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Quasi-Universal Relations for Bankfull Hydraulic Geometry of Single-Thread Gravel-Bed Rivers

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We examine relations for hydraulic geometry of alluvial, single-thread gravel-bed rivers with definable bankfull geometries. Four baseline data sets determine relations for bankfull geometry, i.e. bankfull depth, bankfull width and down-channel slope as functions of bankfull discharge and bed surface median sediment size. In appropriate dimensionless form these relations show a remarkable degree of universality. This universality applies not only within the four sets used to determine the forms, but also to four independent data sets as well. We study the physical basis for this universality in terms of four relations that can be back-calculated from the data: a) a Manning-Strickler-type relation for channel resistance, b) a channel-forming relation expressed in terms of the ratio of bankfull Shields number to critical Shields number, c) a relation for critical Shields number as a function of dimensionless discharge and d) a “gravel yield” relation specifying the (estimated) gravel transport rate at bankfull flow as a function of bankfull discharge and gravel size. We use these underlying relations to explore why the dimensionless bankfull relations are only quasi-universal, and to quantify the degree to which deviation from universality can be expected. Finally, we use the analysis to obtain a first estimate of a partition between skin friction and form drag.

1. Introduction

Single-thread, alluvial gravel-bed rivers represent an important class of natural rivers and a common target for restoration projects. Here “gravel-bed” is used in a loose sense, and refers to stream reaches for which the surface median grain size D_{s50} is greater than 25 mm. Many (but by no means all) such river reaches have a distinct channel and floodplain, such that flow spills from the channel onto the floodplain at a well-defined "bankfull" discharge Q_{bf}. For such reaches it is possible to define a "bankfull channel geometry" (Leopold and Maddock, 1953; Leopold et al., 1964) in terms of a bankfull width B_{bf}, bankfull depth H_{bf} and downchannel bed slope S. Leopold and Maddock (1953) considered the downstream variation of these parameters in terms of power relations of the form

$$B_{bf} = \chi_B Q_{bf}^{n_B} \quad , \quad H_{bf} = \chi_H Q_{bf}^{n_H} \quad , \quad S = \chi_S Q_{bf}^{-n_S} \qquad (1a,b,c)$$

They offered estimates two of these parameters, i.e. n_B and n_H as 0.5 and 0.4, respectively.

Bray (1982) studied the bankfull geometry of single-thread alluvial gravel-bed streams, i.e. the class studied here, and further expanded the analysis to include different streams as well as different reaches of the same stream. Based on data from Canada, Bray (1982) determined the following estimates for the exponents:

$$n_B = 0.527 \quad , \quad n_H = 0.333 \quad , \quad n_S = 0.342 \qquad (2a,b,c)$$

This work has been extended by Hey and Thorne (1986), who suggest the values 0.52 and 0.39 for n_B and n_H respectively, based on an analysis of British gravel-bed streams. Relations of the form of (1a,b,c) are not, however, dimensionally homogeneous, and thus may not reveal the physics underlying the relations. Parker (1979), Andrews (1984), Parker and Toro-Escobar (2002) and Parker *et al.* (2003) developed dimensionless forms for bankfull geometry of single-thread gravel-bed streams, and Ashmore and Parker (1983) developed similar dimensionless relations for anabranches of braided gravel-bed streams. These dimensionless relations have not seen much application to date. For example, contained within the comprehensive survey of Soar and Thorne (2001) is the statement: "…non-dimensional regime-type relationships are considered unsuitable for developing design equations for bankfull width…"

The present analysis is intended to provide further justification for a dimensionless formulation by a) establishing quasi-universal dimensionless relations for hydraulic geometry for single-thread, alluvial gravel-bed streams, and b) developing a physical basis from which these relations can be derived. Four sets of data for alluvial, single-thread gravel-bed streams are used to develop these relations. The non-dimensionalization used here for bankfull width and depth is different from (although ultimately equivalent to) that used in e.g. Parker *et al.* (2003). This modified non-dimensionalization reveals a remarkable constancy in dimensionless bankfull depth, and near-constancy in dimensionless bankfull width, over some four orders of magnitude of variation of dimensionless bankfull discharge.

2. Governing parameters

The following parameters are defined for reaches of alluvial, single-thread gravel-bed rivers: bankfull discharge Q_{bf}, bankfull width B_{bf}, bankfull depth H_{bf}, down-channel bed slope S, median size D_{s50} of the sediment on the surface of the bed and the acceleration of gravity S. The following relations for hydraulic geometry at bankfull flow are postulated:

$$\begin{aligned} B_{bf} &= f_B(Q_{bf}, D_{s50}, g, \text{other parameters}) \\ H_{bf} &= f_H(Q_{bf}, D_{s50}, g, \text{other parameters}) \\ S &= f_S(Q_{bf}, D_{s50}, g, \text{other parameters}) \end{aligned} \quad (3a,b,c)$$

Examples of “other parameters” include gravel supply, the type and density of bank vegetation, bank material type (e.g. Hey and Thorne, 1986) and channel planform. Here the “other parameters” are dropped with the purpose of determining how much universality can be obtained with the shortest possible list of governing parameters. Additional parameters are reconsidered later as factors that can contribute to deviation from universality.

Each of (3a), (3b) and (3c) defines a relation involving four parameters [e.g. B_{bf}, Q_{bf}, D_{s50} and g in the case of (3a)] and two dimensions, length and time. The principles of dimensional analysis allow each relation to be expressed in terms of two dimensionless parameters. Parker (1979), Andrews (1984), Parker and Toro-Escobar (2002) and Parker *et al.* (2003) have used the following forms;

$$\hat{B} = \hat{f}_{\hat{B}}(\hat{Q}) \quad , \quad \hat{H} = \hat{f}_{\hat{H}}(\hat{Q}) \quad , \quad S = \hat{f}_S(\hat{Q}) \quad (4a,b,c)$$

where

$$\hat{B} = \frac{B_{bf}}{D_{s50}} \quad , \quad \hat{H} = \frac{H_{bf}}{D_{s50}} \quad , \quad \hat{Q} = \frac{Q_{bf}}{\sqrt{gD_{s50}}\, D_{s50}^2} \quad (5a,b,c)$$

Here we adopt an alternative but equivalent nondimensionalization for bankfull width and depth, originally suggested by Bray (1982). Defining the dimensionless parameters $\tilde{B}$ and $\tilde{H}$ as

$$\tilde{B} = \frac{g^{1/5} B_{bf}}{Q_{bf}^{2/5}} \quad , \quad \tilde{H} = \frac{g^{1/5} H_{bf}}{Q_{bf}^{2/5}} \quad (6a,b)$$

we seek relations of the following form;

$$\tilde{B} = \hat{f}_{\tilde{B}}(\hat{Q}) \quad , \quad \tilde{H} = \tilde{f}_{\tilde{H}}(\hat{Q}) \quad , \quad S = \hat{f}_S(\hat{Q}) \quad (7a,b,c)$$

More specifically, we anticipate power relations of the form

$$\tilde{B} = \alpha_B \hat{Q}^{n_B} \quad , \quad \tilde{H} = \alpha_H \hat{Q}^{n_H} \quad , \quad S = \alpha_S \hat{Q}^{n_S} \quad (8a,b,c)$$

Note that as opposed to the coefficients in the relations (1a,b,c), which have dimensions that are entirely dependent upon the choice of the exponents, the coefficients in (8a,b,c) are dimensionless.

Dimensionless relations involving the forms $\tilde{B}$ and $\tilde{H}$ are equivalent to corresponding relations involving $\hat{B}$ and $\hat{H}$ because according to (5) and (6),

$$\tilde{B} = \hat{B}\hat{Q}^{-2/5} \quad , \quad \tilde{H} = \hat{H}\hat{Q}^{-2/5} \quad (9a,b)$$

The first motivation for the choice of the forms $\tilde{B}$ and $\tilde{H}$ as opposed to $\hat{B}$ and $\hat{H}$ in the present analysis is related to the possibility of spurious correlation (e.g. Hey and Heritage, 1986). That is, $\hat{B}$, $\hat{H}$ and $\hat{Q}$ all contain grain size D_{s50}, and so allow the possibility of spurious correlation through this parameter, whereas $\tilde{B}$

and $\tilde{H}$ do not contain grain size D_{s50}, which only appears in $\hat{Q}$. As is seen below, however, a more powerful motivation results from the analysis. It is found that within the scatter of the data, $\tilde{H}$ shows essentially no variation with $\hat{Q}$, and $\tilde{B}$ shows only weak variation with $\hat{Q}$.

3. Baseline data set

The baseline data set for bankfull geometry of gravel-bed streams used here is composed of four subsets. These include a) 16 stream reaches in Alberta, Canada contained in Kellerhals *et. al* (1972) (and identified in more detail in Parker, 1979), b) 23 stream reaches in Britain contained in Charlton *et. al* (1978), 23 stream reaches in Idaho, USA (Parker *et. al*, 2003) and 10 reaches of the Colorado River, western Colorado and eastern Utah, USA (Pitlick and Cress, 2000), for a total of 72 reaches. These four sets are respectively referred to as "Alberta," "Britain I,", "Idaho" and "ColoRiver." The terminology "Britain I" is used because a second set of data from Britain is introduced later.

The baseline data set is available at (*to be specified upon acceptance of paper*). The data for B_{bf}, H_{bf}, S and D_{s50} for each of the 10 reaches of the Colorado River represent medians of values for a larger number of subreaches, as extracted from the compendium in Table A-5 of the Appendices of Pitlick and Cress (2000). The data thus differ modestly from the data given in Table 1 of Pitlick and Cress (2002), which are based on averages rather than medians.

The parameters of the baseline set vary over the following ranges:

- bankfull discharge Q_{bf} varies from 2.7 to 5440 m^3/s;
- bankfull width B_{bf} varies from 5.24 to 280 m;
- bankfull depth H_{bf} varies from 0.25 to 6.95 m;
- down-channel bed slope S varies from 0.00034 to 0.031; and
- surface median grain size D_{s50} varies from 27 to 167.5 mm.

Only the data set of Charlton *et al.* (1978) include measured values for sediment specific gravity. The average value for their 23 reaches is 2.63. In all other cases the sediment specific gravity has been assumed to be the standard value for quartz, i.e. 2.65.

4. Quasi-universal relations for hydraulic geometry

Figure 1 shows on a single plot $\tilde{B}$, $\tilde{H}$ and S as functions of $\hat{Q}$. The relations define distinct trends across four decades of variation of $\hat{Q}$. Simple regression yields the following power-law forms for dimensionless bankfull hydraulic geometry;

$$\tilde{B} = 4.63\hat{Q}^{0.0667} \quad \text{i.e.} \quad \alpha_B = 4.63 \quad , \quad n_B = 0.0667 \pm 0.027$$
$$\tilde{H} = 0.382\hat{Q}^{-0.0004} \quad \text{i.e.} \quad \alpha_H = 0.382 \quad , \quad n_H = -0.0004 \pm 0.027$$
$$S = 0.101\hat{Q}^{-0.344} \quad \text{i.e.} \quad \alpha_S = 0.101 \quad , \quad n_S = 0.344 \pm 0.066 \tag{10a,b,c}$$

In the above relations, the uncertainties in the the exponents were computed at the 95% confidence level. At the 95% confidence level the prediction interval is a factor of 3.0 for (10a), 3.0 for (10b) and 14.8 for (10c). These relations turn out upon reduction with (9a,b) to be very close to the relations for $\hat{B}$, $\hat{H}$ and S versus $\hat{Q}$ given in Parker and Toro-Escobar (2002) and Parker *et al.* (2003). This notwithstanding, the present formulation has at least one distinct advantage, i.e. the rather remarkable result of constant $\tilde{H}$.

Figure 1 and regression relation (10b) indicate that for all practical purposes (10b) can be replaced with constant value

$$\tilde{H} \equiv \tilde{H}_o = 0.382 \tag{11}$$

over the entire range of $\hat{Q}$. Specifically, this yields the dimensional form

$$H_{bf} = \frac{0.382}{g^{1/5}} Q_{bf}^{2/5} \tag{12}$$

That is, within the scatter of the data, bankfull depth H_{bf} varies with bankfull discharge Q_{bf} to the 2/5 power, independently of grain size D_{s50}. Thus (12) predicts that a doubling of bankfull discharge results in an increase in bankfull depth by a factor of 1.32. A doubling of grain size D_{s50}, however, is predicted to result in no change in H_{bf}.

It is also seen from Figure 1 that dimensionless bankfull width $\tilde{B}$ does not vary strongly with $\hat{Q}$, with typical values of $\tilde{B}$ near 10. This notwithstanding, $\tilde{B}$ does systematically increase with $\hat{Q}$; (10a) yields the dimensioned form

$$B_{bf} = \frac{4.63}{g^{1/5}} Q_{bf}^{0.4} \left(\frac{Q_{bf}}{\sqrt{gD_{s50}}\, D_{s50}^2} \right)^{0.0667} \tag{13}$$

That is, B_{bf} varies with $Q_{bf}^{0.4667}$ and $D_{s50}^{-0.167}$. According to (13) a doubling of bankfull discharge results in an increase of bankfull width by a factor of 1.38; a doubling of grain size D_{s50} results in a decrease of bankfull width by a factor of 0.89.

The dimensionless relation for slope can be written using dimensioned parameters as

$$S = 0.101 \left(\frac{Q_{bf}}{\sqrt{gD_{s50}}\, D_{s50}^2} \right)^{-0.344} \tag{14}$$

Thus S varies with $Q_{bf}^{-0.344}$ and $D_{s50}^{0.860}$. According to (14), a doubling in bankfull discharge results in a slope decrease by a factor of 0.79; a doubling of grain size results in a slope increase by a factor of 1.81.

The four data sets (Alberta, Britain I, Idaho and ColoRiver) are discriminated in Figure 2. The data points of the four sets all intermingle one among the other, indicating a substantial degree of universal behavior among data from four distinct geographical regions.

