Vertical sorting and the morphodynamics of bed form-dominated rivers: A sorting evolution model. 1. Derivation

Astrid Blom

Water Engineering & Management, Civil Engineering, University of Twente, Netherlands

Jan S. Ribberink

Water Engineering & Management, Civil Engineering, University of Twente, Netherlands

Gary Parker

Department of Civil and Environmental Engineering and Department of Geology, University of Illinois at Urbana-Champaign, Urbana, Illinois, USA

Abstract. A new type of sediment continuity model is proposed for modeling the interaction among grain size-selective sediment transport, vertical sorting, and net aggradation or degradation of a river bed. The model is aimed at conditions dominated by river dunes and bed load transport. Whereas *Blom et al.* [2006] present a reduction of the *Blom and Parker* [2004] modeling framework to equilibrium conditions (i.e. the equilibrium sorting model), we now propose a reduction of the modeling framework for unsteady conditions: the sorting evolution model. The present paper lists the various sub-models of a morphodynamic model system that is based on the sorting evolution model and explains how these sub-models are integrated in the morphodynamic model system. Among these sub-models are models for three types of vertical sediment fluxes: (1) sediment fluxes through dune migration, (2) sediment fluxes through unsteady bed form dimensions, and (3) sediment fluxes through net aggradation or degradation. We propose formulations for these three types of vertical sediment fluxes for unsteady conditions. In the accompanying paper [*Blom et al.*, 2007], the results of the sorting evolution model will be compared to measured data and to results of the widely used Hirano active layer model.

1. Introduction

Morphodynamic model systems are used to gain insight in, for instance, the effects of human interventions on a river system. A morphodynamic model system is here defined as a system that couples modules for calculating flow, sediment transport, and net aggradation or degradation of the river bed. In case sediment sorting processes play a role, the model system needs to include a sediment continuity model describing the interaction among grain size-selective sediment transport, net aggradation or degradation, and the vertical sorting profile. *Hirano* [1970, 1971, 1972] was the first to develop such a sediment continuity model for nonuniform sediment, and proposed to represent the active part of the bed as a distinct homogeneous surface layer.

In reality, however, the active part of the bed is rather represented by a probability density function (PDF) of bed surface elevations, and in most cases is not homogeneous. *Parker et al.* [2000] have introduced a framework for sediment continuity (i.e. the PPL framework) without discrete bed layers, which allows us to take into account that relatively deep bed elevations interact with the flow and are subject to entrainment and deposition less frequently than higher ones. *Blom and Parker* [2004] derive formulations for the grain size-specific and bed elevation-specific entrainment and deposition fluxes as required for the PPL framework, for situations dominated by bed forms and bed load transport. They apply the *Einstein* [1950] formulation for step length to the stoss face of a bed form, which relates deposition of particles over the stoss face of a bed form to entrainment. A lee sorting function describes the grain size-selective deposition of particles over the lee face of a bed form. *Blom and Parker* [2004] take into account the variability in bed form dimensions by accounting for the PDF of bed form trough elevations. All bed forms are assumed to have a triangular shape, and the geometric properties of an individual bed form (e.g., bed form height and bed form length) are assumed to be related to the trough elevation according to simple relations. As such, the likelihood of occurrence of a specific bed form is characterized by the PDF of relative trough elevations.

Blom et al. [2006] have reduced the continuum *Blom and Parker* model to steady or equilibrium conditions, i.e. conditions in which all variables vary around mean values. In the present paper, we consider unsteady conditions and reduce the *Blom and Parker* model to a sorting evolution model. The resulting model describes the time evolution of the sorting profile due to bed form migration, i.e. through grain size-selective deposition down bed form lee faces taking into account the variability in bed form dimensions.

Application of the equilibrium sorting model or the sorting evolution model in a morphodynamic model system requires a number of sub-models. In section 2 we will explain what type of sub-models are required and how they are integrated in the morphodynamic model system. Among these sub-models are models for three types of vertical sediment fluxes: sediment fluxes through dune migration (section 3); sediment fluxes through unsteady bed form dimensions (section 4); and sediment fluxes through net aggradation or

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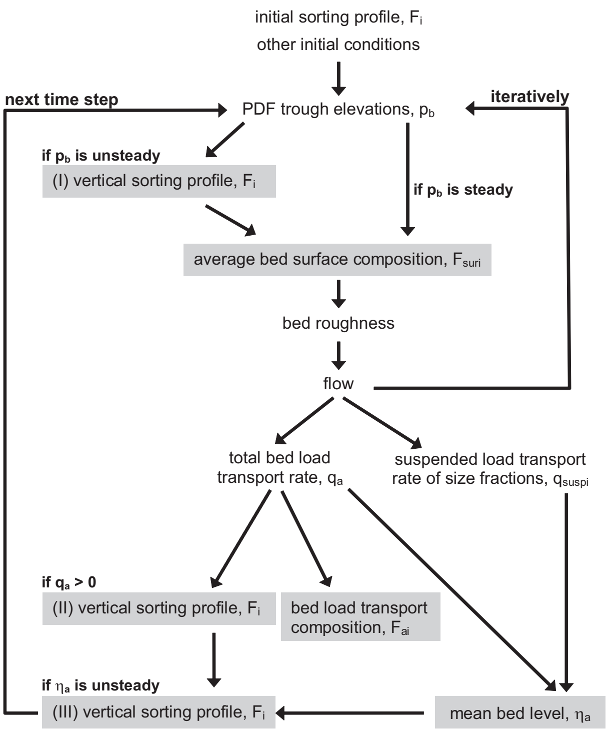

Figure 1. Scheme of a morphodynamic model system for nonuniform sediment when applying the sorting evolution model. Gray boxes represent sub-models that are part of the sorting evolution model. Evolution of the vertical sorting profile occurs through vertical sediment fluxes accompanying (I) a change in time of the PDF of relative trough elevations, (II) dune migration, and (III) net aggradation or degradation.

degradation (section 5). We will propose formulations for these three types of vertical sediment fluxes.

While the present paper explains the various components of a morphodynamic model system based on the sorting evolution model, as well as the derivation of the sorting evolution model, in the accompanying paper [*Blom et al.*, 2007] we will discuss the verification of the model system against measured data from experiments B2 and A2 conducted by *Blom et al.* [2003]. Also, the results of the model system based on the sorting evolution model will be compared to the results of a model system based on the *Hirano* [1971] active layer model and a model system based on the *Ribberink* [1987] two-layer model.

2. The morphodynamic model system

Figure 1 shows an overview of the various sub-models in a morphodynamic model system in the case the sorting evolution model is applied. Sub-models are required for describing:

1. three types of vertical sediment fluxes;
2. the PDF of relative trough elevations;
3. the mean bed surface composition;
4. the hydraulic roughness;
5. the flow;
6. the total bed load transport rate;
7. suspended load transport;

The sorting evolution model

The sorting evolution model is based on the framework for sediment continuity developed by *Parker et al.* [2000]. In this PPL framework the active part of the bed is described by a PDF of bed surface elevations rather than a discrete and homogeneous active layer of sediment. The framework is explained in Appendix A. In the PPL framework sediment conservation of size fraction i at elevation z is presented by equation (A1):

$$\frac{\partial \bar{C}_i}{\partial t} = c_b \bar{P}_s \frac{\partial \bar{F}_i}{\partial t} + c_b \bar{F}_i \frac{\partial \bar{P}_s}{\partial t} = \bar{D}_{ei} - \bar{E}_{ei}$$

where $\bar{C}_i$ denotes the concentration of size fraction i at elevation z ($\bar{C}_i = c_b \bar{P}_s \bar{F}_i$). Note that all parameters are averaged over some representative horizontal distance, i.e. a large number of bed forms. $\bar{F}_i$ denotes the volume fraction content of size fraction i at elevation z, $\bar{P}_s$ denotes the probability distribution of bed surface elevations indicating the probability that the bed surface elevation is higher than z. $\bar{D}_{ei}$ denotes the deposition density of size fraction i defined such that $\bar{D}_{ei}$ is the volume of sediment of size fraction i that is deposited per unit width and time in a bed element with sides dx and dz at elevation z, and $\bar{E}_{ei}$ the entrainment density of size fraction i defined likewise.

