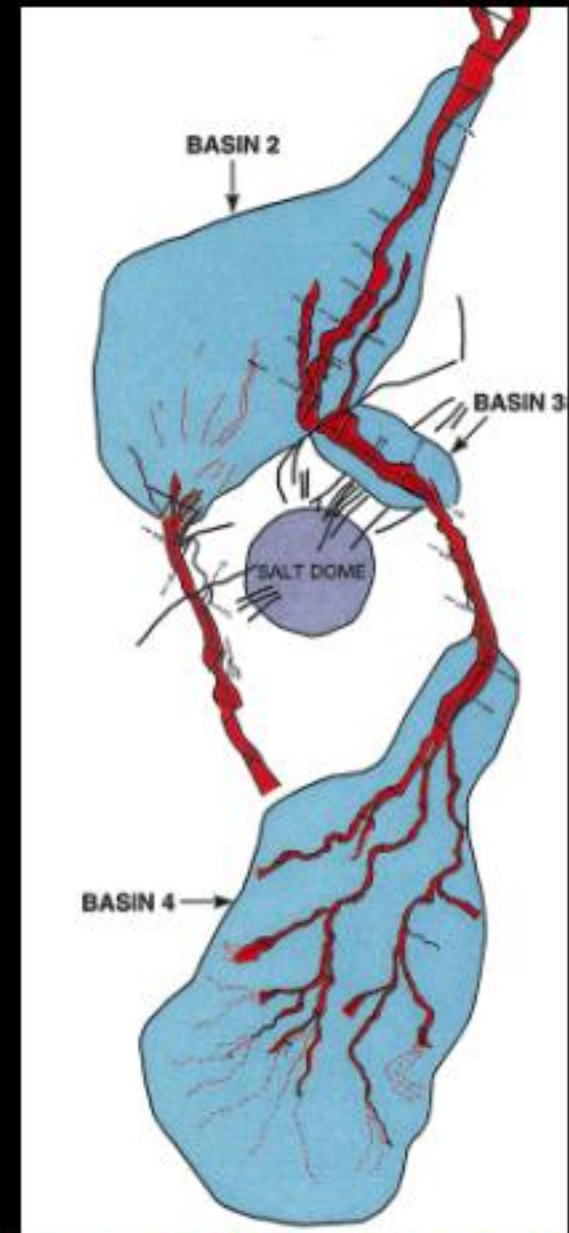
GOM minibasins: East Breaks

ExxonMobil

Gulf of Mexico Bathymetry



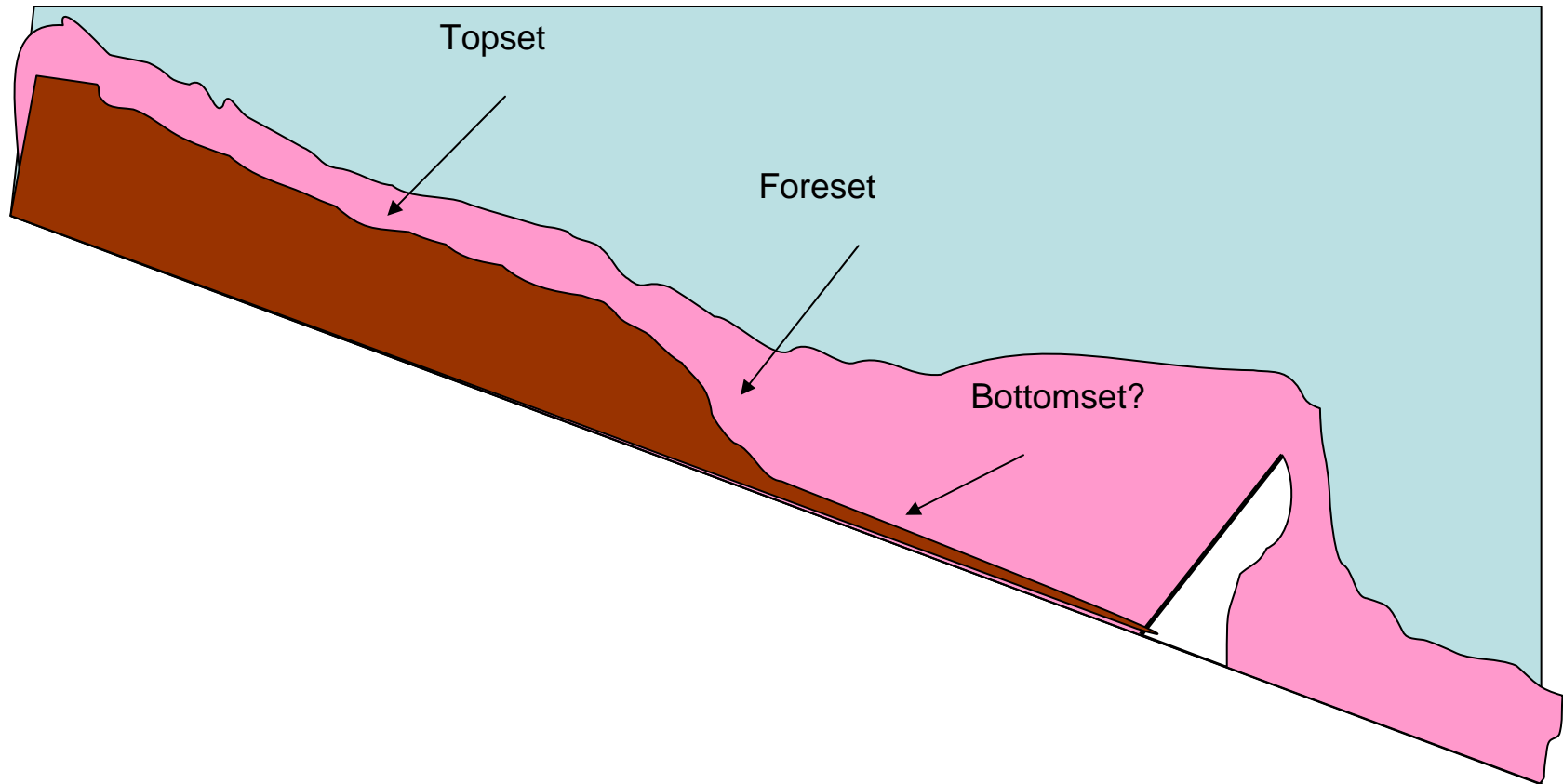
99-13468-001-EBB4 vg