The relations (10a), (11) and (10c) are nevertheless described as "quasi-universal" here because the effects of the "other parameters" in (3) are discernible. Figures 3a and 3b illustrate the clearest deviation from universality; the Britain I rivers are systematically somewhat deeper and narrower than the Alberta rivers. One reason for this may be the more humid climate and consequent denser bank vegetation in the case of the Britain I streams, so increasing the effective "bank strength" relative to the Alberta streams (e.g. Charlton *et al.*, 1978; Hey and Thorne, 1986). Another reason may be the likelihood that the British streams have a lower supply of gravel (after normalizing for water supply) than the Alberta streams. Both of these factors are discussed in more detail below.

The scatter in Figures 1, 2 and 3 also likely embodies an element of measurement error in the parameters in question. Perhaps the parameter that is most subject to measurement error is the surface median grain size D_{s50}; in most cases the samples of bed material from which it was determined likely did not satisfy the rigorous guidelines of Church *et al.* (1987). The down-channel bed slope S is subject to error if the reach used to determine it is not sufficiently long. In addition, bankfull width and depth B_{bf} and H_{bf} are subject to error if they are not based on appropriately-defined reach averaged characteristics, and bankfull discharge Q_{bf} may be difficult to discern from a rating curve if there is not a clear break in the stage-discharge relationship as the flow spills overbank. A number of these issues are discussed in the careful data compilation of Church and Rood (1983).

In Figures 1 and 2 the data for slope show the most scatter, even though there seem to be no systematic differences among the four data sets. As noted above, part of this scatter may be due to measurement error, particularly in the measurement of D_{s50} and S. There is, however, another compelling reason or scatter in the slope relation. Mobile-bed rivers are free to change their bankfull width and depth over short geomorphic time (e.g. 100's or 1000's of years). Slope changes other than those associated with changes in sinuosity, however, require a complete restructuring of the long profile of the river. Such a restructuring must occur over much longer geomorphic time scales, over which such factors as tectonism, climate change and sea level variation make themselves felt [and thus enter as "other parameters" (3)]. This notwithstanding, the slope relation still shows a considerable degree of systematic variation.

Both the predictive quality of the relations (10a), (11) and (10c) and the extent to which “other parameters” are felt can also be studied by plotting values of B_{bf}, H_{bf} and S predicted from (10a), (11) and (10c) versus the reported values. Figure 4a shows predicted versus observed values for B_{bf}. All of the 72 predicted values are between 1/2 and 2 times the reported values. Figure 4b shows predicted versus observed values for H_{bf}; again, all of the 72 predicted values are between 1/2 and 2 times the reported values. Figure 4c shows predicted versus observed values of S; 52 of the 72 predicted values, or 72% are within 1/2 and 2 times the reported values.

Variation within the data sets can be studied in terms of the average value of the ratio $(X)_{pred}/(X)_{rep}$ for each set, where $(X)_{pred}$ denotes the predicted value of parameter X and $(X)_{rep}$ denotes the reported value. These results are given in Table 1, and can be summarized as follows:

- Average values of $(B_{bf})_{pred}/(B_{bf})_{rep}$
  Alberta: 0.83
  Britain I: 1.29
  Idaho: 0.97
  ColoRiver: 0.97
- Average values of $(H_{bf})_{pred}/(H_{bf})_{rep}$
  Alberta: 1.28
  Britain I: 0.81
  Idaho: 1.09
  ColoRiver: 1.08
- Average values of $(S)_{pred}/(S)_{rep}$
  Alberta: 1.16
  Britain I: 1.32
  Idaho: 1.38
  ColoRiver: 1.01

The Alberta streams are systematically wider and shallower, and the Britain I streams narrower and deeper, than that predicted by the regression relations. The average ratios $(B_{bf})_{pred}/(B_{bf})_{rep}$, $(H_{bf})_{pred}/(H_{bf})_{rep}$ and $(S)_{pred}/(S)_{rep}$ are nevertheless in all cases sufficiently close to unity to strengthen the case for quasi-universality of the relations.

5. Comparison of the regression relations against four independent sets of data

Four independent sets of data on gravel-bed rivers are used to test the regression relations presented above. The first of these consists of 24 stream reaches from Colorado compiled by Andrews (1984). This data set is here referred to as “ColoSmall”, not because all the reaches in question are small streams, but because a) none of them pertain to the Colorado River itself and b) all of them are much smaller than any of the reaches included in the “ColoRiver” set. The ranges of parameters for the “ColoSmall” data mostly falls within the corresponding ranges of the baseline data set, but the former set does include some smaller streams.

The second of these consists of 11 stream reaches from Maryland and Pennsylvania, USA (McCandless, 2003), here referred to as “Maryland” for short. The original data set contained 14 reaches, but three of these were excluded because a) the stream was bedrock, or b) the value of D_{s50} was substantially below the range of the baseline set (27 mm to 167.5 mm) or c) the value of S was substantially above the range of the baseline set (0.00034 to 0.031).

The third set of data is the British set of 62 reaches compiled by Hey and Thorne (1986). The specific reaches in this set, which we here term “Britain II” for short, are largely different from those in the Britain I compilation of Charlton *et al.* (1978) used earlier to derive (10a), (11) and (10b). This notwithstanding, the overall characteristics of the “Britain II” streams are, perhaps unsurprisingly in light of the correspondence in geography, quite similar to the “Britain I” streams.

The fourth set of data is a subset of the original set of Rinaldi (2003) for streams in Tuscany, Italy. The subset used here consists of 11 of the 14 reaches classified by Rinaldi (2003) as Type 1 (sinuous) rivers, and is referred to as “Tuscany” for short. The three excluded points are those for which D_{s50} falls below 25 mm, and thus below the range of values of D_{s50} in the baseline data set. Rinaldi (2003) includes three other types: Type 2 (meandering), Type 3 (sinuous with alternate bars) and Type 3/4 (sinuous with alternate bars, - locally braided). The Type 2 streams were excluded because they are sand-bed; the Type 3 and Type 3/4 streams were excluded because several of them do not appear to be clearly single-thread.

The set from Tuscany is for two reasons the most problematic of the three sets. Independently determined bankfull discharge values are unavailable for these reaches, and thus bankfull discharge Q_{bf} has been estimated as the discharge at a flood with a two-year recurrence frequency Q_2. More importantly, these streams are strongly out of equilibrium due to human interference. Indeed, Rinaldi (2003) studied these streams with the goal of quantifying the effects of human interference, including check dams in the uplands, gravel mining, water retention dams in the reaches themselves and engineering works such as straightening, diking and bank revetment. The cumulative effect of these interventions has been channel degradation and narrowing. In the case of the Type 1 streams studied here, the channels have incised into the original floodplain, and are now building a new, lower floodplain which is as yet considerably narrower than the original one. Of interest here, then, is whether or not this intervention is detectable as a systematic deviation from universality in the dimensionless plots of hydraulic geometry.

Figure 5 is an extended version of Figure 1 to which the ColoSmall, Maryland, Britain II and Tuscany data have been added. The regression lines in the figure are (10a), (11) and (10c), i.e. those determined using only the baseline data set. The ColoSmall, Maryland and Britain II data sets intermingle with the four baseline data sets without notably increasing the scatter of the plots. The trend of the Britain II data is consistent with that of the Britain I data. Many of the points for dimensionless width and depth of the Tuscany streams, however, plot low compared to the other seven sets (Alberta, Britain I, Idaho, ColoRiver, ColoSmall, Maryland, Britain II).

Figure 6 shows only the ColoSmall, Maryland, Britain II and Tuscany data along with (10a), (11) and (10c) determined from regressions of the baseline data set. The ColoSmall and Maryland data show little systematic deviation from the (10a), (11) and (10c). The Britain II data does show systematic deviation in the same way as the Britain I data: (10a) overestimates the channel width and (11) underestimates the channel depth. The systematic deviation is larger in the case of the Tuscany data, with (10a) significantly overestimating the width and (11) overestimating the depth. This systematic deviation is explored in more detail in Figures 7a, 7b and 7c, where respectively $(B_{bf})_{pred}$ is plotted against $(B_{bf})_{rep}$, $(H_{bf})_{pred}$ is plotted against $(H_{bf})_{rep}$ and $(S)_{pred}$ is plotted against $(S)_{rep}$.

In Figure 7a all but 4 of of the 97 predicted values of bankfull width for the ColoSmall, Maryland and Britain II sets are between 1/2 and 2 times the reported values. The 4 exceptions are all Britain II reaches, and in all 4 cases (10a) overpredicts the width. Bankfull width is overpredicted for every one of the 11 Tuscany points, and in 6 cases width is overpredicted by over a factor of 2.

In Figure 7b all but 1 of the 97 predicted values of bankfull depth for the ColoSmall, Maryland and Britain II sets, as well as all 11 predicted bankfull depths for the Tuscany set are between 1/2 and 2 times the reported values. The single exception is a Britain II reach, for which (11) underpredicts the depth. In the case of the Tuscany set, however, in 8 out of 11 cases bankfull depth is overpredicted.

In Figure 7c 78 of the 97 predicted values for slope for the ColoSmall, Maryland and Britain II sets, or 80%, are within 1/2 and 2 of the reported values. Of the remaining 19 values, 3 are ColoSmall reaches, 6 are Maryland reaches and 10 are Britain II reaches; all but three of these values correspond to underpredictions of slope. In addition, 7 of the 11 predicted values for slope of the Tuscany set, or 64%, are within 1/2 and 2 of the reported values.

Averages of the ratio of predicted to reported values for the ColoSmall, Maryland, Britain II and Tuscany sets are given in Table 1, and are also reported below. The corresponding values for the Britain I set are also given below for comparison.

- Average values of $(B_{bf})_{pred}/(B_{bf})_{rep}$
  ColoSmall: 1.06
  Maryland: 1.00

Britain II: 1.34 (Britain I: 1.29)
Tuscany: 2.02

- Average values of $(H_{bf})_{pred}/(H_{bf})_{rep}$
  ColoSmall: 1.10
  Maryland: 0.99
  Britain II: 0.91 (Britain I: 0.81)
  Tuscany: 1.19
- Average values of $(S)_{pred}/(S)_{rep}$
  ColoSmall: 0.87
  Maryland: 1.25
  Britain II: 0.99 (Britain I: 1.32)
  Tuscany: 0.87

A comparison of the values given above and in Table 1 allow for some initial conclusions. The first of these is that the ColoSmall, Maryland and Britain II data sets fit within the quasi-universal framework of the baseline data set. The ColoSmall and Maryland data scatter about the regression relations (10a), (11) and (10c) established using the baseline set. The Britain II data also fall within the range of the scatter of the baseline set, but show the same bias toward narrower, deeper channels as the Britain I set.

The second initial conclusion concerns the Tuscany data set. The Tuscany streams show significant deviation from universality. The average value of $(B_{bf})_{pred}/(B_{bf})_{rep}$ for the Tuscany streams is 2.02, a value that is significantly higher than the highest value of 1.34 for any of the other data sets in Table 1. That is, the Tuscany streams are significantly narrower than the other streams. The Tuscany streams are also noticeably shallower than most of the other streams: the average value of $(H_{bf})_{pred}/(H_{bf})_{rep}$ is 1.19, a value that is exceeded only by the Alberta Data, for which it is 1.27.

There are three possible reasons for this deviation. The first of these involves the possibility that measurements in the Tuscany streams were performed in a way that yielded systematic underestimation of bankfull width as compared to the other six data sets. This deviation in measurement procedure would have to be rather extreme, however, to yield a systematic underestimate by a factor of about 1/2. The second of these concerns the use of the flood discharge Q_2 with a two-year recurrence as a surrogate for bankfull discharge Q_{bf}. The 2-year flood has been found to be a reasonable surrogate for bankfull discharge in other geographic locations (e.g. Soar and Thorne, 2001). If the source of the discrepancy between the Tuscany data and the other data were due to a systematic deviation between Q_2 and Q_{bf}, an appropriate adjustment of Q_2 upward or downward ought to bring the predicted values more in line with the reported values. The average of the discrepancy ratio $(B_{bf})_{pred}/(B_{bf})_{rep}$ for the Tuscany streams can be brought down from 2.02 to 1 by estimating the bankfull discharge Q_{bf} as equal to 0.22 Q_2. The same average can be brought down to the largest average value for any other set (1.34 for the Britain II streams) by estimating Q_{bf} as equal to 0.42 Q_2. In either case the downward adjustment the estimate of bankfull discharge is sufficiently severe to suggest that discharge discrepancy is not the main cause.

The third possibility is the one suggested by Rinaldi (2003) himself: anthropogenic interference has caused the Tuscany streams to degrade, and to subsequently form both a channel and a floodplain that are narrower than that before the onset of degradation, and thus not representative of equilibrium conditions. The deviation from universality in the case of the Tuscany streams can thus be tentatively interpreted as a signal of notable human disturbance. This explanation is consistent with a fundamental assumption underlying any universal hydraulic geometry relation. Namely, the appearance of a general trend indicates that the channels have all reached some consistent state of adjustment.

Other than the Tuscany set, the largest deviation from universality is for the case of bankfull width of the Britain II streams, where B_{bf} is on the average overpredicted by (10a) by a factor of 1.34. The Britain II data set of Hey and Thorne (1986) allows for a quantification of this deviation. The authors have classified reaches of the data set on a scale from 1 to 4 in terms of the density of bank vegetation, with 1 denoting the lowest density. In Figure 8 the predicted and reported values of B_{bf} are given with the data discriminated according to vegetation density. Equation (10a) mildly overpredicts the bankfull width for the streams with the least dense bank vegetation, and noticeably overpredicts bankfull width for the streams with the densest bank vegetation The average of the discrepancy ratios $(B_{bf})_{pred}/(B_{bf})_{rep}$ for the four classes of vegetation are as follows: class 1, 0.93; class 2, 1.21; class 3, 1.45 and class 4, 1.66. As previously concluded by Hey and Thorne (1986) in regard to this data set, vegetation appears to exert a measurable control on bankfull width. In the present case this control is expressed as a deviation from universality in the dimensionless relation for bankfull width, with higher bank vegetation favoring narrower channels. The channels closest to universality are those with the lowest density of vegetation.

The above observation concerning bank vegetation is broadly consistent with observations of vegetation effects on multi-thread channels reported by Gran and Paola (2001) and Tal et al. (2004). A further step in the analysis would be to quantify the reduction in width with suitable measures of vegetal influence, including areal stem and root density, vegetation height, etc.