In the sorting evolution model, we distinguish between three types of vertical sorting fluxes (also see Figure 1):

I sorting fluxes through a change in time of the PDF of relative trough elevations;

II sorting fluxes through dune migration, i.e. grain size-selective deposition down a bed form lee face and the variability in trough elevations;

III sorting fluxes through net aggradation or degradation.

$$\bar{D}_{ei} - \bar{E}_{ei} = \left(\bar{D}_{ei} - \bar{E}_{ei}\right)\big|_{I} + \left(\bar{D}_{ei} - \bar{E}_{ei}\right)\big|_{II} + \left(\bar{D}_{ei} - \bar{E}_{ei}\right)\big|_{III} \quad (1)$$

For simplicity, it is assumed that these three types of vertical sorting fluxes do not interact with one another. This means, for instance, that when we consider sediment fluxes through bed form migration, both the PDF of relative trough elevations and the mean bed level are assumed to be steady.

The present paper considers the derivation of formulations for sediment fluxes through bed form migration (type II, section 3) and unsteady PDF of relative trough elevations (type I, section 4). Formulations for sediment fluxes

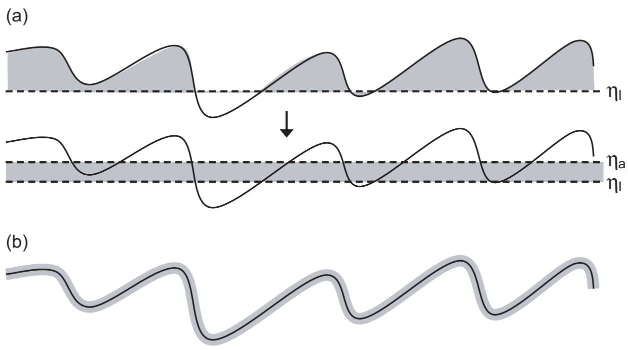

Figure 2. Calculating the mean composition of the bed surface when applying (a) the *Hirano* active layer model using equation 3, and (b) the sorting evolution model using equation 4. The upper plot under (a) illustrates the part of the bed that contributes to calculating the composition of the active layer, whereas the lower plot under (a) illustrates the positioning of the active layer. The upper elevation of the active layer equals the mean bed level, $\bar{\eta}_a$, while its lower elevation, η_I, is determined by the thickness of the active layer, δ ($\eta_I = \bar{\eta}_a - \delta$).

through net aggradation or degradation (type III) are proposed in section 5, but verification of these formulations will be considered in a future paper.

PDF of relative trough elevations

For predictive application of a morphodynamic model system as presented in Figure 1, a sub-model describing the PDF of relative trough elevations, $\tilde{p}_b$, is required. The relative trough elevation, Δ_b, is defined as the vertical distance between the mean bed level and the trough (Figure 5). *Van der Mark et al.* [2005] propose a simple model for the variation in relative trough elevations. Based on a number of data sets from flume experiments, they find that the standard deviation of the relative trough elevation, σ_{Δ_b}, is more or less a linear function of the mean relative trough elevation, μ_{Δ_b}:

$$\sigma_{\Delta_b} = 0.6\,\mu_{\Delta_b} \tag{2}$$

In other words, the deeper the mean trough elevation, the larger is the variation of the trough elevation around its mean value. Apparently, the variation of the trough elevation around its mean value is more or less independent of scale.

This means that the PDF of relative trough elevations can be modeled using equation 2, in combination with a model for the time evolution of the mean relative trough elevation. Such a model, however, is not readily available. For the time-being, we therefore propose to use a model for the time evolution of the mean bed form height, Δ, while assuming the mean relative trough elevation to equal half the mean bed form height ($\Delta_b(t) = 0.5\,\Delta(t)$).

Mean composition of bed surface

Being part of a morphodynamic model system, one of the main quantities a sediment continuity model needs to solve for is the time evolution of the mean composition of the bed surface, $\bar{F}_{suri}$. It is required as input for calculating the following parameters: the hydraulic roughness; the total rate (and composition) of the bed load transport, $\bar{q}_a$ (and $\bar{F}_{ai}$); and the grain size-specific suspended load transport rate, $\bar{q}_{suspi}$. In a morphodynamic model system as presented by Figure 1, these parameters will be predicted using specific sub-models.

When applying the *Hirano* active layer model, the mean composition of the bed surface, $\bar{F}_{suri}$, is assumed to be equal to the composition of the *Hirano* active layer, F_{mi}:

$$\bar{F}_{suri} = F_{mi} \tag{3}$$

When applying the sorting evolution model, in principle the mean composition of the bed surface, $\bar{F}_{suri}$, is calculated from

$$\bar{F}_{suri} = \int_{\eta_{mn}}^{\eta_{mx}} \bar{F}_i\,\bar{p}_e\,dz \tag{4}$$

where $\bar{F}_{suri}$ denotes the mean volume fraction content of size fraction i at the bed surface, weighed over all bed elevations exposed to the flow. Figure 2 illustrates the fundamental difference between equations 3 and 4. One has to realize that the method to determine the mean composition of the bed surface needs to suit the specific sub-model used for calculating the hydraulic roughness, bed load or suspended load transport.

Hydraulic roughness

The part of the hydraulic roughness that is attributed to grains, i.e. skin friction, is obviously closely related to some measure of the composition of the bed surface, e.g., the mean bed surface composition, $\bar{F}_{suri}$. The part of the hydraulic roughness that is attributed to bed forms, i.e. form drag, seems to be closely related to the PDF of bed surface elevations, which characterizes the shape and irregularity of the bed forms. However, a model relating form drag to the PDF of bed surface elevations is not readily available.

Flow

A sub-model describing the flow is required for calculating its effect on (bed load and suspended load) sediment transport. To this end, we can apply the well-known shallow water equations or a simplified equation, such as the formulation for a backwater curve.

Total bed load transport rate

In the present version of the model, the composition of the bed load transport is computed by the sorting evolution model itself. This will be explained in section 3. This notwithstanding, a sub-model for the *total* bed load transport rate is required. Calculating the bed load transport rate usually requires information on the skin friction, some flow parameter, e.g., the dimensionless shear stress, and the mean bed surface composition. *Van der Scheer et al.* [2002] have evaluated the performance of a number of sediment transport models for nonuniform sediment, by comparing the computed rate and composition of the sediment transport to measured data from flume experiments. The study demonstrates that predictions differ greatly between the various sediment transport models, which underlines the large uncertainties accompanying predictions of sediment transport. The sediment transport models by *Wilcock and Crowe* [2003] and *Wu et al.* [2000] showed the best results in reproducing the rate and composition of the transported sediment as measured in various sets of flume experiments. Most sediment transport models appeared to suffer from bad predictions close to conditions of incipient motion. The *Wilcock and Crowe* model showed reasonable results within this range [*Van der Scheer et al.*, 2002].

Suspended load transport

The present version of the sorting evolution model is aimed at conditions dominated by bed load transport. Yet, suspended load transport may be incorporated by taking the following steps: (1) calculate the mean composition of the bed surface, $\bar{F}_{suri}$, using equation (4); (2) neglect the interaction between vertical sediment fluxes through bed load transport and suspended load transport; (3) use a model for predicting the volume of suspended load transport of size fraction i per unit width and time, $\bar{q}_{suspi}$, from the mean bed surface composition, $\bar{F}_{suri}$, and flow parameters:

$$\bar{q}_{suspi} = f_{susp}(\bar{F}_{suri}, \text{flow parameters}) \tag{5}$$

in which f_{susp} represents a model for suspended load transport of nonuniform sediment; and (4) treat vertical sediment fluxes through net aggradation or degradation due to divergences in suspended load transport in the same way as vertical sediment fluxes through net aggradation or degradation due to divergences in bed load transport (section 5).

3. Sediment fluxes through dune migration (type II)

The derivation of sediment fluxes through dune migration under unsteady conditions (type II, see figure 1) continues from the analysis by *Blom and Parker* [2004], which has been summarized in Appendix B. As the present paper is a follow-up of the paper by *Blom and Parker* [2004], the same notation will be used. Reference to equations in *Blom* *and Parker* [2004] will be specified by (BP-*Blom and Parker* equation number).