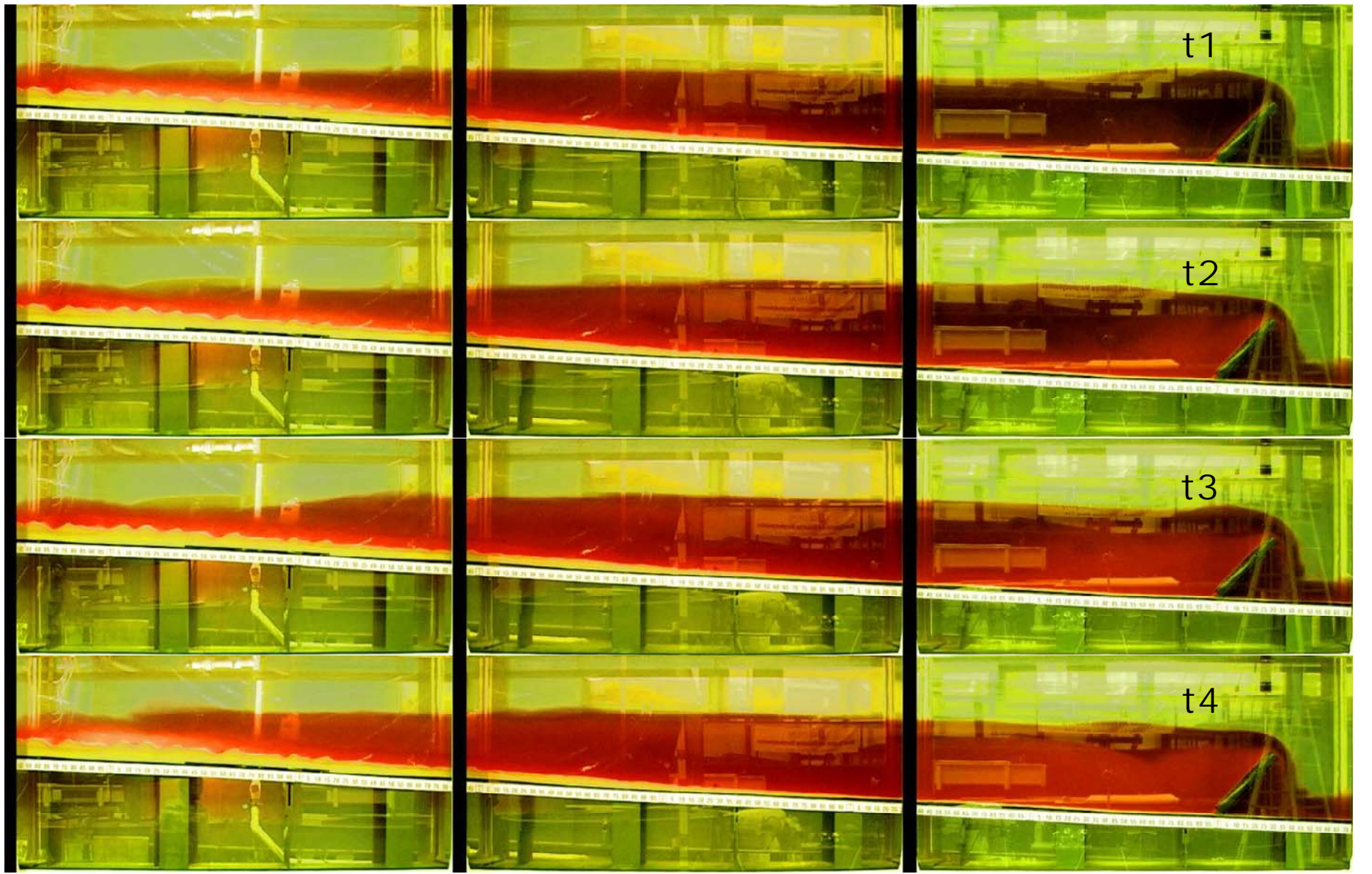


After Beaubouef and Friedmann, 2000

Experimental setup

Saline underflow models mud in turbidity current.
Plastic particles move as bedload and in suspension.