6. Toward the physics underlying the dimensionless relations

Equations (10a), (11) and (10c) presumably reflect the underlying physics of alluvial, single-thread gravel-bed streams. It is thus useful to ask what physical assumptions would yield these same equations as a result. The analysis presented here is of necessity "broad-brush," but is nevertheless intended to identify the factors controlling relations for hydraulic geometry.

We begin by defining suitable parameters. Boundary shear stress at bankfull flow is denoted as $\tau_{b,bf}$, water density is denoted as ρ, sediment density is denoted as ρ_s volume gravel bedload transport rate at bankfull flow is denoted as $Q_{b,bf}$ and cross-sectionally averaged flow velocity is denoted as U_{bf}. Water conservation requires that

$$U_{bf} = \frac{Q_{bf}}{B_{bf} H_{bf}} \tag{15}$$

The normal flow approximation is used here to evaluate the boundary shear stress $\tau_{b,bf}$ and the shear velocity at bankfull flow $u_{*,bf}$;

$$\tau_{b,bf} = \rho g H_{bf} S \quad , \quad u_{*,bf} = \sqrt{\frac{\tau_{b,bf}}{\rho}} = \sqrt{g H_{bf} S} \tag{16a,b}$$

The submerged specific gravity R of the gravel is defined as

$$R = \frac{\rho_s}{\rho} - 1 \tag{17}$$

For natural sediments R is usually close to the value of 1.65 for quartz. The Shields number τ^*_{bf} and Einstein number q^*_{bf}, both at bankfull flow and based in sediment size D_{s50}, are defined as

$$\tau^*_{bf} = \frac{\tau_{b,bf}}{\rho R g D_{s50}} \quad , \quad q^*_{bf} = \frac{Q_{b,bf}}{B_{bf} \sqrt{R g D_{s50}}\, D_{s50}} \tag{18a,b}$$

In addition, a dimensionless bankfull gravel bedload transport rate $\hat{Q}_b$ analogous to the dimensionless water discharge $\hat{Q}$ is defined as

$$\hat{Q}_b = \frac{Q_{b,bf}}{\sqrt{g D_{s50}}\, D_{s50}^2} \tag{19}$$

We assume that the relations that underlie (10a), (11) and (10b) involve a) frictional resistance, b) transport of gravel, c) a channel-forming Shields number, e) a relation for critical Shields number for the onset of gravel motion and e) a relation for gravel "yield" (the reason for the quotes becomes apparent below). Frictional resistance is described in terms of a relation of Manning-Strickler type:

$$\frac{U_{bf}}{u_{*,bf}} = \alpha_r \left(\frac{H_{bf}}{D_{s50}} \right)^{n_r} \tag{20a}$$

where the dimensionless parameters α_r and n_r are to be determined. Reducing with (15) and (16b),

$$\frac{Q_{bf}}{B_{bf} H_{bf} \sqrt{g H_{bf} S}} = \alpha_r \left(\frac{H_{bf}}{D_{s50}} \right)^{n_r} \tag{20b}$$

Gravel transport is described in terms of the Parker (1978) approximation of the Einstein (1950) relation applied to bankfull flow:

$$q_{bf}^{*} = \alpha_G \left(\tau_{bf}^{*}\right)^{3/2} \left(1 - \frac{\tau_c^{*}}{\tau_{bf}^{*}}\right)^{4.5} \tag{21}$$

where τ_c^* is a critical Shields number for the onset of motion and α_G is a coefficient equal to 11.2. Channel form is described in terms of a relation of the form

$$\tau_{bf}^{*} = r\tau_c^{*} \tag{22}$$

as described by Parker (1978), Paola *et al.* (1992) and Parker *et al.* (1998). Equation (20) reduces with (16a) and (18a) to

$$\frac{Q_{b,bf}}{\sqrt{gD_{s50}}\,D_{s50}^2} = \frac{\alpha_G}{R}\frac{B_{bf}}{D_{s50}}\left(\frac{H_{bf}S}{D_{s50}}\right)^{3/2}\left(1 - \frac{1}{r}\right)^{4.5} \tag{23}$$

In the Parker (1978) approximation of the Einstein (1950) bedload relation τ_c^* is taken to be a constant equal to 0.03. Here it is taken to be a (weak) function of $\hat{Q}$ such that the average value for the baseline data set is 0.03;

$$\tau_c^{*} = \alpha_\tau \hat{Q}^{n_\tau} \tag{24}$$

In the above relation the dimensionless constants α_τ and n_τ are to be determined. Between (5c), (16a), (18a) and (22) we find that (24) reduces to

$$\frac{H_{bf}S}{RD_{s50}} = r\alpha_\tau \left(\frac{Q_{bf}}{\sqrt{gD_{s50}}\,D_{s50}^2}\right)^{n_\tau} \tag{25}$$

Finally, a gravel "yield" relation describes how the gravel bedload transport rate at bankfull flow $Q_{b,bf}$ varies with bankfull flow Q_{bf} and grain size D_{s50};

$$\hat{Q}_b = \alpha_y \hat{Q}^{n_y} \tag{26a}$$

where α_y and n_y are dimensionless parameters that we compute below. Reducing (26a) with (5a) and (19),

$$\frac{Q_{b,bf}}{\sqrt{gD_{s50}}\,D_{s50}^2} = \alpha_y \left(\frac{Q_{bf}}{\sqrt{gD_{s50}}\,D_{s50}^2}\right)^{n_y} \tag{26b}$$

Between (23) and (26b),

$$\frac{\alpha_G}{R}\frac{B_{bf}}{D_{s50}}\left(\frac{H_{bf}S}{D_{s50}}\right)^{3/2}\left(1 - \frac{1}{r}\right)^{4.5} = \alpha_y \left(\frac{Q_{bf}}{\sqrt{gD_{s50}}\,D_{s50}^2}\right)^{n_y} \tag{27}$$

The above relations contain the unevaluated dimensionless coefficients α_r, α_τ and α_y and exponents n_r, n_τ and n_y. We now compute these parameters so as to yield precisely the coefficients α_B and α_S, exponents n_B and n_S and the constant $\tilde{H}_o$ determined by regression from the baseline data set, i.e. the values given in (10a), (10c) and (11). Before completing this step, however, some elaboration of the above relations is appropriate.

Equation (20a) is a Manning-Strickler relation of the general form that Parker (1991) has applied to gravel rivers; it is also similar to related logarithmic forms for gravel-bed rivers due to e.g. Limerinos (1970) and Hey (1979). As such, it is appropriate for a broad-brush formulation. There are two reasons why it cannot be accurate in detail. The first of these is the fact that the characteristic grain size on which grain roughness (skin friction) depends is a size coarser than D_{s50}; commonly used sizes are D_{s90} and D_{s84}. The second of these is the likelihood that not all the drag in gravel-bed rivers at bankfull flow is due to skin friction. Bar structures, planform variation and bank vegetation can give rise to at least some form drag (e.g Millar, 1999). The issue of form drag is discussed in more detail below.

The Parker (1978) approximation of the Einstein (1950) bedload transport relation embodied in (21) is also an appropriate broad-brush relation for gravel-bed rivers. There are at least three reasons why it cannot be accurate in detail: a) it does not account for gravel mixtures (e.g. Parker, 1990; Wilcock and Crowe, 2003), b) no attempt has been made to remove the effect of form drag (which would reduce the total bedload transport rate) and c) no attempt has been made to account for preferential "patches" or "lanes" (which would increase the total transport rate; Paola and Seal, 1995).

The original derivation of the relation for channel form (22) presented by Parker (1978) does not account for the effect of form drag or planform variation, both effects that are felt here. This notwithstanding, Paola *et al.* (1992) and Parker *et al.* (1998) have shown its value as a broad-brush relation.

According to (24) the critical Shields number τ_c^* at the onset of motion depends on dimensionless discharge $\hat{Q}$. In the original Parker (1978) approximation of the Einstein (1950) bedload transport relation τ_c^* is a constant equal to 0.03. We demonstrate below, however, that the exponent n_τ in (24) is very small.

Finally, the gravel "yield" relation (26a) does not involve mean annual gravel yield, but rather the gravel transport rate at bankfull flow. One presumably scales with the other, but the details of the scaling have yet to be worked out. The "yield" relation relates to processes at the scale of the drainage basin rather than local in-channel processes. More specifically, it implies that catchments organize themselves to provide gravel during floods such that the gravel discharge scales as a power law of the water discharge. Equation (26a) is the most empirical of the relations used here.

Substituting (10a), (11) and (10c) into (20b), (25) and (27) yields the evaluations

$$\alpha_r = \alpha_B^{-1}\alpha_S^{-1/2}\tilde{H}_o^{-[(3/2)+(5/4)n_S-(5/2)n_B]} \tag{28a}$$

$$n_r = \frac{5}{2}\left(\frac{1}{2}n_S - n_B\right) \tag{28b}$$

$$\alpha_\tau = \frac{\tilde{H}_o \alpha_S}{rR} \tag{29a}$$

$$n_\tau = \frac{2}{5} - n_s \tag{29b}$$

$$\alpha_y = \frac{\alpha_G \left(1 - \frac{1}{r}\right)^{4.5} \alpha_B \tilde{H}_o^{3/2} \alpha_S^{3/2}}{R} \tag{30a}$$

$$n_y = 1 + n_B - \frac{3}{2} n_S \tag{30b}$$

The parameter r is evaluated as follows. Figure 9 shows a plot of τ_{bf}^* as computed from (16a) and (18a), i.e.

$$\tau_{bf}^* = \frac{H_{bf} S}{R D_{s50}} \tag{31}$$

versus $\hat{Q}$ for the baseline data set. The average value $< \tau_{bf}^* >$ for the baseline data set is found to be

$$< \tau_{bf}^* > = 0.0489 \tag{32}$$

Using (22) and the original estimate of τ_c^* of 0.03 in the Parker (1978) approximation of the Einstein (1950) bedload transport relation, we obtain the following estimate for r:

$$r = 1.63 \tag{33}$$

Substitution of (10a), (10c), (11) and (33) into (28) and (29) yields the values for α_r, α_τ, α_Y, n_r, n_τ and n_y;

$$\alpha_r = 3.71 \quad , \quad \alpha_\tau = 0.0143 \quad , \quad \alpha_y = 0.00330$$
$$n_r = 0.263 \quad , \quad n_\tau = 0.0561 \quad , \quad n_y = 0.551 \tag{34a,f}$$

and thus the following evaluations for (20a), (24) and (26);

$$\frac{U_{bf}}{u_{*,bf}} = 3.71 \left(\frac{H_{bf}}{D_{s50}}\right)^{0.263} \tag{35}$$

$$\tau_c^* = 0.0143 \hat{Q}^{0.0561} \tag{36}$$

$$\hat{Q}_b = 0.00330 \hat{Q}^{0.551} \tag{37}$$

In addition, between (22), (33) and (36) it is found that

$$\tau_{bf}^* = 0.0233 \hat{Q}^{0.0561} \tag{38}$$

The exponent in the resistance relation (35) of 0.263 is somewhat larger than the standard Manning-Strickler exponent of 1/6 $\cong$ 0.167. Relations (38) for bankfull Shields number and (37) for critical Shields number show a very weak dependence on $\hat{Q}$. This weak dependence is reflected in the baseline data set: a direct regression of the data of Figure 9 yields a nearly identical relation with a coefficient of 0.0230 and an exponent of 0.0572. The exponent is only significantly different from zero at the 90% level, but not at 95%; $p = 0.078$. This notwithstanding, (36) represents an improvement over a constant critical Shields number of 0.03, for the following reason. Most alluvial gravel-bed rivers can be expected to be competent to move their median surface size D_{s50} at bankfull flow (e.g. Andrews, 1983; Hey and Thorne, 1986). In the case of a constant critical Shields number of 0.03, 21 of 72 reaches in Figure 9, or 29% plot below the threshold of motion at bankfull flow, whereas in the case of (36) only 12 reaches, or 17% plot below the threshold of motion. This empirically-derived weak dependence of τ_c^* on $\hat{Q}$ may represent a consequence of form drag.

The exponent in the gravel "yield" relation of (37) indicates that the gravel transport rate at bankfull flow should increase as about the square root of the bankfull discharge. Thus the volume concentration of transported gravel should decline downstream. Since water discharge usually increases nearly linearly with contributing drainage area, the implication is that "gravel yield" increases with contributing area at a rate that is markedly slower than linear, i.e. roughly as the 0.5 power of contributing area. The explanation and implications of this inference remain to be explored in future work. Irrespective of its origin, (37) expresses a property of how drainage basins organize themselves, rather than local properties in the channel. It is likely, however, that as down-channel slope S drops with increasing flow discharge in accordance with (10c), the adjacent hillslopes often become less steep, so delivering less sediment (and thus less gravel) for the same unit rainfall. This reduced gravel delivery is likely mitigated by downstream fining of the gravel itself.