When describing sediment fluxes through dune migration, we assume the PDF of relative trough elevations, $\tilde{p}_b$, to be steady, as well as the mean bed level, $\bar{\eta}_a$ (also see section 2). This implies that the probability distribution of bed surface elevations relative to the mean bed level, $\tilde{P}_s$, and the probability distribution of bed surface elevations, $\bar{P}_s$, are steady, as well. The fundamental equations of the Parker-Paola-Leclair framework for sediment continuity, i.e. equations (A3) through (A5) now reduce to

$$c_b \bar{P}_s \frac{\partial \bar{F}_i}{\partial t} = \left(\bar{D}_{ei} - \bar{E}_{ei}\right)\big|_{II} \tag{6}$$

$$0 = \left(\bar{D}_e - \bar{E}_e\right)\big|_{II} \tag{7}$$

$$0 = \left(\bar{D} - \bar{E}\right)\big|_{II} \tag{8}$$

The parameters $\bar{F}_i$, $\bar{P}_s$, $\bar{D}_{ei}$, $\bar{E}_{ei}$, $\bar{D}_e$, and $\bar{E}_e$ all depend on the vertical co-ordinate z, the horizontal co-ordinate x, and the time co-ordinate t, whereas $\bar{D}$, and $\bar{E}$ depend only on the horizontal co-ordinate x and the time co-ordinate t. Note that the co-ordinate x varies only over the scale of large numbers of bed forms. For clarity, the co-ordinates have been left out of the equations.

Over the stoss face of a bed form, *Blom and Parker* [2004] distinguish simultaneous entrainment and deposition fluxes, and only deposition fluxes over the lee face:

$$\bar{E}_{ei}\big|_{II} = \bar{E}_{eis} \tag{9}$$

$$\bar{D}_{ei}\big|_{II} = \bar{D}_{eis} + \bar{D}_{eil} \tag{10}$$

which has been explained in Appendix B and clarified by Figure 5. The overall (i.e. averaged over a series of bed forms) entrainment and deposition densities for the stoss and lee faces are given by equations (B4) through (B6).

Like in the equilibrium sorting model [*Blom et al.*, 2006], for simplicity, we assume the bed forms to have a triangular shape with varying trough elevations, so that for each bed form $p_e = p_{se} = p_{le} = J(z)/\Delta$. Besides, like *Blom et al.* [2006] we assume the average bed load transport rate to be identical for each bed form in the series of bed forms ($q_a = \bar{q}_a$). Moreover, we impose the composition of the sediment transported over each crest to be the same ($F_{topi} = \bar{F}_{topi}$), as well as the composition of the lee face deposit ($F_{leei} = \bar{F}_{leei}$). Finally, we make no distinction in sorting between bed forms within one series of bed forms ($F_i = \bar{F}_i$).

Now, in order to find a solution to the time evolution of the vertical sorting profile of nonuniform sediment, $\bar{F}_i$, we need to solve equations (6) and (7). Since the probability distribution of bed surface elevations, $\bar{P}_s$, is steady, the total amount of sediment at each elevation is steady, as well. This will be satisfied when the total amount of sediment entrained from the bed at elevation z (1) is independent of the local bed composition $\bar{F}_i(z)$, (2) is independent of the bed surface elevation z, and (3) has a composition equal to the local bed composition $\bar{F}_i(z)$. These constraints are satisfied when the following equation is met for each individual bed form:

$$E_{snet}\bar{F}_i(z) = E_{siu}(z)\bar{F}_i(z) - E_{siu}(z - \eta_{stepi})\bar{F}_i(z - \eta_{stepi}) \tag{11}$$

where $E_{snet} = q_{top}/\lambda_s$, in which the total bed load transport rate over the bed form crest, q_{top}, is twice the average total bed load transport rate, $\bar{q}_a$:

$$q_{top} = \bar{q}_{top} = 2\,\bar{q}_a \tag{12}$$

Hence, also the bed load transport rate over the bed form crest is the same for each individual bed form. Equation 12 is also found when applying the simple-wave equation to the migration of triangular bed forms.

With equations (7), (11), and (12), equation (6) reduces to a relaxation-type sorting evolution model:

$$\frac{\partial \bar{F}_i}{\partial t} = \frac{2\,\bar{q}_a}{c_b\,\bar{P}_s} \int_{\eta_{bmin}}^{\eta_{bmax}} \frac{p_e}{\lambda} \left[F_{leeloci} - \bar{F}_i\right]\, \tilde{p}_b\, d\eta_b \tag{13}$$

where the volume fraction content of size fraction i in the sediment deposited at elevation z at the lee face, $F_{leeloci}$, is given by (see equations (B8) and (B12)):

$$F_{leeloci} = F_{leei}\,\omega_i(z) = F_{leei}\,J(z)\,\left(1 + \delta_i\, z^*\right) \tag{14}$$

where $z^* = (z - \bar{\eta}_a)/\Delta$ and in which, with (B7), the volume fraction content of size fraction i in the lee deposit, F_{leei}, in (B9) equals

$$F_{leei} = \bar{F}_{leei} = \bar{F}_{topi} \tag{15}$$

Note that λ, Δ, p_e, $F_{leeloci}$, and $\tilde{p}_b$ in equation (13) all depend on the specific trough elevation η_b. The geometrical properties of the individual triangular dunes are described by the following simple rules. Each crest is assumed to have the same absolute distance to the mean bed level as its trough, and the steepness of the lee faces is assumed to equal the angle of repose (ν). The dune length is assumed to be proportional to the dune height and the ratio of the average dune length λ_a to the average dune height Δ_a:

$$\Delta = 2\,\Delta_b \tag{16}$$

$$\lambda = (\lambda_a/\Delta_a)\,\Delta \tag{17}$$

$$\lambda_l = \Delta/\tan(\nu) \tag{18}$$

$$\lambda_s = \lambda - \lambda_l \tag{19}$$

see also equations (BP-50)-(BP-53) and Figure 5. Note that equations (16) through (18) are not supposed to be generally valid and their applicability should be checked against data when applying them.

Equation (13) shows that equilibrium is reached when the overall composition of the sediment deposited at elevation z at the lee face equals the composition of the bed at that elevation. The time scale of the adaptation of the sorting profile will be considered in section 6.

Blom et al. [2006] used the coefficients γ and κ in the lee sorting function, ω_i, as calibration coefficients. Although the calibration was done for equilibrium conditions, we assume the values found for γ and κ to be generally valid:

$$\delta_i = 0.3\,\frac{\phi_i - \bar{\phi}_{mlee}}{\bar{\sigma}_a}\,(\bar{\tau}_b^*)^{-0.5} \tag{20}$$

where the arithmetic mean grain size of the lee deposit, $\bar{\phi}_{mlee}$, is given by $\bar{\phi}_{mlee} = \sum_i^N \phi_i \bar{F}_{leei}$ and the arithmetic standard deviation of the lee deposit, $\bar{\sigma}_a$, is given by $\bar{\sigma}_a^2 = \sum_i^N (\phi_i - \bar{\phi}_{mlee})^2 \bar{F}_{leei}$.

The volume fraction content of size fraction i in the sediment transported over the bed form crest, $\bar{F}_{topi}$, is given by

$$\bar{F}_{topi} = \frac{1}{\bar{E}_{snet}} \int_{\eta_{bmin}}^{\eta_{bmax}} E_{snet} \int_{\eta_b}^{\eta_t} \bar{F}_i\, p_e\, dz\;\tilde{p}_b\, d\eta_b \tag{21}$$

where the overall net entrainment rate, $\bar{E}_{snet}$, equals

$$\bar{E}_{snet} = \int_{\eta_{bmin}}^{\eta_{bmax}} E_{snet}\, \tilde{p}_b\, d\eta_b \tag{22}$$

Equation (21) expresses that the volume fraction content of size fraction i transported over an individual crest is equal to the integral over bed elevations of the vertical sorting profile multiplied by its PDF of bed surface elevations, p_e. This is true since the net entrainment rate over the bed form stoss face, E_{snet}, is uniform over all bed surface elevations, where $E_{snet} = \bar{q}_{top}/\lambda_s$, and the composition of the net entrainment at elevation z is assumed to be equal to the bed composition at that elevation, $\bar{F}_i$. To find the *overall* composition of sediment transported over the bed form crest, $\bar{F}_{topi}$, the composition of the sediment transported over an individual crest is averaged over all trough elevations while weighted by its probability density of occurrence, $\tilde{p}_b$.