3

4

5

6

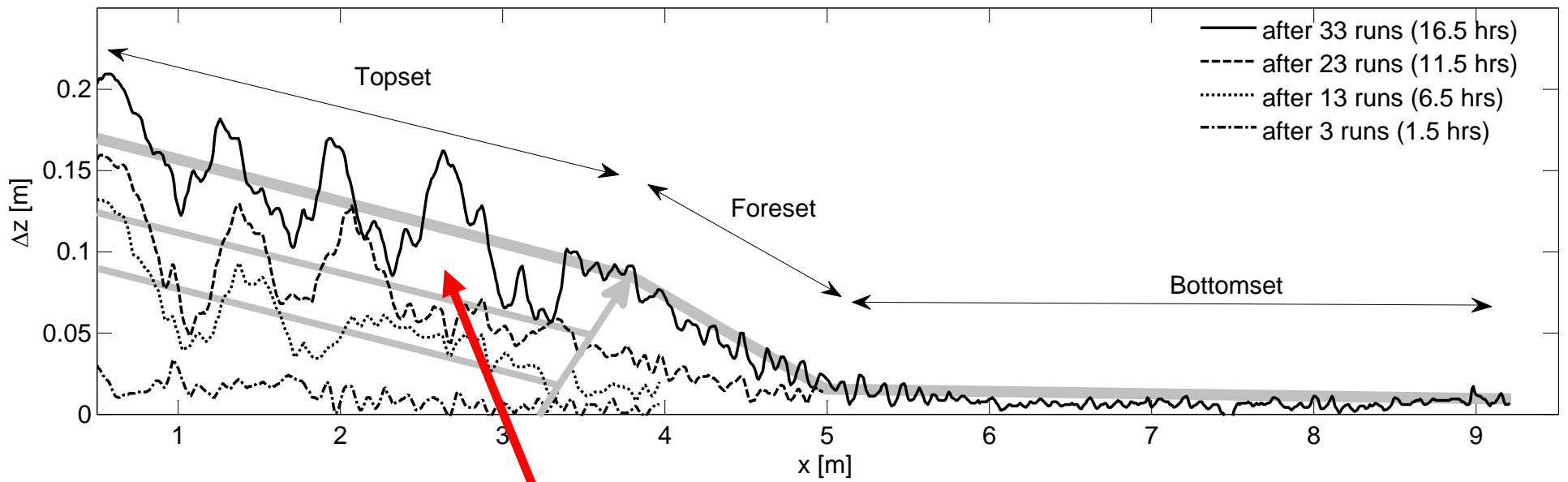
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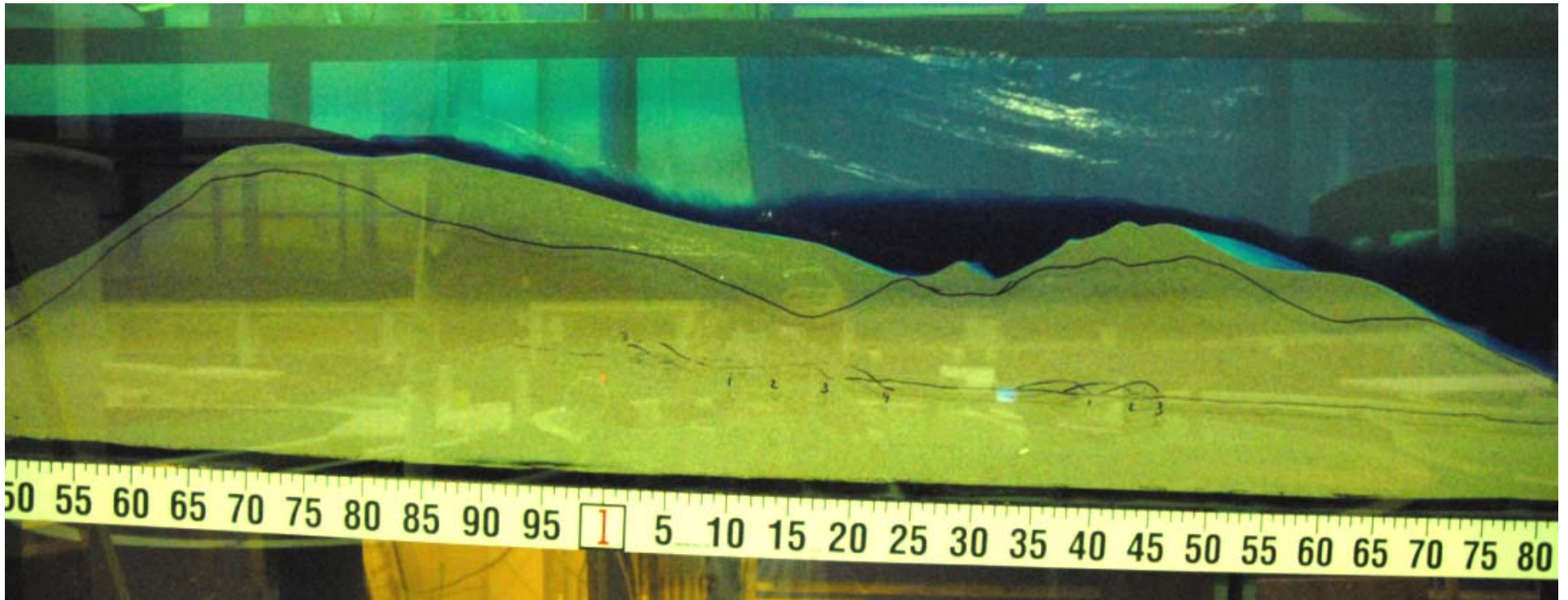
10