7. Quantification of deviation from universality

The derivation of the physical relations underlying hydraulic geometry allows for a quantification of deviations from similarity. This further allows for a characterization of the effect of the "other" parameters in (3a) ~ (3c). In order to do this, the physical relations of the previous section are adopted as primary. The derivation leading to (28) ~ (30) is then inverted so that the coefficients and exponents in the dimensionless relations for hydraulic geometry become functions of the parameter r, and coefficients α_r, α_τ and α_y and the exponents n_r, n_τ and n_y of the physical relations. This yields the following coefficients and exponents describing generalized power relations for hydraulic geometry;

$$\alpha_B = \frac{\alpha_y}{\sqrt{R}\,\alpha_G\left(1-\frac{1}{r}\right)^{4.5}(r\alpha_\tau)^{3/2}} \tag{39a}$$

$$n_B = \frac{1}{5} - \frac{1}{2}n_\tau - \frac{2}{5}n_r \tag{39b}$$

$$\tilde{H}_o = \left[\frac{\alpha_G \left(1 - \frac{1}{r}\right)^{4.5} r\alpha_\tau}{\alpha_y \alpha_r} \right]^{\frac{1}{1+n_R}} \tag{40}$$

$$\alpha_S = R\alpha_\tau \left[\frac{\alpha_G \left(1 - \frac{1}{r}\right)^{4.5} r\alpha_\tau}{\alpha_y \alpha_r} \right]^{-\left(\frac{1}{1+n_R}\right)} \tag{41a}$$

$$n_S = \frac{2}{5} - n_\tau \tag{41b}$$

Here we examine the effect of variation of the following parameters on the deviation from universality: r, α_r and α_y. The first of these, i.e. the ratio of bankfull Shields number to critical Shields number, can be thought of as a measure of "bank strength," in that channels with stronger banks can maintain higher values of τ_{bf}^* relative to τ_c^*. Using information in Rice (1979) and Ashmore (1979), Parker (1982) deduced a mean value of τ_{bf}^* of 0.0420, and thus a value of r of about 1.4 for anabranches of the braided gravel-bed Sunwapta River, Jasper National Park, Canada, which flows on an unvegetated valley flat. This value represents a lower limit in the absence of vegetation and cohesive sediment to add bank strength. The average value of r of 1.63 deduced for the baseline data set presented here is considerably higher. The Britain II data can be used to provide a qualitative measure of the effect of bank vegetation density on r. Figure 10 shows a plot of the average value of r for each vegetation density class of the Britain II data. Here r is calculated in the same way as for the baseline data, i.e. from (31), (22) and an estimated value of τ_c^* of 0.03. The parameter r takes the following values in order of vegetation density: 1.49 (class 1, lowest vegetation density); 1.63 (class 2), 1.92 (class 3) and 2.67 (class 4, highest vegetation density). For reference, the value of r determined from the baseline data set is 1.63. Here r is allowed to vary from 0.9 to 1.1 times the baseline value of 1.63

Channel resistance decreases as the parameter α_r in the Manning-Strickler relation (20a) increases. This can be seen by defining a dimensionless resistance coefficient C_f as

$$C_f = \frac{\tau_{b,bf}}{\rho U_{bf}^2} \tag{42}$$

Between (15), (16a), (20b) and (42) we find that

$$C_f = \alpha_r^{-2} \left(\frac{H_{bf}}{D_{s50}} \right)^{-2n_r} \tag{43}$$

Here α_r is allowed to vary from 0.8 to 1.2 times its baseline value of 3.71. At the lower value the resistance coefficient C_f is increased by a factor of 1.56; at the higher value C_f is decreased by a factor of 0.69.

Gravel supply increases linearly with increasing parameter α_y in the "gravel yield" relation (26a). Here α_y is allowed to vary from 0.5 to 1.5 times its baseline value of 0.00330.

Varied values of r, α_r and α_y cause the coefficients α_B and α_S in (39a) and (41a), respectively, and the parameter $\tilde{H}_o$ in (40) to vary. The effects of this variation are summarized in Table 2 and Figures 11a, 11b and 11c.

The effect of varying r is illustrated in Figure 11a. Increasing r (i.e. increasing "bank strength") from 0.9 to 1.1 times the baseline value results in an bankfull channel that is increasingly narrower and has an increasingly lower bed slope. A comparison with the data in Figure 11a suggests that bank strength is one reason why the Alberta reaches are wider and shallower than the Britain I reaches.

The effect of varying α_r is studied in Figure 11b. Decreasing α_r from 1.2 to 0.8 times the baseline value, and thus increasing the channel resistance coefficient from 0.69 to 1.56 times that which would be predicted using the baseline value of α_y, results in a bankfull channel that is increasingly deeper and has an increasingly lower slope. Changing α_r has no effect on channel width.

The effect of varying α_y is shown in Figure 11c. Increasing α_y (and thus gravel supply) from 0.5 to 1.5 times the baseline value results in a bankfull channel that is increasingly wider, shallower and steeper. A comparison with the data in Figure 11c suggests that another reason why the Alberta streams are wider and shallower than the Britain I streams may be that they have a higher gravel supply.

8. Predictor for bankfull discharge

In general bankfull discharge should be determined from a rating curve of discharge versus stage. Bankfull discharge is indicated by the "rollover" in the plot of stage ξ versus flow discharge Q indicated in Figure 12. In practice, however, such information is often not available.

Equation (20b) along with the evaluations of α_r and n_r of (34a) and (34d), respectively, provide a means for estimating bankfull discharge Q_{bf} from measured channel parameters B_{bf}, H_{bf}, S and D_{s50}. In Figure 13 the values of Q_{bf} predicted from (20b) are compared against the measured values for the four baseline data sets used to derive (20b). We find that 93% of the predicted values are seen to be between 1/2 and 2 times the reported values. The scatter in the data of Figure 12 is very small for measured discharges above 500 m^3/s. Most of these points refer to the Colorado River. The values for bankfull discharge for the ten reaches of the Colorado River are characteristic values determined with the use of a form of Manning's relation calibrated site-specifically to the field data (Pitlick and Cress, 2000). Evidently this procedure has reduced the scatter.

An independent test of (20b) is given in Figure 14 using the ColoSmall, Maryland, Britain II data sets. All 97 predicted values are seen to be between 1/2 and 2 times the reported values.

9. Form drag

The resistance to flow in a river can be partitioned into skin friction, i.e. that part of the drag that acts directly on the grains themselves, and form drag, i.e. that part associated with bedforms such as bars, channel planform irregularities etc. Parker and Peterson (1980) have argued that form drag in gravel-bed streams is significant at low flow, but may be neglected at the flood flows that move gravel because the bars are effectively drowned. Millar (1999), on the other hand, has argued that form drag may be measurable at flood flows as well. The present analysis provides a basis for quantifying the partition between skin friction and form drag in gravel-bed streams.

An appropriate relation for the resistance coefficient C_{fs} due to skin friction alone (here applied to bankfull conditions) is

$$C_{fs}^{-1/2} = 8.1\left(\frac{H_{bf}}{k_s}\right)^{1/6} \tag{44}$$

where H denotes flow depth and k_s is a roughness height given as

$$k_s = 2D_{s90} \tag{45}$$

and D_{s90} is the surface size such that 90 percent is finer (Parker, 1991; Wong, 2003). Total channel resistance is given by (43). The fraction of resistance φ_f due to form drag at bankfull flow is then given by the relation

$$\varphi_f = \frac{C_f - C_{fs}}{C_f} \tag{46}$$

where C_f is evaluated from (43) and C_{fs} is evaluated from (44) and the baseline values for α_r and n_r.

The above relations allow for a specification of φ_f as a function of H_{bf}/D_{s50} upon specification of the ratio D_{s90}/D_{s50}. This parameter is a function of, among other things, sediment supply. Mueller and Pitlick (2005a) report values of both D_{s50} and D_{s90} for 32 gravel-bed reaches in Idaho extracted from the compendium of King et al. (2004). Many of the stream reaches in this set overlap with those in the Idaho data of Parker et al. (2003) used as baseline data here. The values of D_{s90}/D_{s50} in the data set of Mueller and Pitlick (2005a) ranges from a low value of 1.69 to a high value of 13.8, with a median value of 2.99. With this in mind the value D_{s90}/D_{s50} = 3 is used as an example. The resulting prediction for form drag is shown in Figure 15. The fraction of resistance that is form drag is predicted to decrease from 0.57 to 0.21 as H_{bf}/D_{s50} increases from 4 to 100, a range that captures the great majority of the reaches studied here. A refinement of the broad-brush analysis presented above would involve removing this form drag in the calculation of gravel transport.

Equation (38) indicates that the Shields number at bankfull flow τ_{bf}^* is a weak function of dimensionless discharge $\hat{Q}$ and nothing else. Mueller and Pitlick (2005a), however, have shown a tendency for τ_{bf}^* to increase with bed slope S as well. The applied linear regression to their data set to obtain the trend

$$\tau_{bf}^* = 1.91S + 0.037 \tag{47}$$

A plot of τ_{bf}^* versus S using the four baseline data sets of this paper is shown in Figure 16. While the scatter is considerable, the tendency for τ_{bf}^* to increase with increasing bed slope S is clear. A linear regression applied to the same baseline data results in the relation

$$\tau_{bf}^* = 2.00\,S + 0.038 \tag{48}$$

As noted above, most of the stream reaches in the data set used by Mueller and Pitlick (2005a) overlap with those in the Idaho baseline data set used above. On the other hand, some 68 percent of the reaches in the baseline set (Alberta, Britain I and ColoRiver) do not overlap with those used by Mueller and Pitlick (2005a). The good correspondence between (x1) and (x2) in Figure 16 thus suggests that the trend is real.

Mueller and Pitlick (2005a) have speculated on the reasons why τ_{bf}^* tends to increases with increasing bed slope. One contributor to this effect might be form drag. Figure 15 suggests that the fraction of resistance that is form drag increases with decreasing values of H_{bf}/D_{s50}. Figure 17 illustrates that for the baseline data used here H_{bf}/D_{s50} correlates negatively with bed slope S. The implication is that form drag increases with increasing slope. If the bankfull Shields number associated with skin friction alone remains insensitive to slope, the total bankfull Shields number (including skin friction and form drag) should increase with increasing slope.

10. Application to stream restoration

The relations presented above may be of use in stream restoration schemes for single-thread, alluvial gravel-bed streams. In the absence of gage records, (20b) along with (34a) and (34b), i.e.

$$Q_{bf} = 3.71 B_{bf} H_{bf} \sqrt{g H_{bf} S}\left(\frac{H_{bf}}{D_{s50}}\right)^{0.263} \tag{49}$$

can be used to estimate bankfull discharge from measured channel characteristics. We emphasize that the above equation should not be applied outside the range of its derivation, or to streams that are not alluvial or do not have a definable floodplain.

The scatter in the data of Figure 13 means that the predictions given by (49) cannot be expected to be too precise. Here this is quantified using an "augmented data set" consisting of the base data set plus the sets Maryland, Britain II and ColoSmall, and so including 169 reaches. (The Tuscany data set is excluded, as these streams appear to be out of equilibrium, as noted above). Let X_{pred} and X_{rep} denote predicted and reported values, respectively. Set A consists of reaches for which a predicted value X_{pred} falls in the range $4/5\ X_{rep} \leq X_{pred} \leq 5/4\ X_{rep}$; the corresponding ranges for Sets B and C are $2/3\ X_{rep} \leq X_{pred} \leq 3/2\ X_{rep}$ and $1/2\ X_{rep} \leq X_{pred} \leq 2\ X_{rep}$, respectively. In the case of Equation (49), 53 percent of the predicted values fall within Set A, 83 percent fall within Set B and 97 percent fall within Set C.

Subject to these same restrictions, (12), (13) and (14) may be used to estimate the bankfull width, bankfull depth and bed slope of a channel with a specified bankfull discharge Q_{bf}. Again, the predictions cannot be expected to be too precise. In respect to the predictor (13) for bankfull width, 57 percent of the predictions using the augmented data set fall in Class A, 80 percent fall within Class B and 98 percent fall within Class C. The corresponding percentages for the predictor (12) for bankfull depth are as follows; 71 percent in Class A, 90 percent in Class B and 99 percent in Class C. The corresponding percentages for the predictor (14) for channel bed slope are as follows; 27 percent in Class A, 54 percent in Class B and 77 percent in Class C.

Uncertainty in specifying an appropriate value of bankfull discharge Q_{bf} for a channel design adds to the uncertainty in predicting bankfull width and depth. A more useful application of these relations is to predict changes in channel dimensions in response to changes in discharge. For example, if Q_{bf} is changed through e.g. river regulation, the dimensions to which the channel would evolve in response can be estimated. This natural evolution might take 100's to 1000's or years, and slope adjustments are constrained over these time scales to changes in channel sinuosity. The process might be speeded by "pre-fitting" a restored channel to the estimated equilibrium dimensions. This "pre-fitting" should be as broad-brush as the relations presented here (as reflected in the scatter of e.g. Figure 1), and the channel is likely to change further on its own. More specifically, rivers have a tendency to reject the imposition of idealized meander planforms (e.g. Kondolf, 2001), and are often better left to design themselves in this regard. More certain estimates of channel adjustment require explicit consideration of sediment transport and specification of future sediment supply, if that information is available.

Equations (39a), (40) and (41a) may be used in conjunction with (8a) ~ (8c) to provide rough estimates of channel change in response to changes in gravel supply, “bank strength” (through e.g. vegetation) and bulk channel resistance (through e.g. change in sinuosity) on channel bankfull geometry.

Discussion

It is the dimensionless formulation used here that allows backing out the physics behind the relations for hydraulic geometry. This underlying physics in turn allows the study of, for instance, the dependence of hydraulic geometry on sediment supply or “bank strength.” Such information cannot be obtained using dimensionally inhomogeneous equations obtained by means of regression applied directly to parameters of differing dimensions.

The analysis presented here indicates that a specification of channel-based relations for flow resistance, gravel transport and channel form alone are insufficient to derive both the coefficients and exponents governing hydraulic geometry. It has been known for some time that one more constraint is required. Many authors have taken this constraint to be an optimization condition applied to the channel itself. It has been variously proposed that channels adjust their cross-sections to a) minimize variance b) minimize unit stream power, c) minimize total stream power, d) maximize the friction coefficient, e) maximize the sediment transport rate or efficiency and f) minimize the Froude number. Surveys of these proposed constraints are given in Soar and Thorne (2001) and Millar (2005).

We suggest an alternative avenue. We propose that the extra constraint is external to the channel itself, and instead describes how the catchment itself functions. The constraint used here is an empirical one back-calculated from the hydraulic relations. It specifies how the gravel transport rate at bankfull flow varies with bankfull discharge. Gravel transport rate per unit area can be expected to decrease with increasing drainage area, indicating greater sediment production, closer hillslope/stream coupling, and less sediment storage in the upper parts of watersheds. In order to convert this empirical result to one with a physical basis it is necessary to model the interaction between channels and sediment supply at the scale of the drainage basin.