Similar to the formulation for the overall composition of sediment transported over the bed form crest, $\bar{F}_{topi}$, the volume fraction content of size fraction i in the bed load transport at the stoss face at elevation z, $\bar{F}_{qsi}$, is given by

$$\bar{F}_{qsi}(z) = \frac{1}{\hat{E}_{snet}(z)} \int_{\eta_{bmin}}^{\eta_{bmax}} \frac{E_{snet}}{\int_{\eta_b}^{z} p_e \, dz} \int_{\eta_b}^{z} \bar{F}_i \, p_e \, dz \, \tilde{p}_b \, d\eta_b \quad (23)$$

where the entrainment rate, $\hat{E}_{snet}$, is given by

$$\hat{E}_{snet}(z) = \int_{\eta_{bmin}}^{\eta_{bmax}} J(z) \, E_{snet} \; \tilde{p}_b \, d\eta_b \quad (24)$$

Now, the bed load transport composition, $\bar{F}_{ai}$, is found by averaging the grain size-specific and elevation-specific bed load transport rate, $\bar{F}_{qsi}$, over all elevations of the active bed:

$$\bar{F}_{ai} = \int_{\eta_{mn}}^{\eta_{mx}} \bar{F}_{qsi} \, \bar{p}_e \, dz \quad (25)$$

where η_{mn} and η_{mx} denote the lower and upper limits of the active bed, respectively. Note that in equation 25, for simplicity, the contribution of the composition of the bed load transport over the lee face has been neglected, as the horizontal length of the lee face is much shorter than the length of the stoss face.

For a series of *regular* bed forms, equations (13) and (21) reduce to

$$\frac{\partial \bar{F}_i}{\partial t} = \frac{2 \, \bar{q}_a \, \bar{p}_e}{c_b \, \bar{P}_s \, \bar{\lambda}} \left[\bar{F}_{leeloci} - \bar{F}_i \right] \quad (26)$$

and

$$\bar{F}_{topi} = \int_{\eta_b}^{\eta_t} \bar{F}_i \, \bar{p}_e \, dz \quad (27)$$

Thus, the set of equations derived in the present section comprises a sorting evolution model for rivers characterized by nonuniform sediment and bed forms. The resulting model computes the time evolution of both the vertical sorting profile and volume fraction contents of size fractions in the bed load transport from the following input parameters: the initial sorting profile; the time evolution of the PDF of relative trough elevations, $\tilde{p}_b$; the time evolution of the total bed load transport rate, $\bar{q}_a$ (not its composition); and the ratio of the average bed form length to the average bed form height, λ_a/Δ_a.

4. Sediment fluxes through unsteady bed form dimensions (type I)

Bed form dimensions, and therefore the PDF of relative trough elevations, vary in time with changing hydraulic conditions. For instance, during a flood event the increase in bed shear stress may cause bed form crests to become higher and troughs to become deeper, while the mean bed level may remain steady. Such a change in time of the PDF of relative trough elevations, $\tilde{p}_b$, results from net entrainment and deposition fluxes from and to bed surface elevations. As mentioned in section 2, we simply assume these fluxes to be independent of the sediment fluxes through both dune migration and net aggradation or degradation. In other words, when the PDF of relative trough elevations, $\tilde{p}_b$, changes in time, the mean bed level is assumed to be steady and vertical sediment fluxes through bed form migration are assumed to be negligible.

Thus, at each time step we need to predict the PDF of relative trough elevations using an external sub-model. Having predicted $\tilde{p}_b$ at the new time step t_2, we can determine the probability distribution of bed surface elevations at the new time step, $\bar{P}_s(t_2)$, from

$$\bar{p}_e = \int_{\eta_{bmin}}^{\eta_{bmax}} \frac{J}{\Delta} \, \tilde{p}_b \, d\eta_b \quad (28)$$

$$\bar{P}_s = 1 - \int_{-\infty}^{z} \bar{p}_e \, dz \quad (29)$$

where $\bar{p}_e$ denotes the PDF of bed surface elevations.

Figure 3 illustrates how at elevations where $\bar{P}_s(t_2) > \bar{P}_s(t_1)$, sediment has been deposited. At elevations where $\bar{P}_s(t_2) < \bar{P}_s(t_1)$, sediment has been entrained. We simply assume that sediment entrained from bed surface elevation z has the same composition as present at that elevation. We can now determine the average volume fraction content of size fraction i in the total amount of sediment entrained from the bed, $\bar{\bar{F}}_{iP}$:

$$\bar{\bar{F}}_{iP} = \frac{\int_{\eta_{mn}}^{\eta_{mx}} I(z) \left[\bar{C}_i(t_1) - \bar{C}_i(t_2) \right] \, dz}{\int_{\eta_{mn}}^{\eta_{mx}} \sum_i^N I(z) \left[\bar{C}_i(t_1) - \bar{C}_i(t_2) \right] \, dz} = \\ = \frac{\int_{\eta_{mn}}^{\eta_{mx}} I(z) \left[\bar{P}_s(t_1) - \bar{P}_s(t_2) \right] \bar{F}_i(t_1) \, dz}{\int_{\eta_{mn}}^{\eta_{mx}} I(z) \left[\bar{P}_s(t_1) - \bar{P}_s(t_2) \right] \, dz} \quad (30)$$

where

$$I(z) = \begin{cases} 1 & \text{if } \bar{P}_s(t_2) < \bar{P}_s(t_1) \\ 0 & \text{if } \bar{P}_s(t_2) \geq \bar{P}_s(t_1) \end{cases}$$

and where η_{mn} and η_{mx} denote the lower and upper levels of the active bed at either time t_1 or t_2, that is, when the active bed covers the widest range of bed elevations.

At the bed elevations where deposition occurs, the composition of the deposited sediment is assumed to be equal to the average composition of the total amount of sediment entrained from the bed, $\bar{\bar{F}}_{iP}(t)$. If $\bar{P}_s(t_2) \leq \bar{P}_s(t_1)$, the bed composition at the new time step equals

$$\bar{F}_i(t_2) = \bar{F}_i(t_1) \quad (31)$$

If $\bar{P}_s(t_2) > \bar{P}_s(t_1)$, the bed composition at the new time step equals

$$\bar{F}_i(t_2) = 1/\bar{P}_s(t_2)[\bar{P}_s(t_1)\bar{F}_i(t_1) + \quad (32)$$

$$\left(\bar{P}_s(t_2) - \bar{P}_s(t_1)\right) \bar{\bar{F}}_{iP}(t)] \quad (33)$$

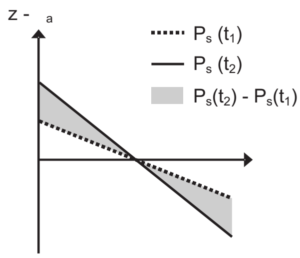

Figure 3. The change in time of the probability distribution of bed surface elevations, $\bar{P}_s$.

Note that this method for accounting for a change in time of the PDF of relative trough elevations is a rather artificial method that strongly simplifies the actual physical processes. For instance, the method does not incorporate grain size-selective processes. In reality, however, grain size-selective processes will surely play a role. For instance, the winnowing of fines from the trough surface and subsurface may cause the coarse bed layer below migrating bed forms to subside and the range of elevations of the active bed to gradually increase.

5. Sediment fluxes through net aggradation or degradation (type III)

Divergences in bed load and/or suspended load transport result in net aggradation or degradation of the river bed. In this section, it will be explained how we calculate the net aggradation or degradation of the river bed and the change in the vertical sorting profile through the resulting sediment fluxes. We also refer to the type III sediment fluxes in Figure 1).

In order to include net aggradation or degradation in the sorting evolution model, we make a number of assumptions. As mentioned in section 2, we neglect the interaction among vertical sediment fluxes through (I) a change in time of the PDF of relative trough elevations, (II) bed form migration, and (III) net aggradation or degradation. Furthermore, we neglect the interaction between vertical sediment fluxes through divergences in bed load transport and those through suspended load transport. The fundamental equations of the PPL framework (equations (A3) through (A5)), now yield

$$c_b \bar{P}_s \frac{\partial \bar{F}_i}{\partial t} + c_b \bar{F}_i \bar{p}_e \frac{\partial \bar{\eta}_a}{\partial t} = \left(\bar{D}_{ei} - \bar{E}_{ei}\right)\big|_{III} \tag{34}$$

$$c_b \bar{p}_e \frac{\partial \bar{\eta}_a}{\partial t} = \left(\bar{D}_e - \bar{E}_e\right)\big|_{III} \tag{35}$$

$$c_b \frac{\partial \bar{\eta}_a}{\partial t} = \left(\bar{D} - \bar{E}\right)\big|_{III} = -\frac{\partial\left(\bar{q}_a + \bar{q}_{susp}\right)}{\partial x} \tag{36}$$

where $\bar{q}_{susp}$ denotes the total volume of suspended load transport per unit width and time excluding pores and averaged over a series of bed forms. Note that from equation (36) we can predict the change in time of the mean bed level. We now assume the vertical sediment fluxes through net aggradation or degradation to be distributed over bed elevations according to their exposure to the flow, whence the bed elevation-specific entrainment and deposition fluxes are given by

$$\left(\bar{D}_e - \bar{E}_e\right)\big|_{III} = -\bar{p}_e \frac{\partial\left(\bar{q}_a + \bar{q}_{susp}\right)}{\partial x} \tag{37}$$