Well, we got our deepwater delta

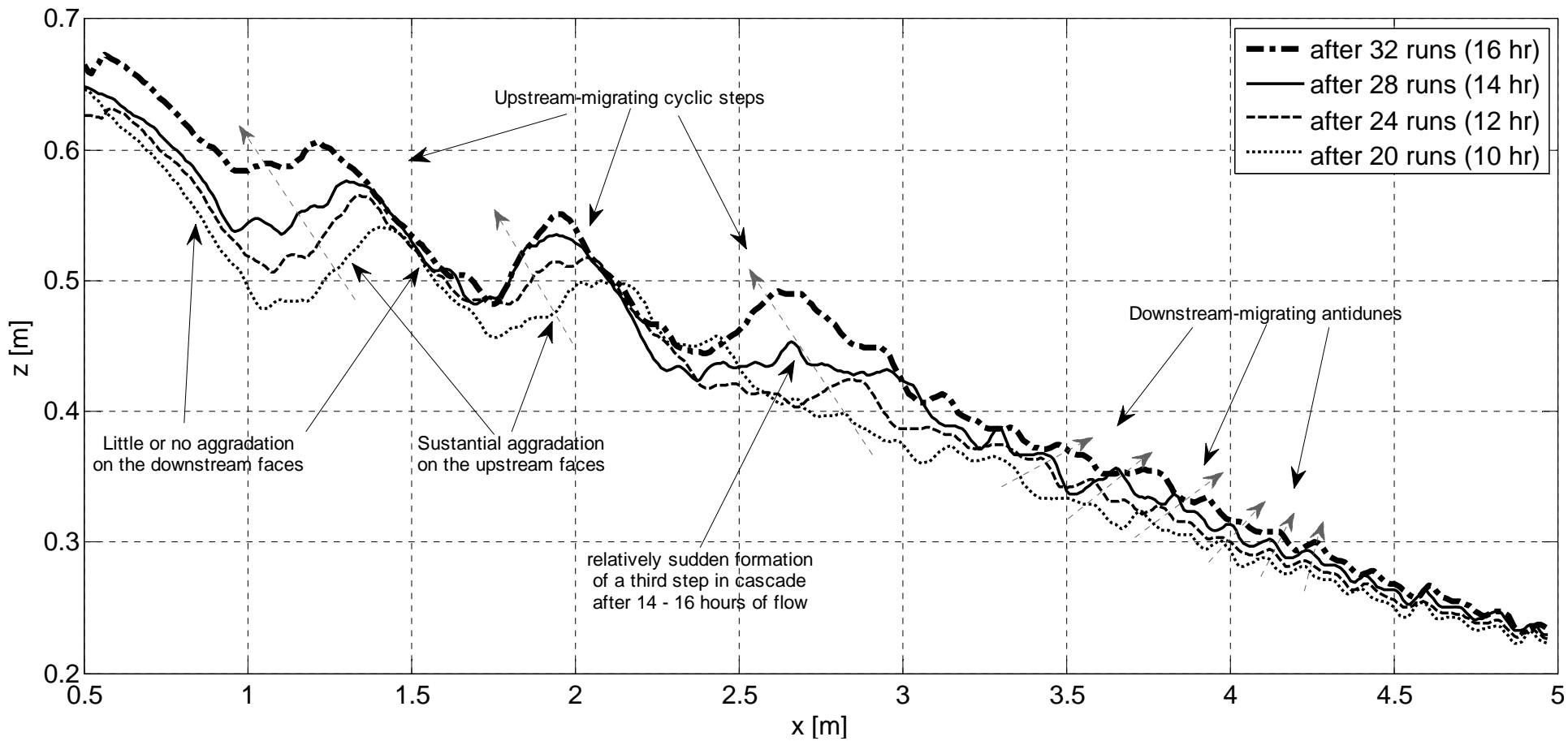
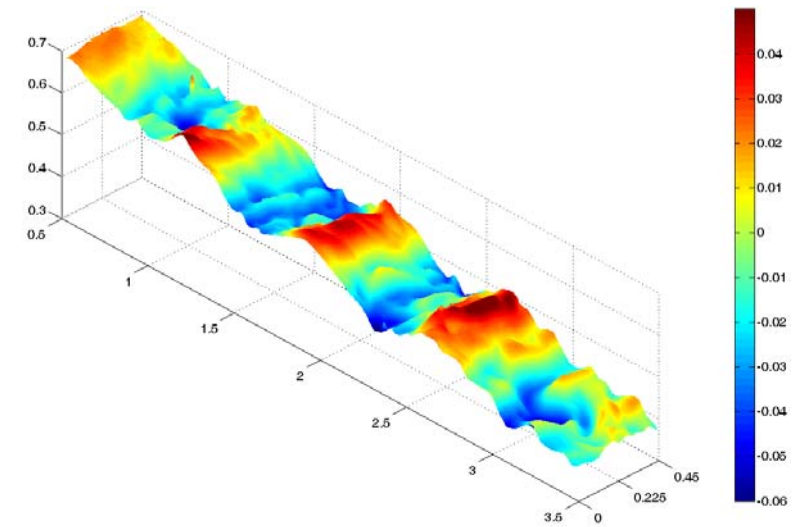


But look here!

We got cyclic steps too!



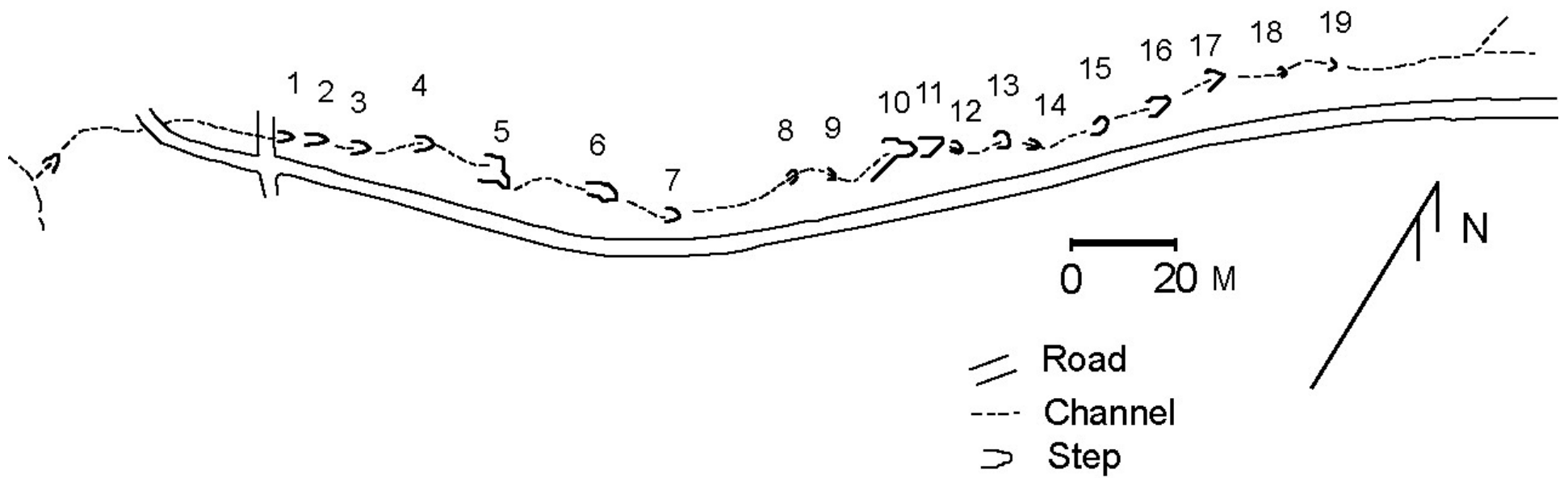
And they migrate upstream!