Equation (37) indicates that the gravel transport rate at bankfull flow $Q_{b,bf}$ increases with bankfull discharge Q_{bf} to about the half power. Mueller and Pitlick (2005b), however, have estimated a linear relation between annual gravel yield and bankfull flow for the Halfmoon Creek basin, a headwater catchment in Colorado. The reason for the discrepancy is not known at this time. It may be, however, that a decrease in the ratio of gravel yield to bankfull discharge would be realized if the analysis of Mueller and Pitlick (2005b) were carried farther downstream into regions of lower bed slope. The discrepancy highlights the fact that the relations derived here apply as overall averages, and thus may be at variance with site-specific data.

11. Conclusions

A baseline data set consisting of stream reaches from Alberta, Canada, Idaho, USA, Britain and the Colorado River, Colorado, USA is used to determine dimensionless bankfull hydraulic relations for alluvial, single-thread gravel-bed streams with definable channels and floodplains. These dimensionless relations show a remarkable degree of universality. Application of the regression relations to two other data sets, one from Maryland, USA and one from Britain, confirms this tendency toward universality.

The relations are, however, only quasi-universal in that some systematic deviation from universality can be detected. For example, the Alberta streams tend to be wider and shallower, and both set of British streams tend to be narrower and deeper than predicted by the regression relations. In the case of the British streams the deviation appears to be associated with differing density of bank vegetation, and thus “bank strength.”

The regression relations are used to back-calculate the underlying physical relations governing bankfull hydraulic geometry. This back-calculation results in a) a Manning-Strickler relation for channel resistance, b) a relation in which the critical Shields number for the onset of gravel motion varies weakly with dimensionless flow discharge and c) a relation for “gravel yield” which relates the dimensionless gravel transport rate at bankfull flow to dimensionless bankfull discharge. Having specified these relations, the coefficients and exponents of the dimensionless bankfull hydraulic relations are generalized so as to determine the effect of changing “bank strength,” channel resistance and gravel supply. This calculation suggests that the difference between the Alberta and British streams results from a combination of differing density of bank vegetation (the British streams likely having a higher density) and differing gravel supply (the Alberta stream likely having a higher supply).

Application of the dimensionless relations for bankfull hydraulic geometry to three more data sets (Maryland, USA, Britain and Colorado, USA) confirms their quasi-universality. A greater degree of deviation from universality is found when the regression relations are applied to a set of streams in Tuscany, Italy. These streams have undergone recent degradation in response to human interference, and as a result have incised into their former floodplains. The rivers at present have relatively narrow channels in narrow floodplains forming below the original floodplain. The effect of human interference is detectable when the data are compared against the regression relations.

The Manning-Strickler relation back-calculated from the data provides a means for estimating bankfull discharge from measured values of bankfull depth, bankfull width, down-channel bed slope and surface median size. The predictive relation performs well against both the baseline data set and the data sets from Maryland and Britain that were not used to determine the relation.

The analysis allows an estimation of the effect of form drag in gravel-bed streams at bankfull flow. This estimation suggests that form drag becomes progressively more important as the ratio of bankfull depth to surface median size decreases.

Finally, the analysis suggests that the piece of information missing from previous analyses to close the formulation for bankfull hydraulic geometry is not some kind of extremal constraint applied to a cross-section, but rather a relation that expresses how a catchment organizes itself to deliver gravel downstream, i.e. a "gravel yield" relation.

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Notation

B_{bf}	bankfull width
$\hat{B}$	$= B_{bf}/D_{s50}$
$\tilde{B}$	$= g^{1/5}B_{bf}/Q_{bf}^{2/5}$
C_f	resistance coefficient
C_{fs}	coefficient of of resistance due to skin friction
D_{s50}	bed surface size such that 50% are finer
D_{s90}	bed surface size such that 90% are finer
g	gravitational acceleration
H_{bf}	bankfull depth
$\hat{H}$	$= H_{bf}/D_{s50}$
$\tilde{H}$	$= g^{1/5}H_{bf}/Q_{bf}^{2/5}$
k_s	bed roughness height
n_B	exponent in dimensional hydraulic relation (1a) or dimensionless hydraulic relation (8a)
n_H	exponent in dimensional hydraulic relation (1b) or dimensionless hydraulic relation (8b)
n_r	exponent in (20a)
n_S	exponent in dimensional hydraulic relation (1c) or dimensionless hydraulic relation (8c)
n_y	exponent in (26a)
n_τ	exponent in (24)
U_{bf}	mean flow velocity at bankfull flow
$u_{*,bf}$	shear velocity at bankfull flow
Q_{bf}	bankfull discharge
$\hat{Q}$	$= Q_{bf}/(\sqrt{gD_{s50}}D_{s50}^2)$
$Q_{b,bf}$	volume bedload transport rate at bankfull flow
$\hat{Q}_b$	$= Q_{b,bf}/(\sqrt{gD_{s50}}D_{s50}^2)$
Q_2	flood discharge with a two-year recurrence interval
q_{bf}^*	dimensionless Einstein number characterizing gravel transport rate at bankfull flow, defined in (18b)
R	$= (\rho_s - \rho)/\rho$; submerged specific gravity of sediment
r	ratio between bankfull Shields number and critical Shields number, defined in (22)
S	channel bed slope
$(X)_{pred}$	predicted value of any parameter X
$(X)_{rep}$	reported value of any parameter X
α_B	dimensionless coefficient in (8a)
α_G	dimensionless coefficient in (21)
α_H	dimensionless coefficient in (8b)
α_r	dimensionless coefficient in (20a)

α_S	dimensionless coefficient in (8c)
α_y	dimensionless coefficient in (26a)
α_τ	dimensionless coefficient in (24)
φ_f	fraction of resistance that is form drag, defined in (46)
$\tau_{b,bf}$	bed shear stress at bankfull flow
τ^*_{bf}	dimensionless Shields number at bankfull flow, defined in (18a)
τ^*_c	dimensionless Shields number at the threshold of motion
ρ_s	material density of sediment
ρ	density of water

TABLE 1: Average values for $(X)_{pred}/(X)_{rep}$ for seven data sets, where X = bankfull width B_{bf}, bankfull depth H_{bf} and down-channel slope S.

Average of discrepancy ratio	$(B_{bf})_{pred}/(B_{bf})_{rep}$	$(H_{bf})_{pred}/(H_{bf})_{rep}$	$(S)_{pred}/(S)_{rep}$
Alberta	0.83	1.27	1.16
Britain I	1.30	0.81	1.32
Idaho	0.97	1.08	1.38
ColoRiver	0.98	1.07	1.00
ColoSmall	1.06	1.10	0.87
Maryland	1.00	0.99	1.25
Britain II	1.34	0.91	0.99
Tuscany	2.02	1.19	0.87

TABLE 2: Effect of variation of the parameters r, α_r and α_y on the parameters $\tilde{H}_o$, α_B and α_y.

r	r factor	$\tilde{H}_o$	α_B	α_S
1.79	1.1	0.696	2.19	0.0578
1.63	1	0.400	4.63	0.101
1.47	0.9	0.184	12.97	0.218
α_y	α_y factor	$\tilde{H}_o$	α_B	α_S
0.00531	1.5	0.290	6.95	0.139
0.00354	1	0.400	4.63	0.101
0.00177	0.5	0.692	2.32	0.0581
α_r	α_r factor	$\tilde{H}_o$	α_B	α_S
4.11	1.2	0.346	4.63	0.134
3.43	1	0.400	4.63	0.1001
2.74	0.8	0.477	4.63	0.0707

FIGURE CAPTIONS

Figure 1 Dimensionless bankfull width $\tilde{B}$, dimensionless bankfull depth $\tilde{H}$ and down-channel bed slope S as functions of dimensionless bankfull discharge $\hat{Q}$. The Alberta, Britain I, Idaho and Colorado subsets of the baseline data set have been lumped together. Also shown are power relations derived from regression on the lumped data set.

Figure 2 Dimensionless bankfull width $\tilde{B}$, dimensionless bankfull depth $\tilde{H}$ and down-channel bed slope S as functions of dimensionless bankfull discharge $\hat{Q}$, in which the Alberta, Britain I, Idaho and Colorado data subsets of the baseline data set are distinguished by different symbols.

Figure 3 a) Plot of $\tilde{B}$ versus $\hat{Q}$ for the baseline data set, in which the Alberta, Britain I, Idaho and Colorado data subsets are distinguished by different symbols.

b) Plot of $\tilde{H}$ versus $\hat{Q}$ for the baseline data set, in which the Alberta, Britain I, Idaho and Colorado data subsets are distinguished by different symbols.

Figure 4 a) Predicted versus reported bankfull width B_{bf} for the baseline data set.

b) Predicted versus reported bankfull depth H_{bf} for the baseline data set.

c) Predicted versus reported down-channel bed slope S for the baseline data set.

Figure 5 Dimensionless bankfull width $\tilde{B}$, dimensionless bankfull depth $\tilde{H}$ and down-channel bed slope S as functions of dimensionless bankfull discharge $\hat{Q}$, in which the baseline data set has been augmented by the ColoSmall, Maryland, Britain II and Tuscany subsets. All subsets are distinguished by different symbols.

Figure 6 Dimensionless bankfull width $\tilde{B}$, dimensionless bankfull depth $\tilde{H}$ and down-channel bed slope S as functions of dimensionless bankfull discharge $\hat{Q}$ for the ColoSmall, Maryland, Britain II and Tuscany data subsets, along with the power regression lines determined from the baseline data set.

Figure 7 a) Predicted versus reported bankfull width B_{bf} for the ColoSmall, Maryland, Britain II and Tuscany data subsets.

b) Predicted versus reported bankfull depth H_{bf} for the ColoSmall, Maryland, Britain II and Tuscany data subsets.
c) Predicted versus reported down-channel bed slope S for the ColoSmall, Maryland, Britain II and Tuscany data subsets.

Figure 8 Predicted versus reported bankfull width B_{bf} for the Britain II data stratified according to vegetation density. Class 1 refers to the lowest, and Class 4 refers to the highest vegetation density.

Figure 9 Plot of the bankfull Shields number τ_{bf}^* for the baseline data set. Also included are a) the line $\tau_{bf}^* = 0.0489$ corresponding to the average value for the baseline data set, b) relation (38) for τ_{bf}^*, c) the estimate of critical Shields number $\tau_c^* = 0.03$ and d) the relation (36) for critical Shields number.

Figure 10 Plot of the parameter r estimating the ratio of bankfull Shields number to critical Shields number as a function of vegetation density for the Britain II data. Class 1 refers to the lowest, and Class 4 refers to the highest vegetation density.

Figure 11 a) Dimensionless bankfull width $\tilde{B}$, dimensionless bankfull depth $\tilde{H}$ and down-channel bed slope S as functions of dimensionless bankfull discharge $\hat{Q}$, showing the predictions of the generalized hydraulic geometry relations as the parameter r is varied from 0.9 to 1.1. Increasing r is associated with increasing “bank strength.” Also shown is the baseline data set discriminated according to subset.

b) Dimensionless bankfull width $\tilde{B}$, dimensionless bankfull depth $\tilde{H}$ and down-channel bed slope S as functions of dimensionless bankfull discharge $\hat{Q}$, showing the predictions of the generalized hydraulic geometry relations as the parameter α_r is varied from 0.8 to 1.2. Increasing α_r is associated with decreasing channel resistance. Also shown is the baseline data set discriminated according to subset.

c) Dimensionless bankfull width $\tilde{B}$, dimensionless bankfull depth $\tilde{H}$ and down-channel bed slope S as functions of dimensionless bankfull discharge $\hat{Q}$, showing the predictions of the generalized hydraulic geometry relations as the parameter α_y is varied from 0.5 to 1.5. Increasing α_y is associated with increasing gravel supply. Also shown is the baseline data set discriminated according to subset.

Figure 12 Definition diagram for determining bankfull discharge from a stage-discharge curve.

Figure 13 Predicted versus reported bankfull discharge for the baseline data set, discriminated according to subset.

Figure 14 Predicted versus reported bankfull discharge for the Maryland, Britain II and ColoSmall data sets.

Figure 15 Estimated fraction of the resistance coefficient that is form drag versus the ratio H_{bf}/D_{s50}, based on the assumption that D_{s90}/D_{s50} is equal to 3.

Figure 16 Plot of bankfull Shields number τ_{bf}^{*} versus bed slope S for the baseline data set. Also included is the relation (47) due to Mueller and Pitlick (2005a) and the linear regression relation (48) obtained from the baseline data set.

Figure 17 Plot of H_{bf}/D_{s50} versus S for the baseline data set.

Figures for: QUASI-UNIVERSAL RELATIONS FOR BANKFULL HYDRAULIC GEOMETRY OF SINGLE-THREAD GRAVEL-BED RIVERS

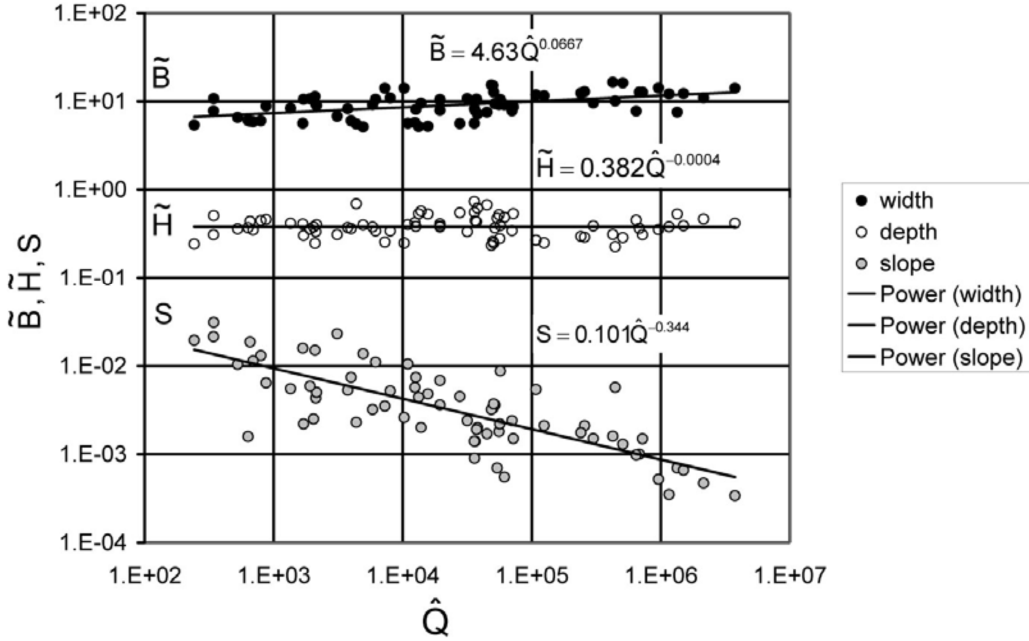


Figure 1 Dimensionless bankfull width $\tilde{B}$, dimensionless bankfull depth $\tilde{H}$ and down-channel bed slope S as functions of dimensionless bankfull discharge $\hat{Q}$. The Alberta, Britain I, Idaho and Colorado subsets of the baseline data set have been lumped together. Also shown are power relations derived from regression on the lumped data set.