Furthermore, we assume the composition of the vertical sediment fluxes through net aggradation or degradation to be independent of bed surface elevation, so that the grain size-specific and bed elevation-specific entrainment and deposition fluxes are given by

$$\left(\bar{D}_{ei} - \bar{E}_{ei}\right)\big|_{III} = -\bar{p}_e \frac{\partial\left(\bar{q}_{ai} + \bar{q}_{suspi}\right)}{\partial x} \tag{38}$$

This assumption, viz. that the composition of the vertical sediment fluxes through net aggradation or degradation is independent of bed surface elevation, is not necessarily true. Present research by the first author is aimed at investigating whether this assumption is justified. Combination of (34) and (38) yields

$$\frac{\partial \bar{F}_i}{\partial t} = -\frac{\bar{p}_e}{c_b \bar{P}_s}\left(\frac{\partial\left(\bar{q}_{ai} + \bar{q}_{suspi}\right)}{\partial x} + c_b \bar{F}_i \frac{\partial \bar{\eta}_a}{\partial t}\right) \tag{39}$$

where the change in mean bed level, $\partial\bar{\eta}_a/\partial t$, can be calculated from equation (36).

Thus, equation (39) allows us to calculate the effect of net aggradation or degradation upon the vertical sorting profile (also see Figure 1). The formulations proposed in this section will be verified in a future paper.

6. Time scales

When applying the sorting evolution model in a morphodynamic model system, we can distinguish the following time scales:

1. time scale of dune migration, T_c;
2. time scale of adaptation of dune dimensions, T_p;
3. time scale of vertical sorting, T_f;
4. time scale of large-scale morphodynamic changes, T_m;

which is illustrated in Figure 4. The time scale of dune migration, T_c, is defined as the time required for a bed form to cover its average bed form length, $\bar{\lambda}$ ($T_c = \bar{\lambda}/c$).

Since all parameters in the sorting evolution model are averaged over a series of bed forms, sediment deposited at elevation z is assumed to be mixed immediately with all material present at this elevation. This implies that, for applying the sorting evolution model, it is required that the time scale of dune migration is much smaller than the time scales of adaptation of dune dimensions, vertical sorting, and morphodynamic changes:

$$T_c \ll \min\{T_p, T_f, T_m\}$$

This is consistent with the description of large-scale morphodynamic changes in many existing morphodynamic model systems. Namely, for using our common sediment transport models, which are derived for equilibrium conditions, it is required that the time scale of morphodynamic changes is much larger than the one of dune migration:

$$T_c \ll T_m$$

Likewise, for using models for hydraulic roughness, bed form height, and bed form length, which are mostly valid for steady conditions, it is required that the time scale of adaptation of dune dimensions is larger than the one of dune migration, and that the time scale of morphodynamic changes is larger than the one of adaptation of dune dimensions:

$$T_c \ll T_p$$
$$T_p \ll T_m$$

Let us now consider the relation between the sorting time scale, T_f, and the time scale of adaptation of dune dimensions, T_p. It seems that the latter is either smaller than the sorting time scale or of the same order of magnitude:

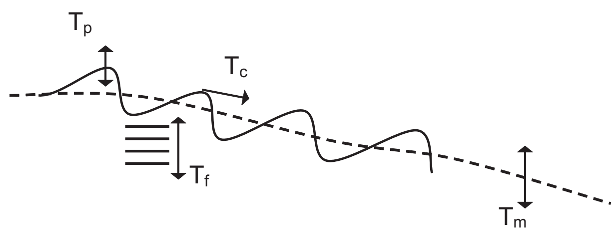

Figure 4. Time scales involved when applying the sorting evolution model in a morphodynamic model system: (1) the time scale of dune migration, T_c, (2) the time scale of adaptation of dune dimensions, T_p, (3) the time scale of vertical sorting, $T_f(z)$, and (4) the time scale of large-scale morphodynamic changes, T_m.

1. $T_p \ll T_f$
In conditions with high bed shear stresses that are well above the critical bed shear stresses of all grain sizes in the mixture, deeper bed layers that are reached by the flow only occasionally will slowly change in composition. The PDF of bed surface elevations may have reached equilibrium much faster. This was the case in, for instance, flume experiment B2 by *Blom et al.* [2003].

2. $T_p \simeq T_f$
In conditions with relatively low bed shear stresses and widely graded sediment mixtures that are predominated by partial transport (i.e. the coarsest size fractions are not or barely in transport), the time evolution of the PDF of bed surface elevations is directly related to the time evolution of the sorting profile. Coarse bed layers may develop that hinder the entrainment of bed material and thus the growth of bed forms. Fine sediment may be winnowed from below a coarse bed layer, which may result in a very slow adaptation of the bed form height. This was the case in, for instance, flume experiments A1 and B1 by *Blom et al.* [2003].

Equation (13) describes the time evolution of sorting through dune migration. It is a relaxation-type equation, but a formulation for the time scale of the adaptation of sorting is not straightforward. This is due to the fact that most parameters in (13) depend on time and the trough elevation, η_b. Yet, under the assumptions that net aggradation or degradation is negligible, the PDF of relative trough elevations is steady, the total bed load transport rate is steady, and the composition of the sediment transported over the crest is steady, the time scale of the adaptation of sorting is of the order of

$$T_f(z) = \frac{\bar{\lambda}}{2\,\bar{q}_a}\frac{c_b\bar{P}_s(z)}{\bar{p}_e(z)} \tag{40}$$

Equation (4) tells us that

1. the larger the relative amount of sediment at bed elevation z (represented by $c_b\bar{P}_s$), the slower is the adaptation of sorting;

2. the larger the exposure to the flow of elevation z ($\bar{p}_e(z)$), the faster is the adaptation of sorting;

3. the larger the average bed form length, $\bar{\lambda}$, the slower is the adaptation of sorting. Namely, the larger the average bed form length, $\bar{\lambda}$, the smaller is the amount of bed forms over some fixed distance, and the smaller are the entrainment and deposition rates;

4. the larger the total bed load transport rate, $\bar{q}_a$, the faster is the adaptation of sorting.

Note that the time scale of vertical sorting, T_f, is a function of bed elevation z. At deeper elevations of the active bed the bed composition adjusts more slowly to changing conditions than at higher bed elevations. This is due to (1) the very low elevations of the active bed being reached by the flow only occasionally (represented by a small value of $\bar{p}_e$), and (2) more bed material being present at lower bed elevations than at higher elevations (represented by a large value of $c_b\bar{P}_s$).

Note that in case of a distinct surface layer, equation 4 reduces to

$$T_f = \frac{\bar{\lambda}}{2\,\bar{q}_a}c_b\Delta \tag{41}$$

which equals the *Hirano* [1971] time scale of the adaptation of the composition of the active layer.

When the time scale of morphodynamic changes, T_m, is of the same order of magnitude as the ones of adaptation of dune dimensions, T_p and vertical sorting, T_f, we need to take into account the time evolution of both the adaptation of dune dimensions and the vertical sorting profile when computing changes in morphodynamics. The sorting evolution model is particularly adequate for this purpose.

A special situation occurs when the time scale of morphodynamic changes is much larger than the sorting time scale and the time scale of adaptation of dune dimensions:

$$T_m \gg \max\{T_p, T_f\} \qquad\qquad \min\{T_p, T_f\} \gg T_c \tag{42}$$

If, in this case, one is interested in processes at the time scale of morphodynamic changes, T_m, we may assume that the PDF of relative trough elevations, $\tilde{p}_b$, and the vertical sorting profile, $\bar{F}_i$, have reached a state of quasi-equilibrium at every point in time. In these quasi-equilibrium conditions, we may apply equilibrium similarity profiles for the PDF of relative trough elevations $\tilde{p}_b$, and the sorting profile, $\bar{F}_i$. In that case, the equilibrium sorting model [*Blom et al.*, 2006] can be applied instead of the sorting evolution model.