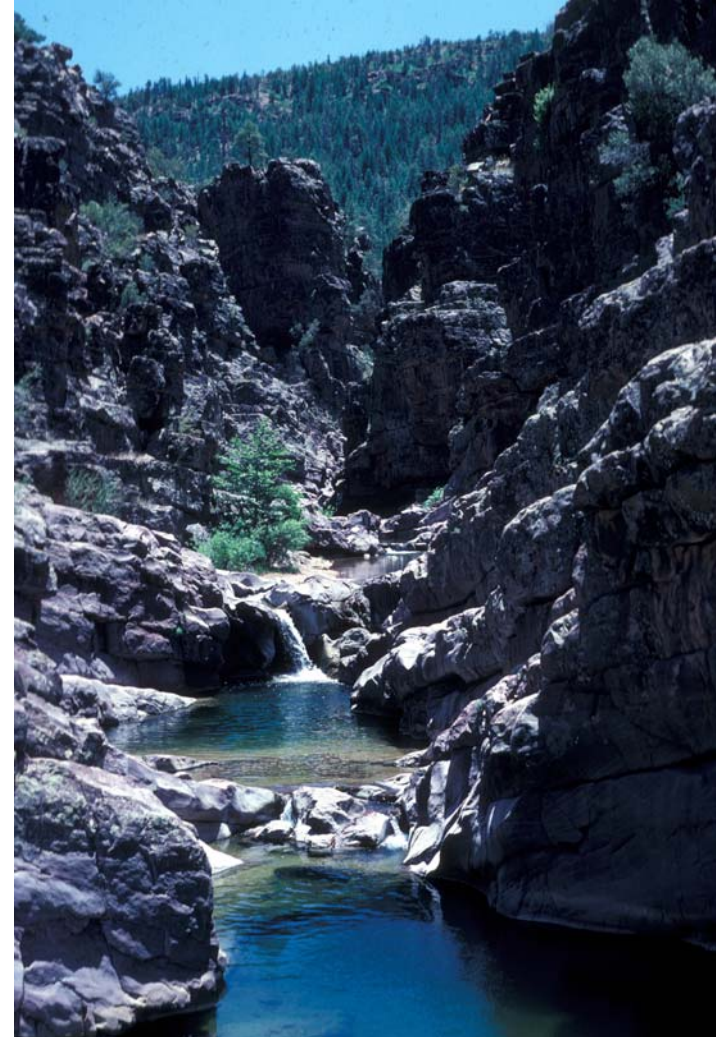
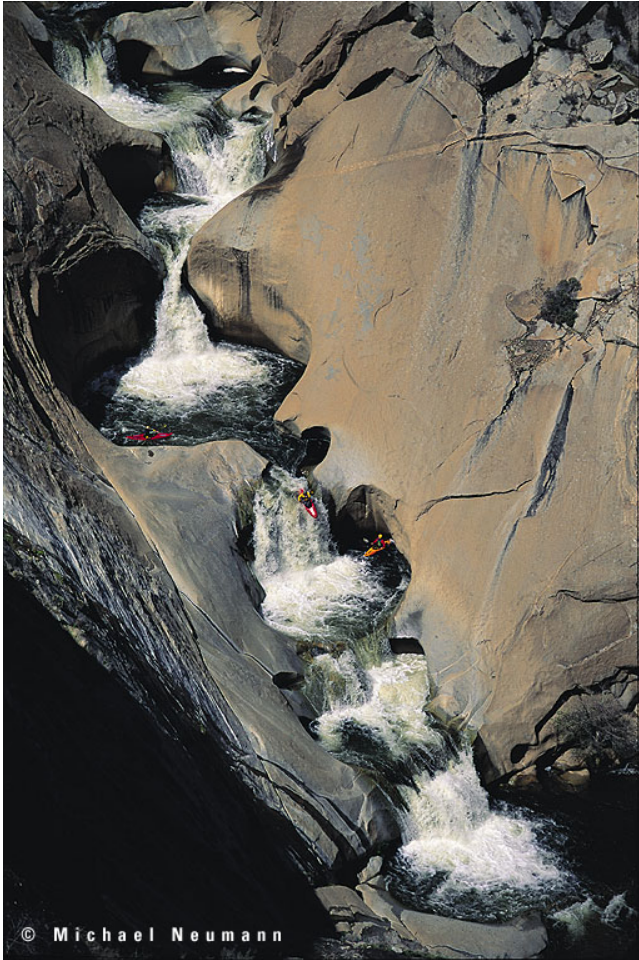


Internal delta experiments, B. Spinewine & O. Sequeros UIUC July07. Speeded up by a factor 5, Vertical scale of