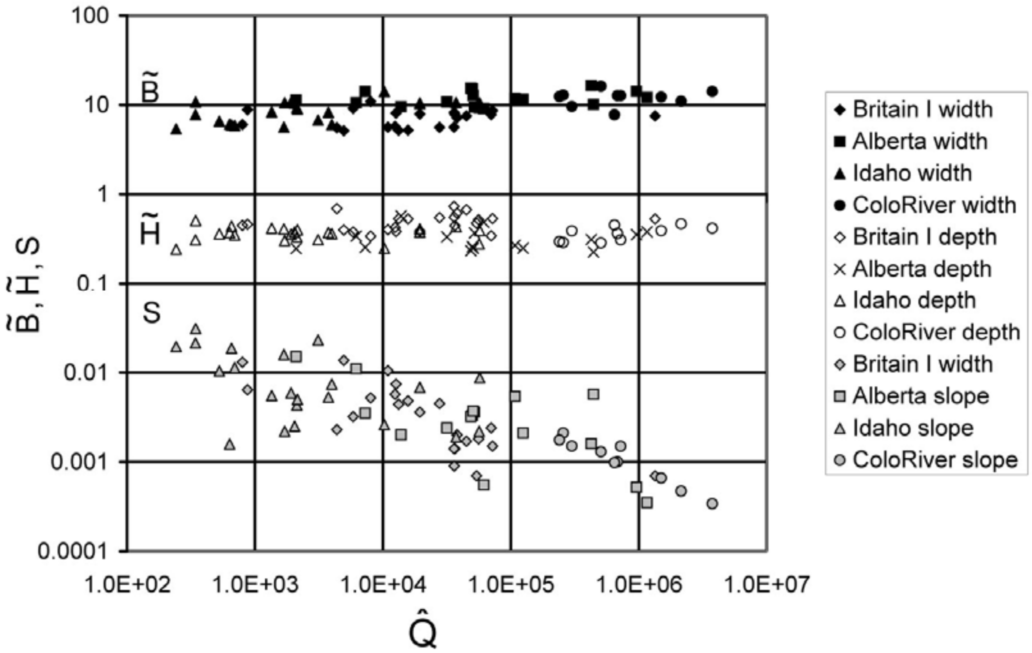


Figure 2 Dimensionless bankfull width $\tilde{B}$, dimensionless bankfull depth $\tilde{H}$ and down-channel bed slope S as functions of dimensionless bankfull discharge $\hat{Q}$, in which the Alberta, Britain I, Idaho and Colorado data subsets of the baseline data set are distinguished by different symbols.

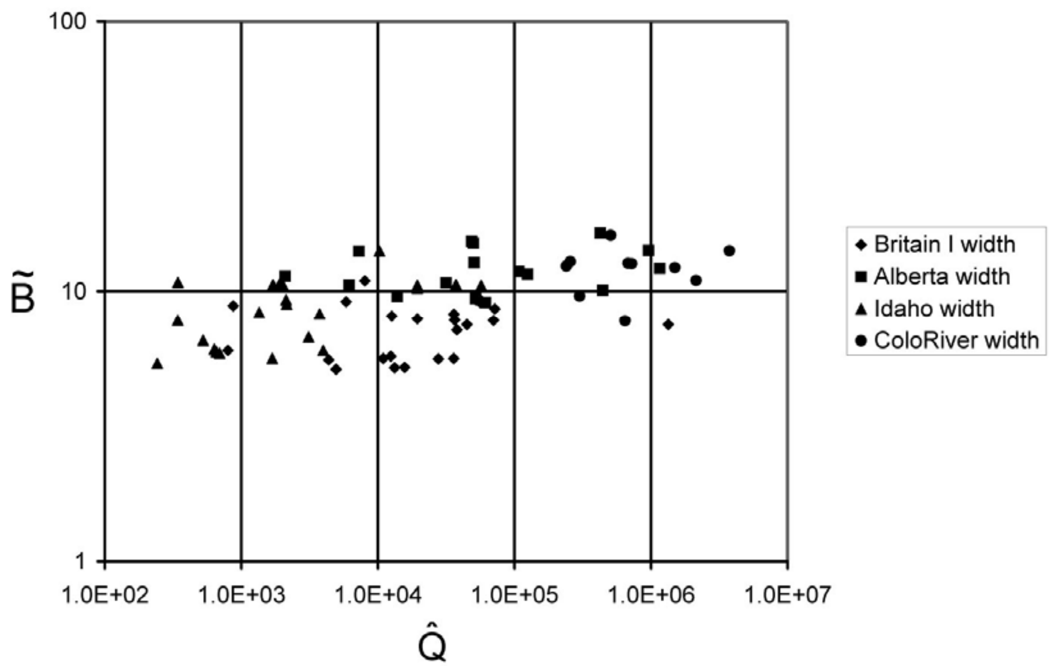


Figure 3 a) Plot of $\tilde{B}$ versus $\hat{Q}$ for the baseline data set, in which the Alberta, Britain I, Idaho and Colorado data subsets are distinguished by different symbols.

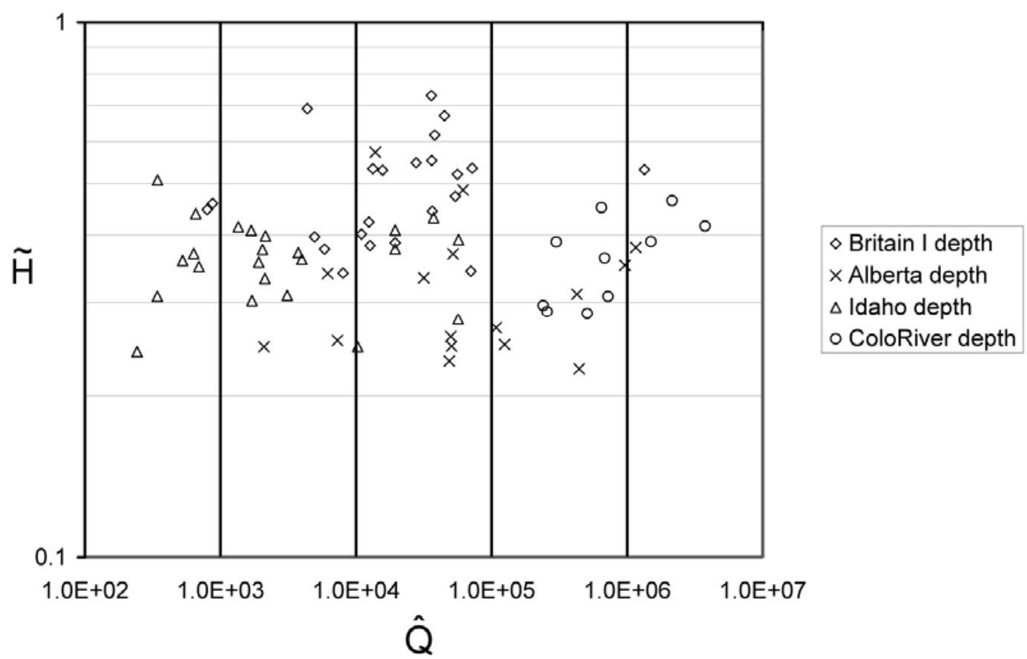


Figure 3 b) Plot of $\tilde{H}$ versus $\hat{Q}$ for the baseline data set, in which the Alberta, Britain I, Idaho and Colorado data subsets are distinguished by different symbols.

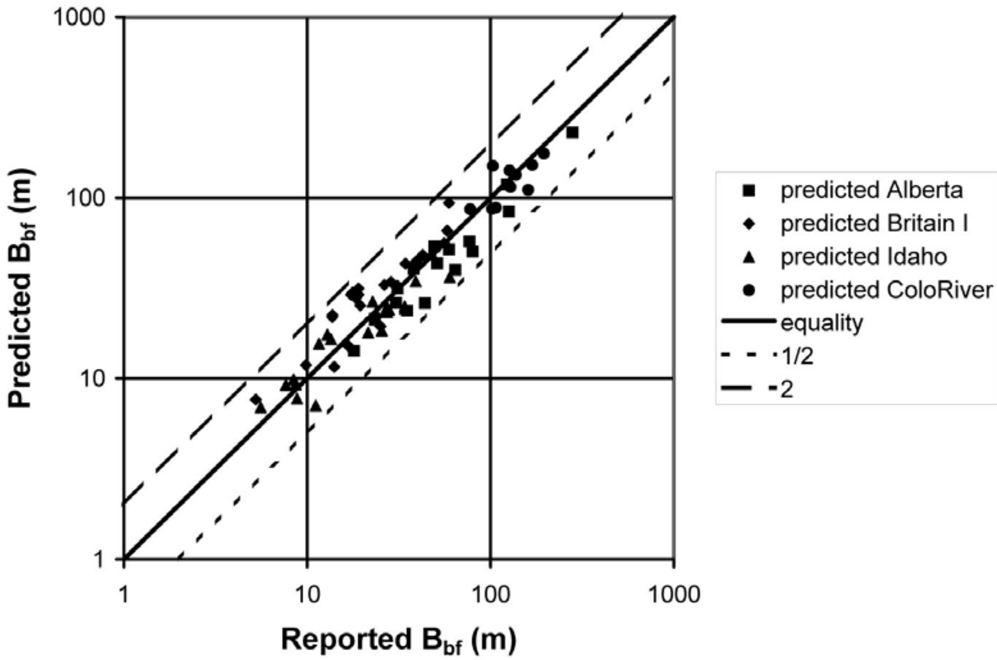


Figure 4 a) Predicted versus reported bankfull width B_{bf} for the baseline data set.

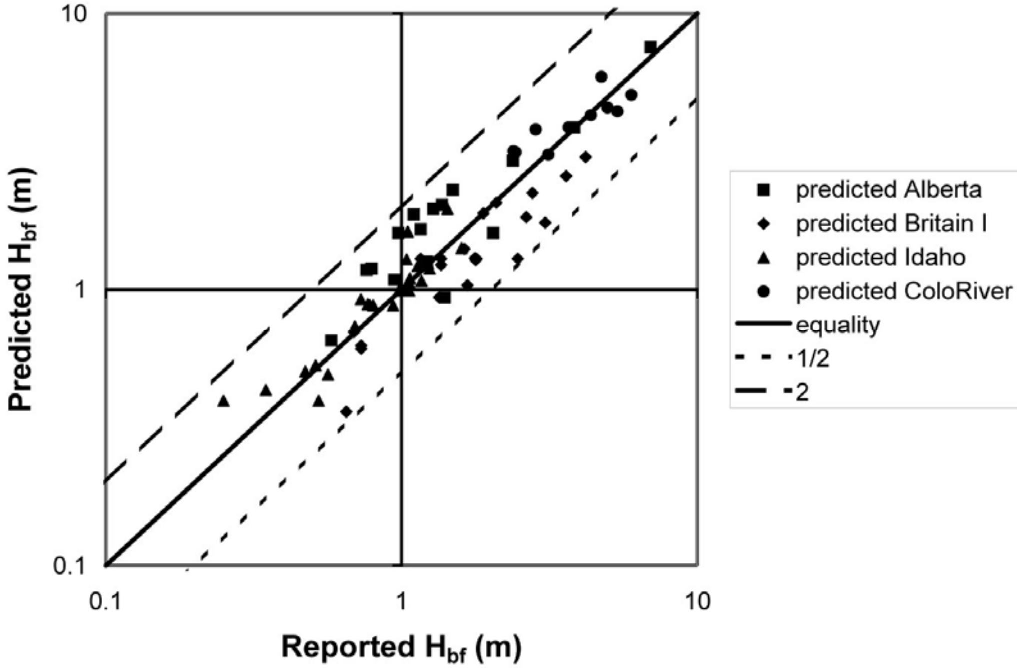


Figure 4 b) Predicted versus reported bankfull depth H_{bf} for the baseline data set.

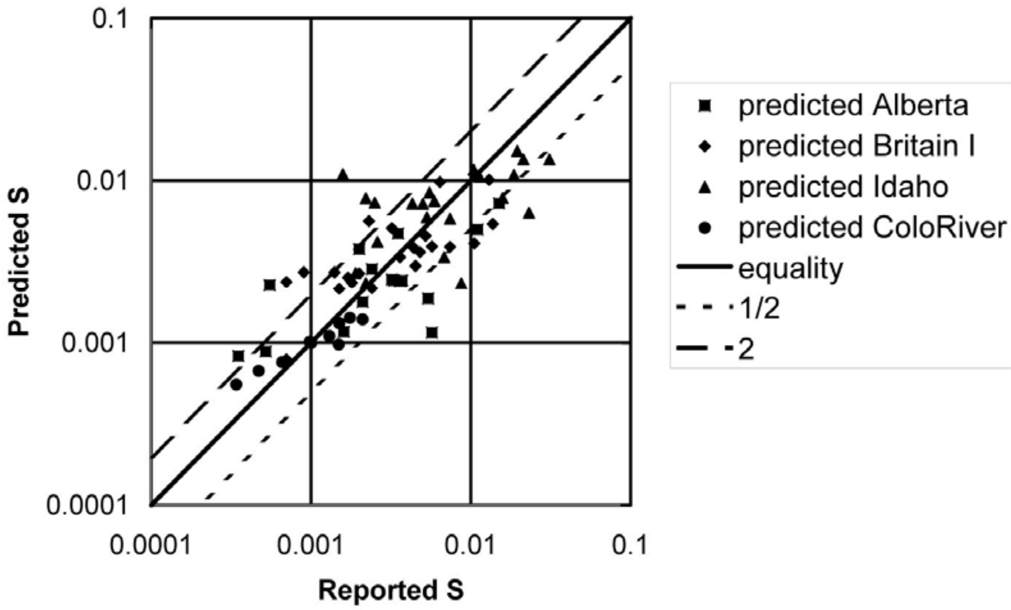


Figure 4 c) Predicted versus reported down-channel bed slope S for the baseline data set.