7. Discussion

It is emphasized that the sorting evolution model's main sorting mechanism is the grain size-selective deposition over the lee face. The model does not allow for grain size-selective entrainment over the stoss face, as all particles present at a certain elevation of the active bed are assumed to be transported over the bed form crest. Particles present on the stoss face, but too coarse to be transported, are not allowed to settle down as the bed form migrates. Instead, these coarse particles are assumed to be transported over the bed form crest and are then deposited onto the lower elevations of the active bed through the mechanism of grain size-selective deposition down the lee face. Moreover, also the winnowing of fines from the trough surface and subsurface plays a role in the formation of a coarse bed layer and is not included in the model. The mechanisms of winnowing of fines and the settling of immobile coarse particles need to be incorporated in a later version of the model, so as to improve the description of the formation of a coarse bed layer.

The present formulations for the grain size-specific and elevation-specific entrainment and deposition rates have been derived for rivers wherein the bed forms are characterized by a lee face (ripples or dunes). *Hassan and Church* [1994] remark how in such rivers the reworking or redistribution of sediment is dominated by the migrating bed forms, whereas in gravel bed rivers with an armour layer under lower-regime plane-bed conditions, the reworking is more sporadic and primarily results from local scour and fill. This has been confirmed nicely in flume experiments conducted by *Wong Egoavil* [2006].

Formulations for the grain size-specific and elevation-specific entrainment and deposition rates under plane-bed conditions can be derived by following the procedure suggested by *Parker et al.* [2000]. This procedure differs from the method in the present study in that it does not consider particle step lengths. Present research by *Parker et al.* (personal communication, 2002), among which are flume experiments, aims at modeling the PDF of bed surface elevations under plane-bed conditions.

8. Conclusions and recommendations

We have reduced the new continuum sorting model as derived by *Blom and Parker* [2004] to a model that solves for the time evolution of both the vertical sorting profile and the bed load transport composition, without applying a model for the grain size-specific and elevation-specific entrainment over bed forms. The difficulties of applying such an entrainment model have been discussed by *Blom and Parker* [2004]. The resulting sorting model is called the sorting evolution model. Like the reduction to the equilibrium sorting model [*Blom et al.*, 2006], the reduction to the sorting evolution model is based on the assumption that the composition of the net entrainment at a certain bed elevation is equal to the bed composition at this elevation. The sorting evolution model is a relaxation-type model and a time scale of sorting has been defined.

The following data is required as input to the model: the initial sorting profile; the time evolution of the PDF of relative trough elevations; the time evolution of the bed load transport rate; and the time evolution of the ratio of the average dune length to the average dune height.

It is important to note that, except for the yet calibrated lee sorting function, no parameters in the sorting evolution model have been calibrated upon.

The method to account for the effects of a change in time of the PDF of relative trough elevations in the sorting evolution model is a rather artificial one. In the present method, grain size-selective processes are not accounted for, while these processes surely play a role. Further research on this topic is recommended.

A method for incorporating net aggradation or degradation in the sorting evolution model has been proposed, but will be verified in a future paper.

We recommend further research into the following topics: (1) the incorporation of suspended load transport in a morphodynamic model system for nonuniform sediment, (2) the derivation of a model for skin friction based on the mean composition of the bed surface, (3) the derivation of a model for form drag based on the PDF of bed surface elevations.

Notation

c_b sediment concentration within the bed ($c_b = 1 - \lambda_b$).

$\bar{C}_i$ concentration of size fraction i at elevation z, averaged over a series of bed forms.

d_i geometric grain size of size fraction i, m.

d_{mlee} geometric mean grain size of the lee deposit, m.

d_{ref} geometric reference grain size ($d_{ref} = 1$ mm).

$\bar{D}$ volume of deposited sediment per unit area and time, summed over all size fractions and averaged over a series of bed forms, m s^{-1}.

$\bar{D}_e$ deposition density defined like $\bar{D}_{ei}$ but summed over all size fractions, s^{-1}.

$\bar{D}_{ei}$ deposition density of size fraction i defined such that $\bar{D}_{ei}dxdz$ is the volume of size fraction i deposited in a bed element with sides dx and dz at elevation z, per unit width and time, averaged over a series of bed forms, s^{-1}.

D_l deposition rate at the lee face, m s^{-1}.

D_{si} volume of size fraction i locally deposited onto the stoss face, per unit area and time, m s^{-1}.

$\bar{E}$ volume of entrained sediment per unit area and time, summed over all size fractions and averaged over a series of bed forms, m s^{-1}.

$\bar{E}_e$ entrainment density defined like $\bar{E}_{ei}$ but summed over all size fractions, s^{-1}.

$\bar{E}_{ei}$ entrainment density of size fraction i, defined such that $\bar{E}_{ei}dxdz$ is the volume of size fraction i entrained from a bed element with sides dx and dz at elevation z, per unit width and time, averaged over a series of bed forms, s^{-1}.

E_{snet} net entrained volume of all size fractions on the stoss face, per unit area and time, m s^{-1}.

E_{si} volume of size fraction i locally entrained from the stoss face, per unit area and time, m s^{-1}.

E_{siu} volume of size fraction i locally entrained from the stoss face, per unit area and time, if only sediment of size fraction i would be present, m s^{-1}.

$\bar{F}_{ai}$ volume fraction content of size fraction i in the bed load transport, averaged over a series of bed forms.

$\bar{F}_i$ volume fraction content of size fraction i in the bed at elevation z, averaged over a series of bed forms.

$\bar{F}_{leei}$ volume fraction content of size fraction i in the lee deposit, averaged over a series of bed forms.

$F_{leeloci}$ volume fraction content of size fraction i in the sediment deposited at elevation z at the lee face.

$\bar{F}_{topi}$ volume fraction content of size fraction i in the bed load transport over the bed form crest, averaged over a series of bed forms.

I Heaviside function which equals 1 when $\bar{P}_s(t_2) < \bar{P}_s(t_1)$.

J Heaviside function which equals 1 when considering an elevation covered by the bed form.

N total number of size fractions.

$\tilde{p}_b$ adapted probability density function of trough elevations relative to the mean bed level for a series of bed forms, indicating the probability density that the trough elevation equals z, weighted by the horizontal distance involved, m^{-1}.

$\bar{p}_e$ probability density function of bed surface elevations for a series of bed forms, indicating the probability density that the bed surface elevation equals z, m^{-1}.

p_e probability density function of bed surface elevations for an individual bed form, m^{-1}.

$\bar{P}_s$ probability distribution of bed surface elevations for a series of bed forms, indicating the probability the bed surface elevation is higher than z.

q volume of bed load transport per unit width and time (excluding pores), m^2 s^{-1}.

$\bar{q}_a$ volume of bed load transport per unit width and time (excluding pores), averaged over a series of bed forms (excluding pores), m^2 s^{-1}.

$\bar{q}_{top}$ volume of bed load transport at the bed form crest per unit width and time (excluding pores), averaged over a series of bed forms (excluding pores), m^2 s^{-1}.

t time co-ordinate, s.

T_c time scale of dune migration, i.e. the time required for a bed form to cover its average bed form length $\bar{\lambda}$, s.

T_p time scale of the evolution of the PDF of trough elevations, s.

T_f time scale of vertical sorting, s.

T_m time scale of morphodynamic changes, s.

x horizontal co-ordinate, m.

z vertical co-ordinate, m.

$\tilde{z}$ vertical co-ordinate relative to the mean bed level $\bar{\eta}_a$, m.

z^* dimensionless vertical co-ordinate relative to the mean bed level $\bar{\eta}_a$.

$\hat{z}^*$ dimensionless vertical co-ordinate relative to the mean bed level $\bar{\eta}_a$.
γ constant in lee sorting function.
δ_i lee sorting parameter.
Δ bed form height, m.
Δ_a bed form height averaged over a series of bed forms, m.
Δ_b trough elevation relative to the mean bed level, i.e. the relative trough elevation, m.
η local bed surface elevation, m.
$\bar{\eta}_a$ bed surface elevation averaged over a series of bed forms (mean bed level), m.
η_b bed form trough elevation, m.
η_{bmax} highest bed form trough elevation, m.
η_{bmin} lowest bed form trough elevation, m.
η_t bed form crest elevation, m.
η_{mn} lower limit of the active bed, m.
η_{mx} upper limit of the active bed, m.
η_{stepi} step length in z-direction for size fraction i, m.
κ constant in the lee sorting function.
λ bed form length, m.
λ_a bed form length averaged over a series of bed forms, m.
λ_b porosity.
Λ_i step length of size fraction i, m.
μ_{Δ_b} mean value of relative trough elevation, m.
ν angle of repose, °.
σ_a arithmetic standard deviation of the composition of the lee deposit.
σ_{Δ_b} standard deviation of relative trough elevation, m.
$\bar{\tau}_b$ bed shear stress averaged over a series of bed forms, N m^{-2}.
ϕ_i arithmetic grain size of size fraction i.
ϕ_{mlee} arithmetic mean grain size of the lee deposit.
ω_i lee sorting function, specifying to what extent a specific size fraction that is transported over the bed form crest is deposited at elevation z of the lee face.