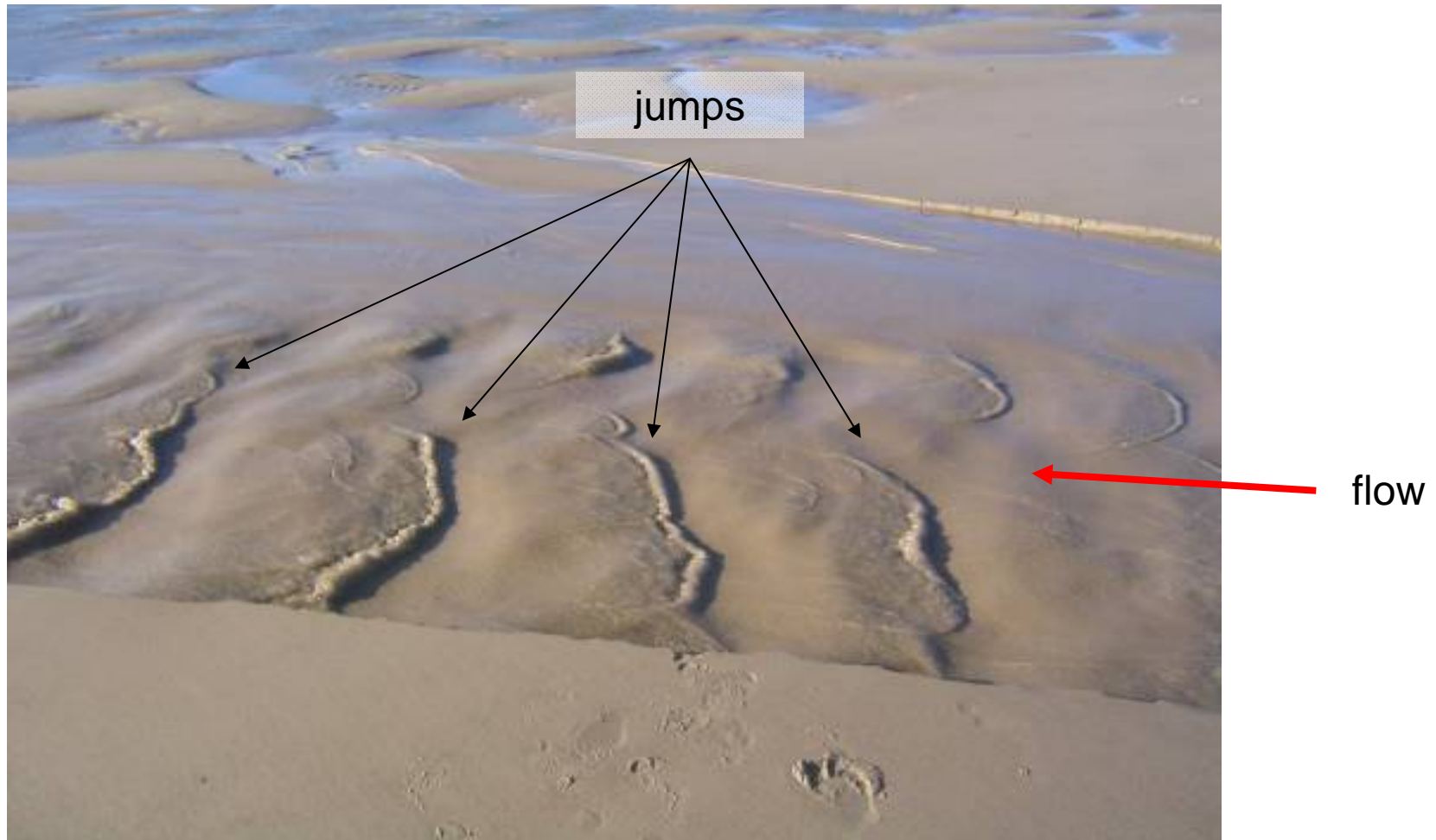
Cyclic steps!



Cyclic steps!