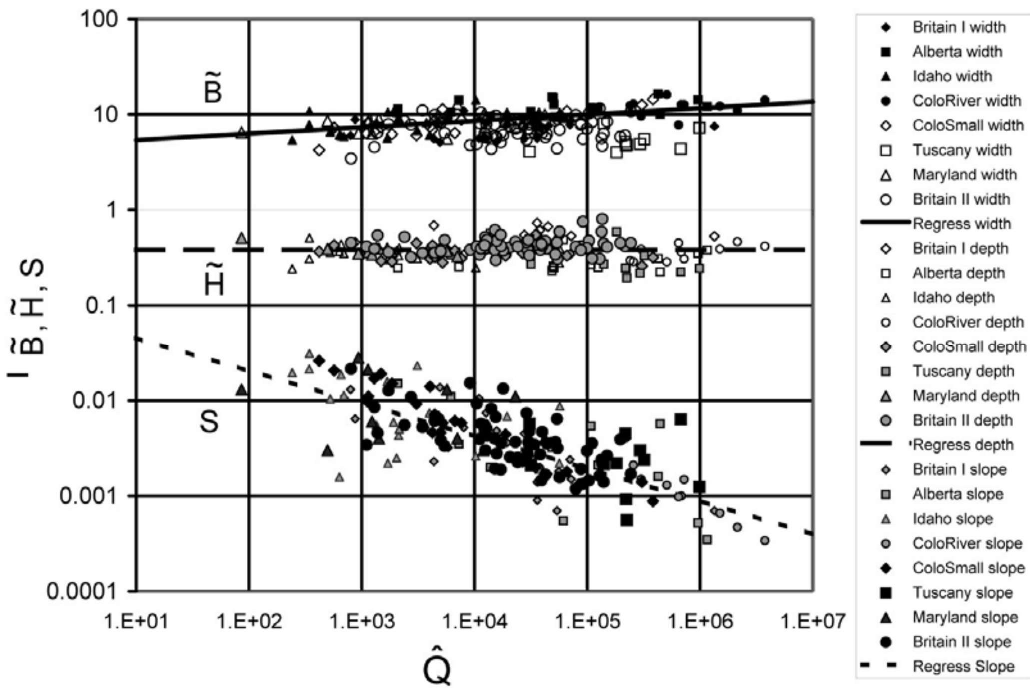
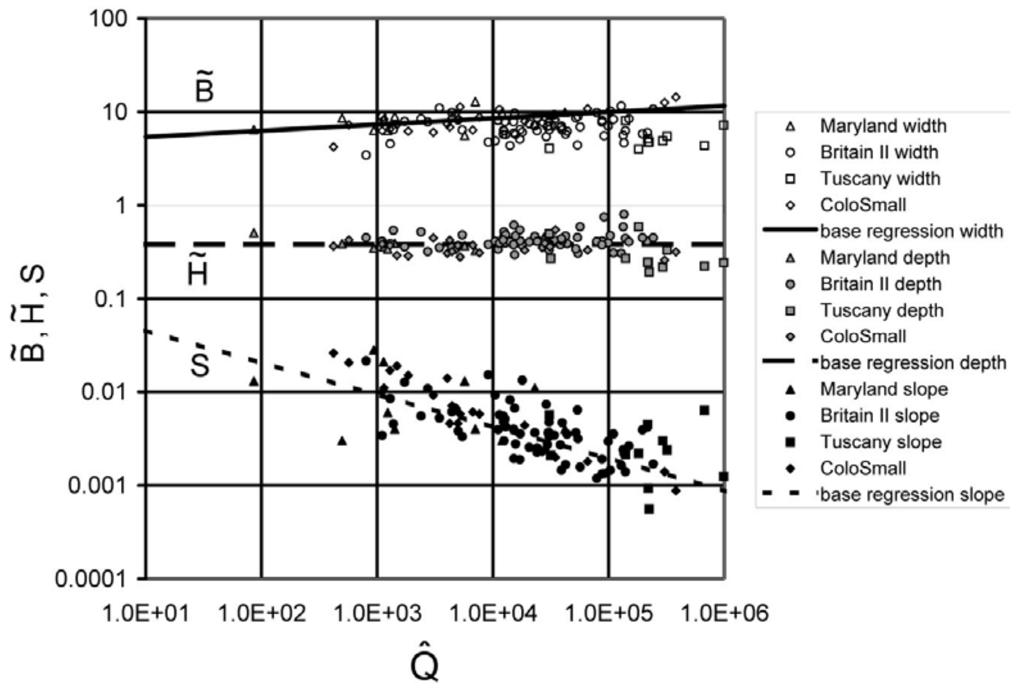


Figure 5 Dimensionless bankfull width $\tilde{B}$, dimensionless bankfull depth $\tilde{H}$ and down-channel bed slope S as functions of dimensionless bankfull discharge $\hat{Q}$, in which the baseline data set has been augmented by the Maryland, Britain II and Tuscany subsets. All subsets are distinguished by different symbols.



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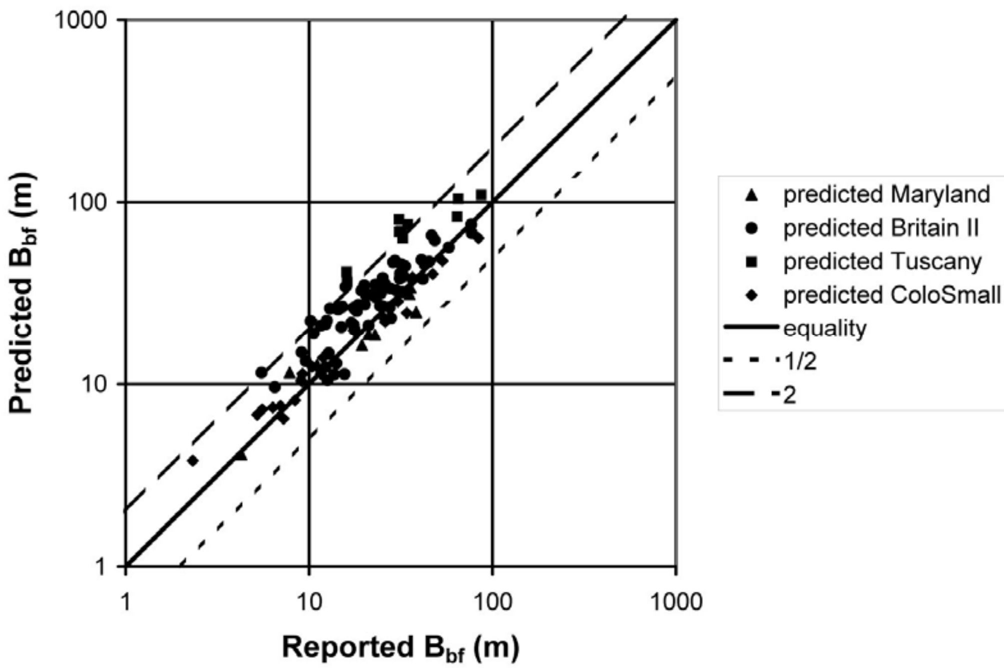


Figure 7 a) Predicted versus reported bankfull width B_{bf} for the Maryland, Britain II and Tuscany data subsets.

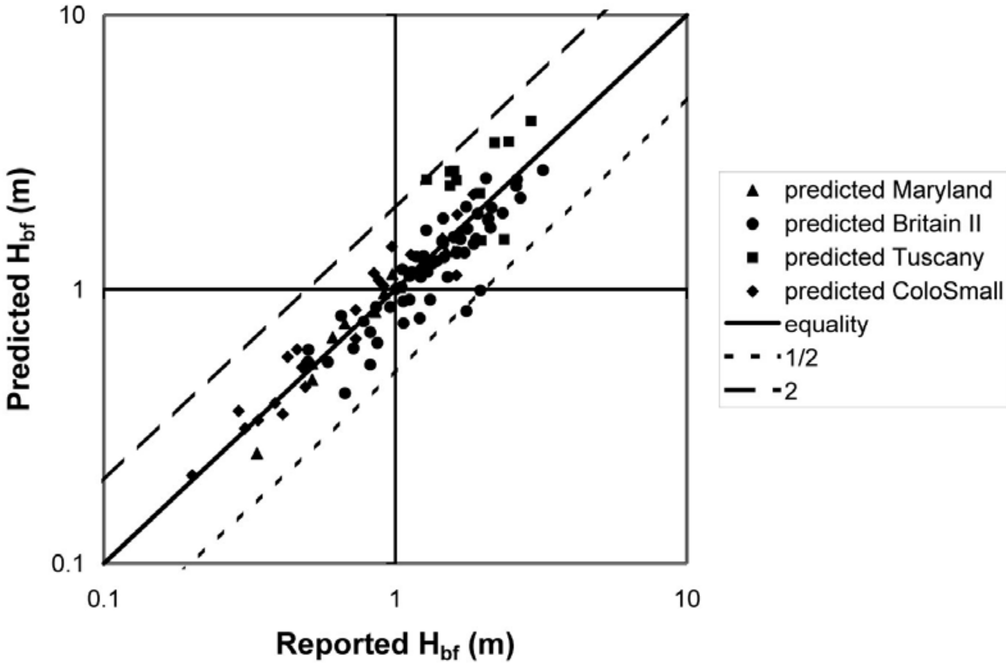


Figure 7 b) Predicted versus reported bankfull depth H_{bf} for the Maryland, Britain II and Tuscany data subsets.

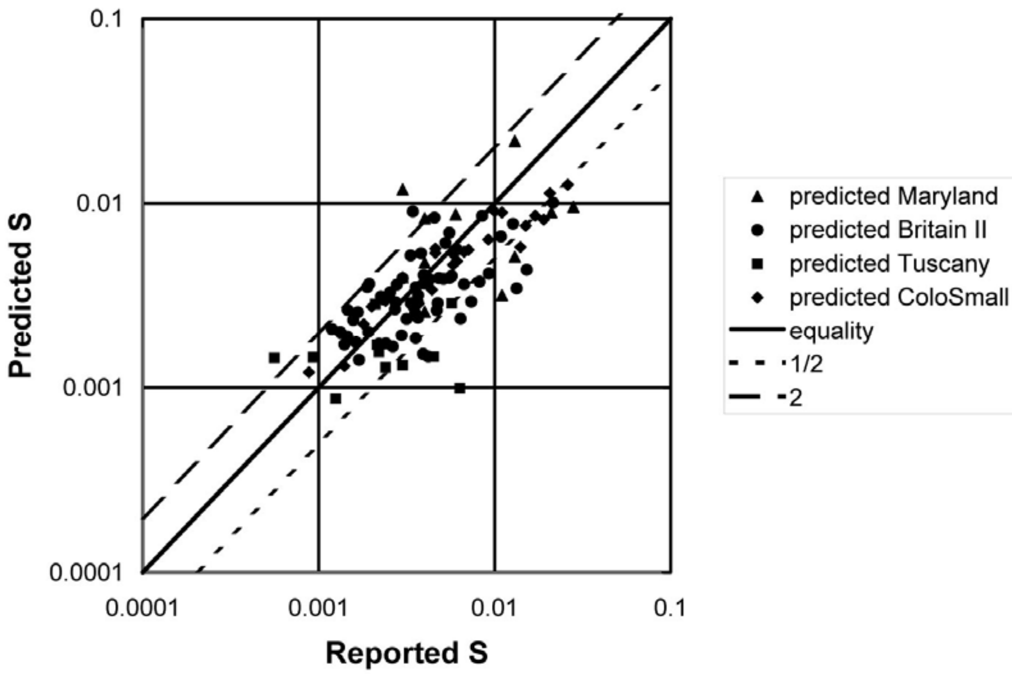
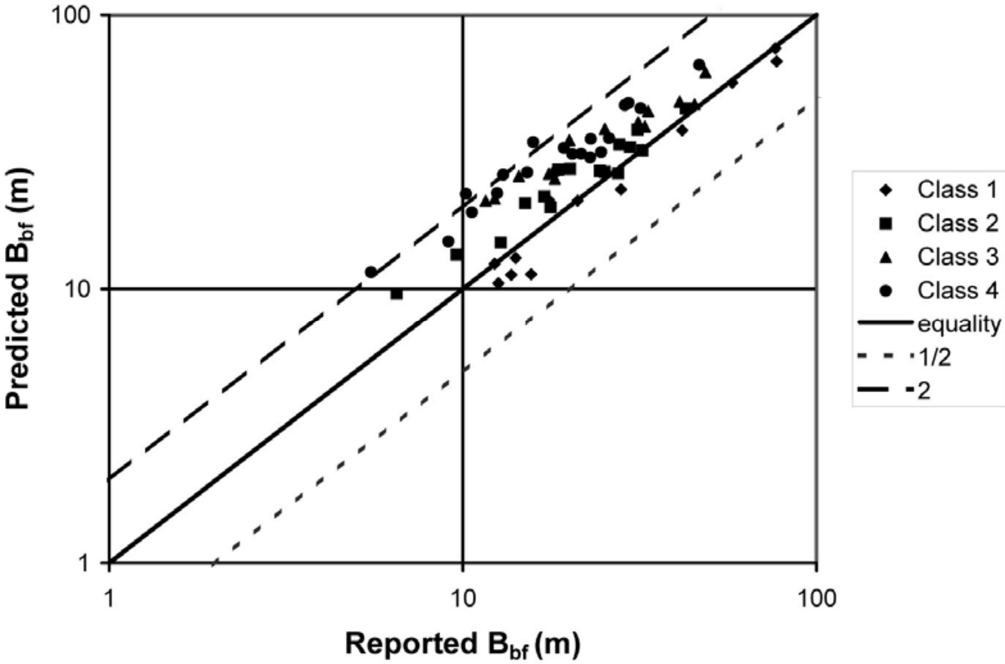


Figure 7 c) Predicted versus reported down-channel bed slope S for the Maryland, Britain II and Tuscany data subsets.