Subscript
i number of the size fraction.
l lee face.
s stoss face.

An asterisk denotes a dimensionless parameter. An overbar denotes that a parameter is horizontally averaged over a series of bed forms. A double overbar indicates a parameter is horizontally averaged over a series of bed forms and over a specific range of bed elevations. A tilde indicates a parameter is relative to the mean bed level.

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References

Blom, A., and G. Parker (2004), Vertical sorting and the morphodynamics of bedform-dominated rivers: A modeling framework, *J. Geophys. Res.*, *109*, F02007, doi:10.1029/2003JF000069.

Blom, A., J. S. Ribberink, and H. J. de Vriend (2003), Vertical sorting in bed forms: Flume experiments with a natural and a trimodal sediment mixture, *Water Resour. Res.*, *39*(2), 1025, doi:10.1029/2001WR001088.

Blom, A., G. Parker, J. S. Ribberink, and H. J. de Vriend (2006), Vertical sorting and the morphodynamics of bed-form-dominated rivers: An equilibrium sorting model, *J. Geophys. Res.*, *111*, F01006, doi:10.1029/2004JF000175.

Blom, A., J. S. Ribberink, and G. Parker (2007), Vertical sorting and the morphodynamics of bedform-dominated rivers: A sorting evolution model. 2. Application, *submitted to J. Geophys. Res.*

Einstein, H. A. (1950), The bed-load function for sediment transportation in open channel flows, *Technical Report 1026*, US Dept. Agric., Soil Conserv. Serv., T.B.

Hassan, M. A., and M. Church (1994), Vertical mixing of coarse particles in gravel bed rivers: a kinematic model, *Water Resour. Res.*, *30*(4), 1173–1185.

Hirano, M. (1970), On phenomena of river-bed lowering and armouring below reservoirs, in *Proc. 14th. Hydraul. Lecture Meeting*, 13-14 Feb., Civ. Eng. Ass. Hydr. Committee, Hatsumei Kaikan.

Hirano, M. (1971), River bed degradation with armouring, *Transactions Jap. Soc. Civ. Eng.*, *3*(2), 194–195.

Hirano, M. (1972), Studies on variation and equilibrium state of a river bed composed of nonuniform material, *Transactions Jap. Soc. Civ. Eng.*, *4*, 128–129.

Parker, G., C. Paola, and S. Leclair (2000), Probabilistic Exner sediment continuity equation for mixtures with no active layer, *J. Hydraul. Eng.*, *126*(11), 818–826.

Ribberink, J. S. (1987), Mathematical modelling of one-dimensional morphological changes in rivers with non-uniform sediment, Ph.D. thesis, Delft University of Technology, the Netherlands.

Van der Mark, C. F., A. Blom, S. J. M. H. Hulscher, S. F. Leclair, and D. Mohrig (2005), On modeling the variability of bedform dimensions, in *Proc. IAHR Symposium on River, Coastal and Estuarine Morphodynamics*, edited by G. Parker and M. H. Garcia, pp. 831–841, Taylor & Francis Group, London, Urbana, Illinois, USA.

Van der Scheer, P., J. S. Ribberink, and A. Blom (2002), Transport formulas for graded sediment: behaviour of transport formulas and verification with data, *Technical Report CiT: 2002R-002, ISSN 1568-4652*, University of Twente.

Wilcock, P. R., and J. C. Crowe (2003), Surface-based transport model for mixed-size sediment, *J. Hydraul. Eng.*, *129*(2), 120–128.

Wong Egoavil, M. (2006), Model for erosion, transport and deposition of tracer stones in gravel-bed streams, Ph.D. thesis, University of Minnesota, Minneapolis, USA.

Wu, W., S. S. Y. Wang, and Y. Jia (2000), Nonuniform sediment transport in alluvial rivers, *J. Hydraul. Res.*, *38*(6), 427–434.

A. Blom, University of Twente, Civil Engineering, P.O. Box 217, 7500 AE Enschede, Netherlands. (a.blom@utwente.nl)

J. S. Ribberink, University of Twente, Civil Engineering, P.O. Box 217, 7500 AE Enschede, Netherlands. (j.s.ribberink@utwente.nl)

G. Parker, Department of Civil and Environmental Engineering and Department of Geology, University of Illinois at Urbana-Champaign, Urbana IL 61801, USA. (parkerg@uiuc.edu)

Appendix A: The PPL framework

This section presents the main equations of the Parker-Paola-Leclair (PPL) framework for sediment continuity

[*Parker et al.*, 2000]. Sediment conservation of size fraction i at elevation z is given by

$$\frac{\partial \bar{C}_i}{\partial t} = c_b \bar{P}_s \frac{\partial \bar{F}_i}{\partial t} + c_b \bar{F}_i \frac{\partial \bar{P}_s}{\partial t} = \bar{D}_{ei} - \bar{E}_{ei} \quad \text{(A1)}$$

where $\bar{C}_i$ denotes the concentration of size fraction i at elevation z ($\bar{C}_i = c_b \bar{P}_s \bar{F}_i$), $\bar{F}_i$ denotes the volume fraction content of size fraction i at elevation z, and $\bar{P}_s$ denotes the probability distribution of bed surface elevations indicating the probability that the bed elevation is higher than z. $\bar{D}_{ei}$ denotes the deposition density of size fraction i defined such that $\bar{D}_{ei} dx dz$ is the volume of sediment of size fraction i that is deposited per unit width and time in a bed element with sides dx and dz at elevation z, and $\bar{E}_{ei}$ denotes the entrainment density of size fraction i defined likewise, and c_b the concentration of sediment in the bed ($c_b = 1 - \lambda_b$, where λ_b denotes the porosity). The bar indicates that the parameter is averaged over some horizontal distance, e.g., a large number of bed forms, x denotes the horizontal co-ordinate on the scale of series of bed forms, z denotes the vertical co-ordinate, and t denotes the time co-ordinate.

Applying a co-ordinate transformation ($\tilde{x} = x$, $\tilde{t} = t$, and $\tilde{z} = z - \bar{\eta}_a$ wherein $\tilde{z}$ denotes the deviation from the mean bed level, $\bar{\eta}_a$), and the chain rule yields

$$\frac{\partial \bar{P}_s}{\partial t} = \frac{\partial \tilde{P}_s}{\partial t} + \bar{p}_e \frac{\partial \bar{\eta}_a}{\partial t} \quad \text{(A2)}$$

where $\tilde{P}_s$ denotes the probability distribution of bed surface elevations relative to the mean bed level, $\bar{\eta}_a$. The PDF of bed surface elevations, $\bar{p}_e$, expresses the probability density that the bed surface elevation equals z or the likelihood of elevation z being exposed to the flow ($\bar{p}_e = -\partial \bar{P}_s / \partial z = -\partial \tilde{P}_s / \partial \tilde{z}$). With equation (A2), equation (A1) becomes

$$c_b \bar{P}_s \frac{\partial \bar{F}_i}{\partial t} + c_b \bar{F}_i \frac{\partial \tilde{P}_s}{\partial t} + c_b \bar{F}_i \bar{p}_e \frac{\partial \bar{\eta}_a}{\partial t} = \bar{D}_{ei} - \bar{E}_{ei} \quad \text{(A3)}$$

Adding up equation (A3) over all grain sizes yields

$$c_b \frac{\partial \tilde{P}_s}{\partial t} + c_b \bar{p}_e \frac{\partial \bar{\eta}_a}{\partial t} = \bar{D}_e - \bar{E}_e \quad \text{(A4)}$$

where $\bar{D}_e$ denotes the deposition density defined such that $\bar{D}_e dx dz$ is the volume of all size fractions deposited in a bed element with sides dx and dz at elevation z per unit width and time ($\bar{D}_e = \sum_i^N \bar{D}_{ei}$ where N denotes the total number of size fractions) and $\bar{E}_e$ denotes the entrainment density defined likewise.