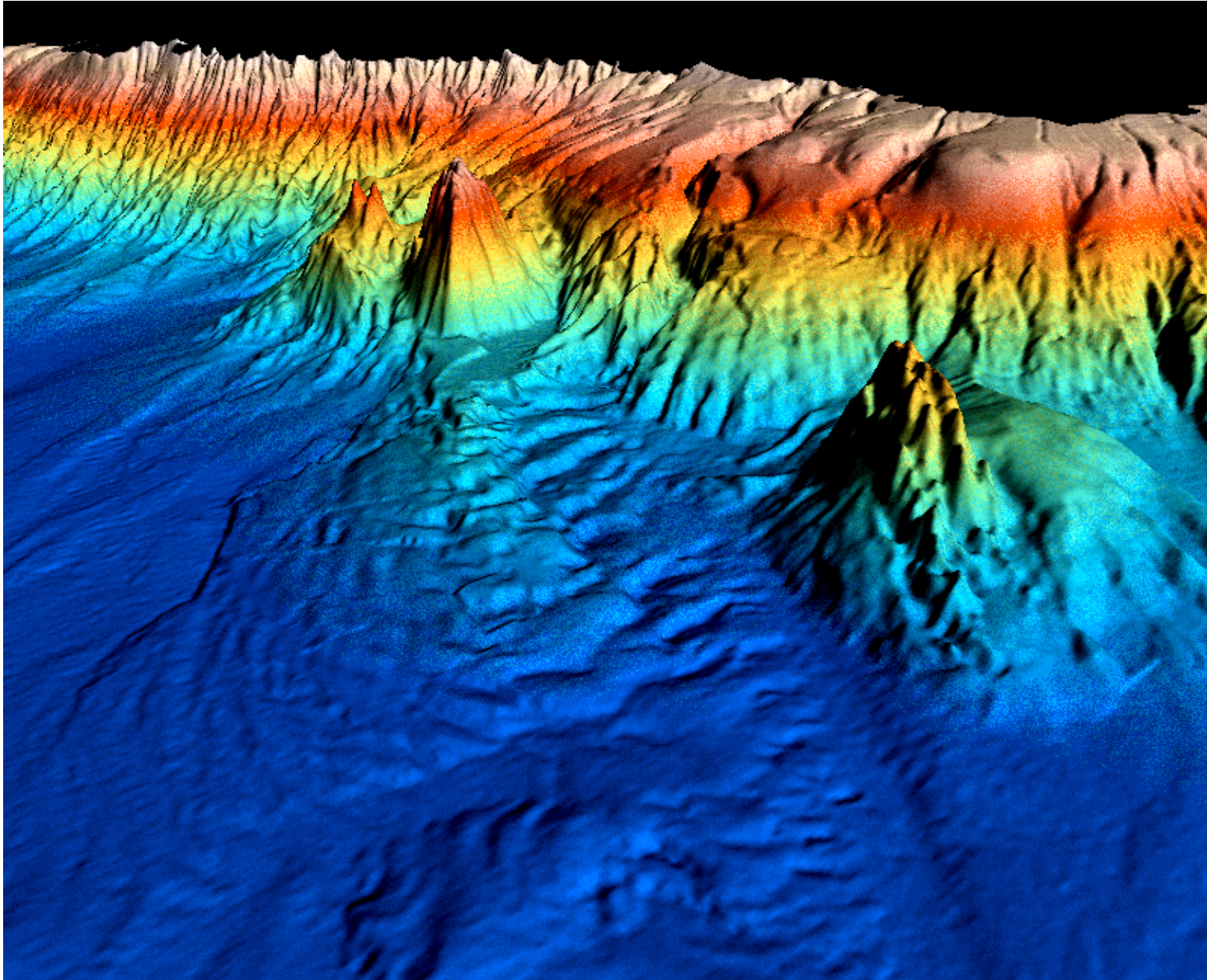


Cyclic steps!

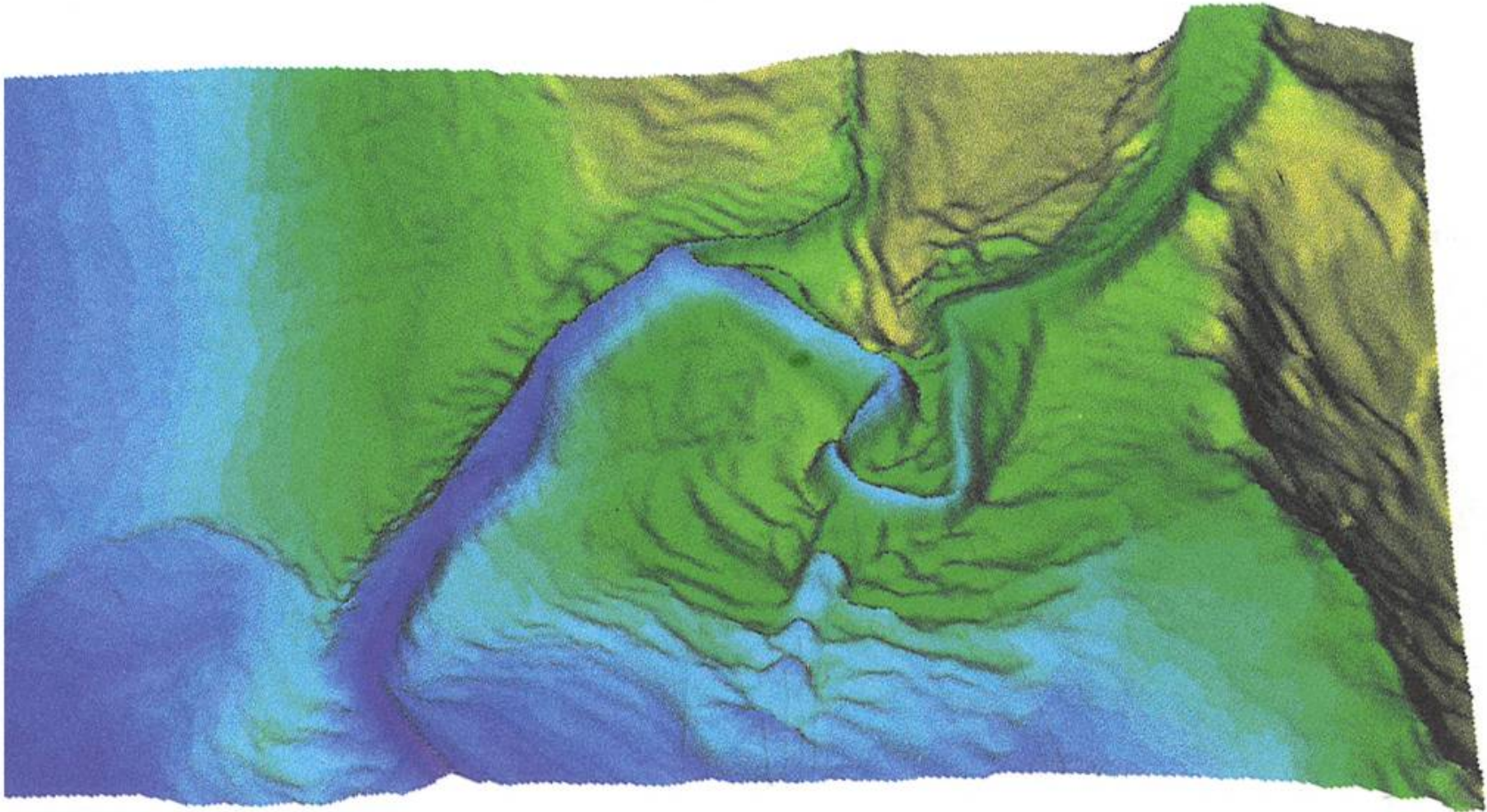


Trains of cyclic steps in a coastal outflow channel on a beach in Calais, France. Image courtesy H. Capart.