Integration of equation (A4) over all bed elevations yields

$$c_b \frac{\partial \bar{\eta}_a}{\partial t} = \bar{D} - \bar{E} \left(= -\frac{\partial \bar{q}_a}{\partial x} \right) \quad \text{(A5)}$$

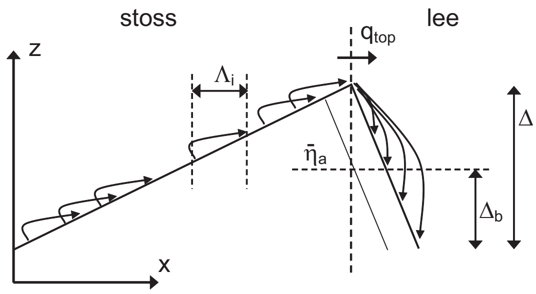

Figure 5. Bed form parameters, and division of bed form in stoss and lee sides with accompanying entrainment and deposition fluxes.

where $\bar{D}$ denotes the volume of all size fractions deposited per unit area and time ($\bar{D} = \int_{-\infty}^{\infty} \bar{D}_e \, dz$), and $\bar{E}$ the entrainment rate defined likewise. In equation (A5) we recognize the commonly applied sediment continuity equation, where $\bar{q}_a$ denotes the bed load transport rate averaged over a series of bed forms.

Appendix B: The BP framework

The work by *Blom and Parker* [2004] adapts the derivation of formulations for the grain size-specific and bed elevation-specific entrainment and deposition fluxes as required for the PPL framework to the case of a field of dunes. This section presents the fundamental equations of the Blom-Parker framework. They distinguish between an entrainment flux and a deposition flux at the stoss face, $\bar{E}_{eis}$ and $\bar{D}_{eis}$, and a deposition flux at the lee face, $\bar{D}_{eil}$ (Figure 5).

The parameter E_{siu} is introduced as the volume of sediment of size fraction i picked up from the bed per unit length, width, and time, in case only size fraction i is present. The weighted entrainment rate $E_{si}(x)$ denotes the volume of sediment of grain size d_i locally entrained from the stoss face per unit area and time:

$$E_{si}(x) = E_{siu}(x) \, F_i(x) \quad \text{(B1)}$$

and the weighted deposition rate D_{si} of size fraction i at x equals the weighted entrainment rate of this size fraction one step length upstream of x ($D_{si}(x) = E_{si}(x - \Lambda_i)$):

$$D_{si}(x) = E_{siu}(x - \Lambda_i) \, F_i(x - \Lambda_i) \quad \text{(B2)}$$

where the Einstein step length is given by $\Lambda_i = \alpha d_i$ (Figure 5).

The transport rate of size fraction i at co-ordinate x at the stoss face, q_{si}, is given by

$$q_{si}(x) = \int_0^{\Lambda_i} E_{siu}(x - y) F_i(x - y) \, dy \quad \text{(B3)}$$

The total volume transport rate per unit width at x is denoted as $q_s(x)$, where $q_s(x) = \sum_i^N q_{si}(x)$. The volume fraction content of size fraction i in the transported sediment on the stoss face, F_{qsi}, then equals q_{si}/q_s. For the derivation of these equations we refer to the derivation of equations (BP-15), (BP-16), and (BP-19) in *Blom and Parker* [2004].

Blom and Parker [2004] derive the following expressions for the entrainment and deposition densities averaged over a series of irregular bed forms:

$$\bar{E}_{eis}(z) = \int_{\eta_{bmin}}^{\eta_{bmax}} \frac{\lambda_s}{\lambda} p_{se}(z) E_{siu}(z) \, \bar{F}_i(z) \, \tilde{p}_b \, d\eta_b \quad \text{(B4)}$$

$$\bar{D}_{eis}(z) = \int_{\eta_{bmin}}^{\eta_{bmax}} \frac{\lambda_s}{\lambda} p_{se}(z) E_{siu}(z - \eta_{stepi}(z)) \cdot \bar{F}_i(z - \eta_{stepi}(z)) \, \tilde{p}_b \, d\eta_b \quad \text{(B5)}$$

$$\bar{D}_{eil}(z) = \int_{\eta_{bmin}}^{\eta_{bmax}} \frac{\lambda_l}{\lambda} p_{le}(z) D_l \, F_{leeloci}(z) \, \tilde{p}_b \, d\eta_b \quad \text{(B6)}$$

where the subscript s indicates the stoss face, the subscript l indicates the lee face, η_{stepi} denotes the vertical step length at elevation z on the stoss face for size fraction i, D_l denotes the deposition rate at the lee face, $F_{leeloci}$ denotes the volume fraction content of size fraction i in the sediment deposited at elevation z at the lee face, $\tilde{p}_b$ denotes the PDF of trough elevations relative to the mean bed level for a series of bed forms, indicating the probability density that the trough elevation equals z, weighted by the horizontal distance involved, η_b denotes the trough elevation,

η_{bmax} denotes the highest trough elevation, and η_{bmin} denotes the lowest trough elevation. Note that the integral in these equations denotes the procedure of averaging over all trough elevations. For the derivation of these equations we refer to equations (BP-59)-(BP-61) of *Blom and Parker* [2004].

The deposition rate at the lee face, D_l, the volume fraction content of size fraction i in the sediment deposited at the lee face, $F_{leeloci}$, and the volume fraction content of size fraction i in the deposit at the bed form lee face, F_{leei}, are given by equations (BP-28), (BP-36), and (BP-31):

$$D_l = \frac{q_{top}}{\lambda_l} - \frac{\lambda}{\lambda_l}\frac{\partial q_a}{\partial x} \tag{B7}$$

$$F_{leeloci} = F_{leei}\,\omega_i \tag{B8}$$

$$F_{leei} = \frac{1}{\lambda_l D_l}\left(q_{topi} - \lambda\frac{\partial q_{ai}}{\partial x}\right) \tag{B9}$$

where the bed load transport rate of size fraction i at the bed form crest, q_{topi}, and the bed-form-averaged bed load transport rate of size fraction i, q_{ai}, are given by equations (BP-B16) and (BP-B17):

$$q_{topi} = \lambda_s \int_{\eta_t-\eta_{stepi}}^{\eta_t} E_{siu}(z)F_i(z)p_{se}(z)\,dz \tag{B10}$$

$$q_{ai} = \frac{\lambda_s^2}{\lambda}\int_{\eta_b}^{\eta_t}\int_0^{\eta_{stepi}} E_{siu}(z-z')F_i(z-z')p_{se}(z)\cdot p_{se}(z')\,dz'\,dz + \frac{\lambda_l}{2\lambda}D_l\lambda_l F_{leei} \tag{B11}$$

where the bed-form-averaged bed load transport rate is given by $q_a = \sum_i^N q_{ai}$ and the bed-form-averaged volume fraction content of size fraction i in the bed load transport $F_{ai} = q_{ai}/q_a$. The parameters q_{top} and F_{topi}, indicating the bed load transport at the bed form crest, are defined likewise. The lee sorting function, ω_i, determines to what extent a specific size fraction that is transported over the bed form crest is deposited at a certain elevation of the lee face, and is given by equation (BP-38):

$$\omega_i = J\,(1 + \delta_i\,\hat{z}^*) \tag{B12}$$

The Heaviside function $J(z)$ equals 1 when considering an elevation covered by the specific bed form. For triangular dunes $\hat{z}^* = z^* = (z - \bar{\eta}_a)/\Delta$. As a very first step toward a generic formulation, the following expression is proposed for the lee sorting parameter, δ_i:

$$\delta_i = \gamma\,\frac{\phi_i - \phi_{mlee}}{\sigma_a}\,(\tau_b^*)^{-\kappa} \tag{B13}$$

also see equation (BP-40). Herein ϕ_{mlee} denotes the arithmetic mean grain size of the lee deposit and ϕ_i denotes the arithmetic grain size of size fraction i:

$$\phi_i = -{}^2\log\left(\frac{d_i}{d_{ref}}\right) \tag{B14}$$

where d_i denotes grain size and in which the reference grain size d_{ref} equals 1 mm. Note that equation (B14) equals the conventional manner in which the arithmetic mean grain size is calculated. Yet, this formulation is the mathematically correct notation, as logarithms cannot be taken of non-dimensionless parameters. As a result, the arithmetic grain size (ϕ) is correctly dimensionless. In equation B13, τ_b^* denotes the dimensionless bed shear stress averaged over the bed form length. The constant γ weights the relative importance of the grain-size term on the right-hand side of (B13), while the value of κ sets the relative importance of the dimensionless bed shear stress term. For the derivation of the above equations we refer to *Blom and Parker* [2004].