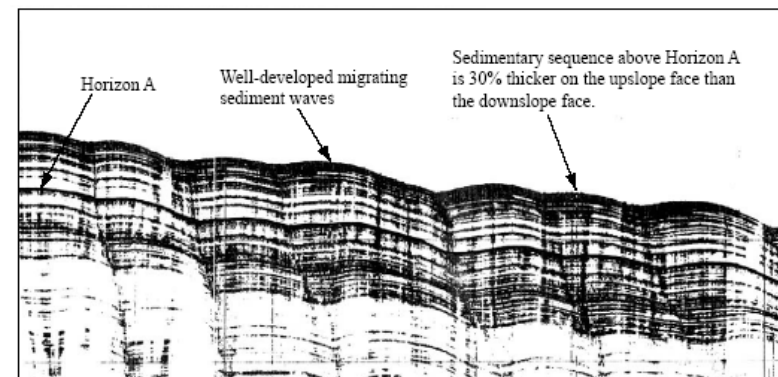
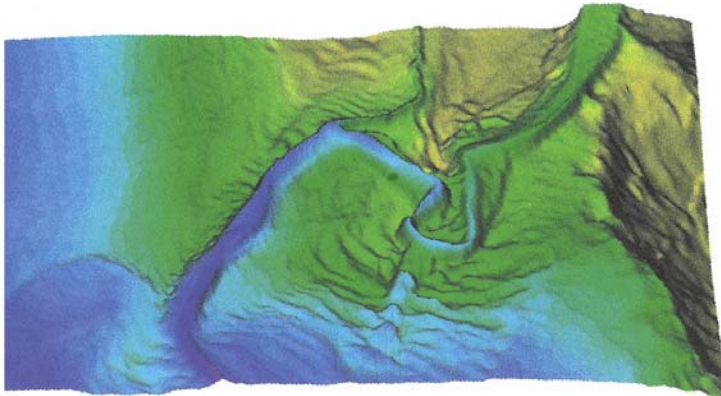
Cyclic steps!

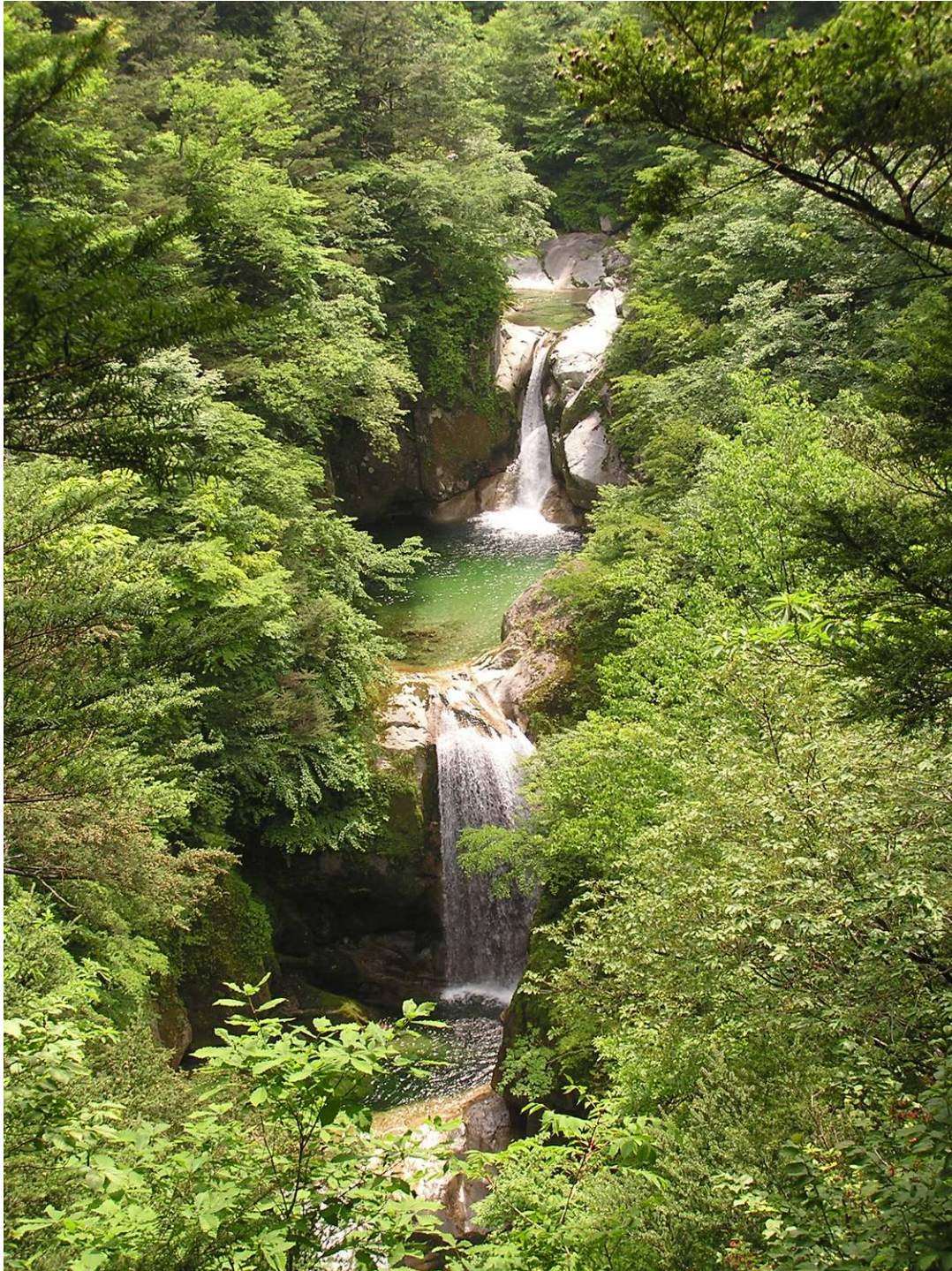


Cyclic steps!



A UNIVERSAL BEDFORM ASSOCIATED WITH INSTABILITY OF FROUDE-SUPERCritical FLOW OVER AN ERODIBLE BED





**IF YOU ARE
STILL AWAKE I'D
BE HAPPY TO
ADDRESS
COMMENTS AND
QUESTIONS.**