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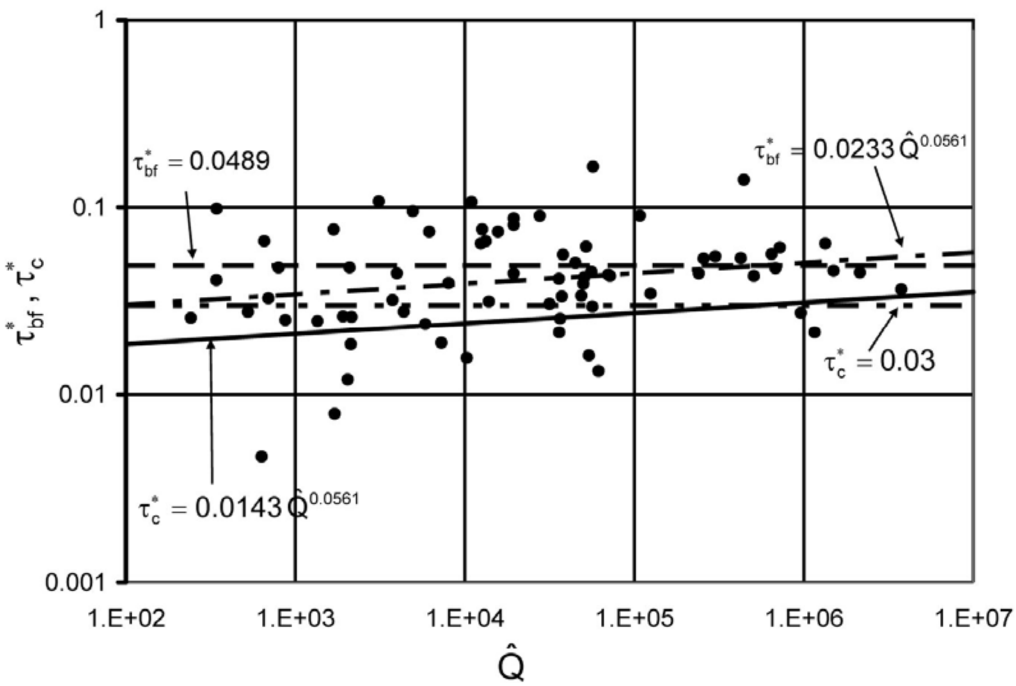
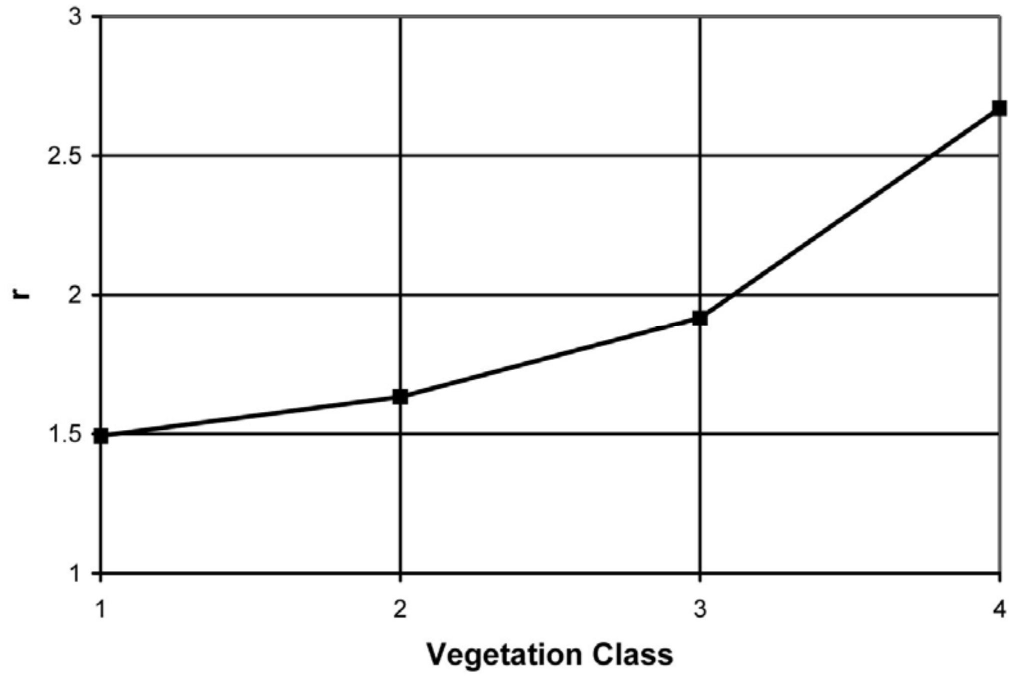


Figure 9 Plot of the bankfull Shields number τ^*_{bf} for the baseline data set. Also included are a) the line $\tau^*_{bf} = 0.0489$ corresponding to the average value for the baseline data set, b) relation (38) for τ^*_{bf}, c) the estimate of critical Shields number $\tau^*_c = 0.03$ and d) the relation (36) for critical Shields number.



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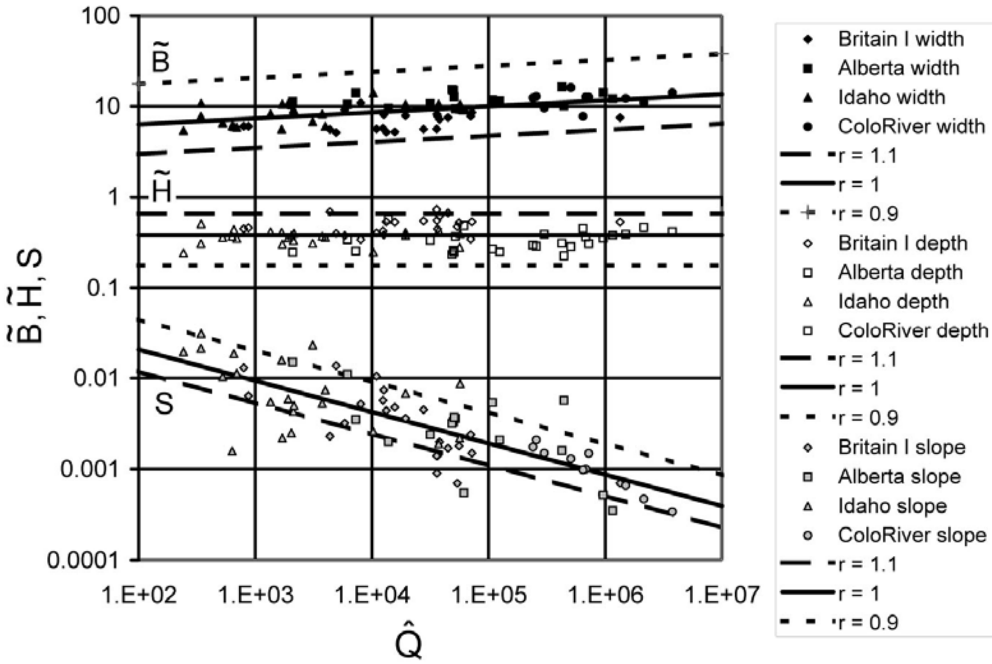

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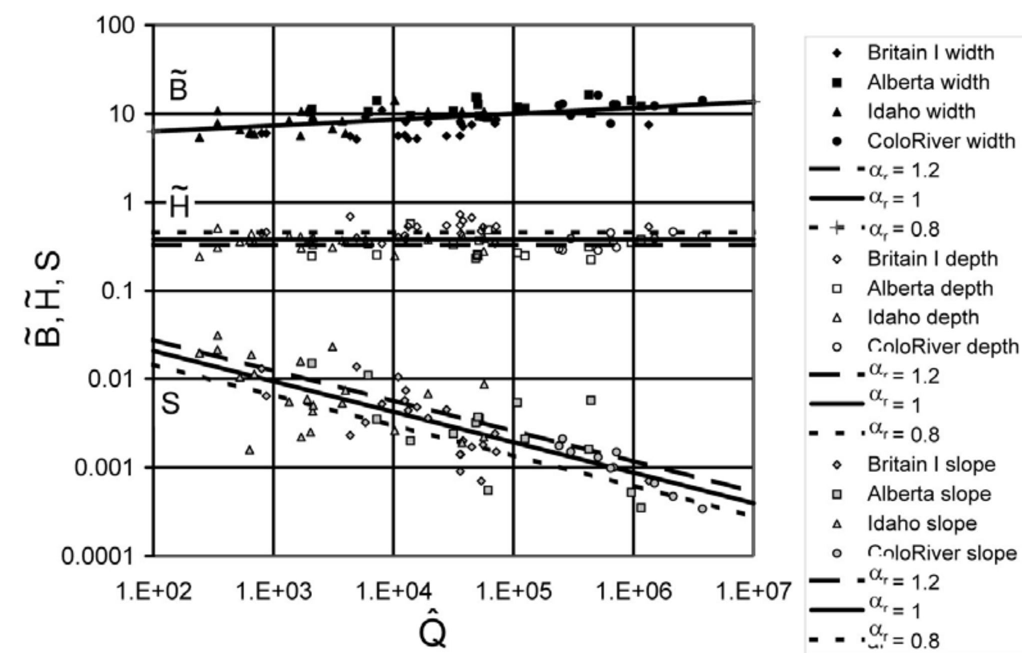
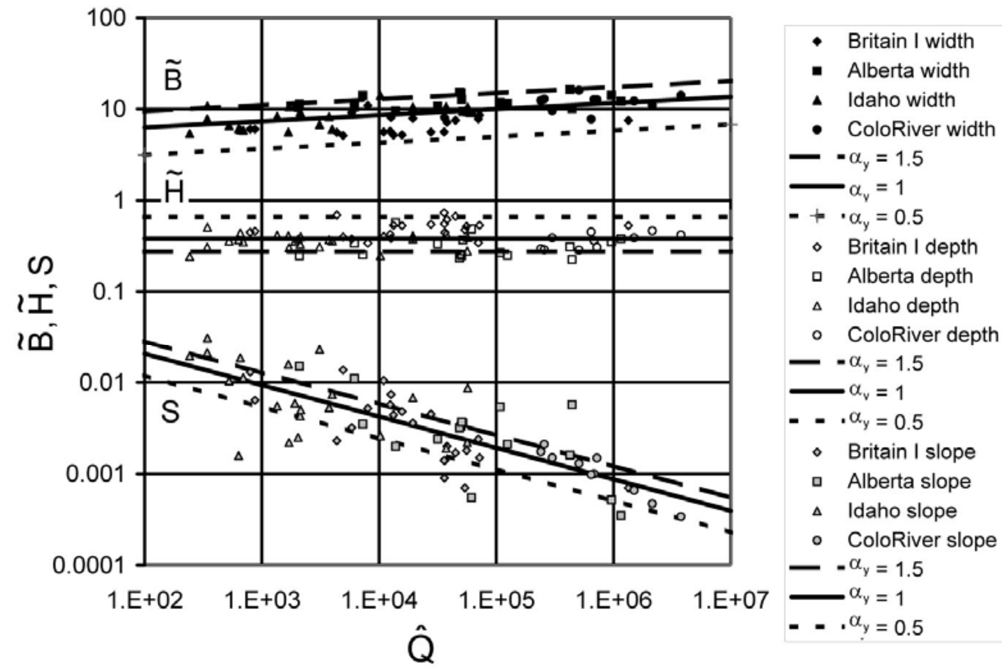


Figure 11 b) Dimensionless bankfull width $\tilde{B}$, dimensionless bankfull depth $\tilde{H}$ and down-channel bed slope S as functions of dimensionless bankfull discharge $\hat{Q}$, showing the predictions of the generalized hydraulic geometry relations as the parameter α_r is varied from 0.8 to 1.2. Increasing α_r is associated with decreasing channel resistance. Also shown is the baseline data set discriminated according to subset.



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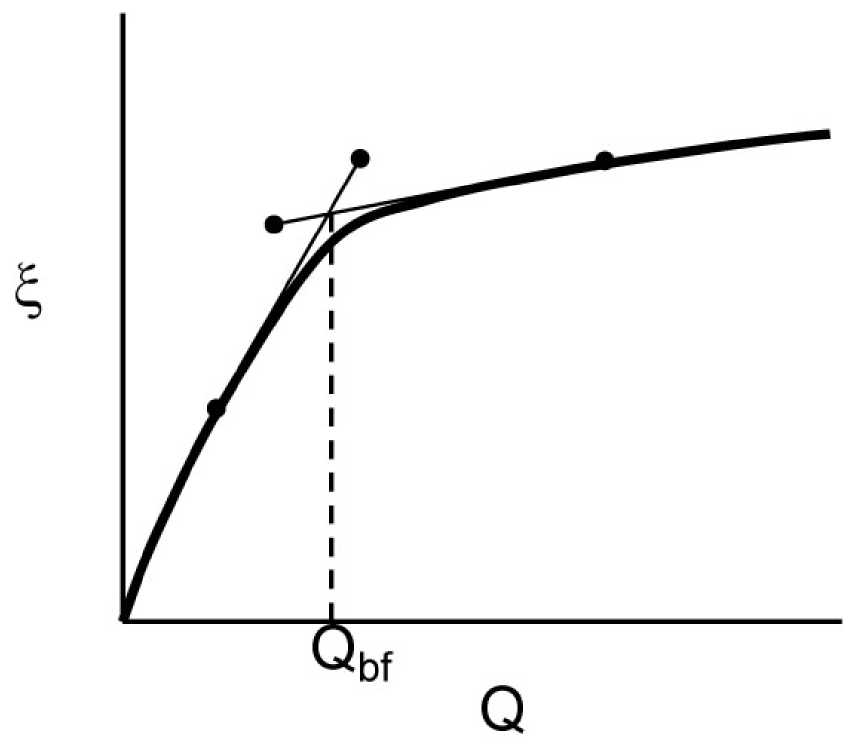

Figure 12 Definition diagram for determining bankfull discharge from a stage-discharge curve.

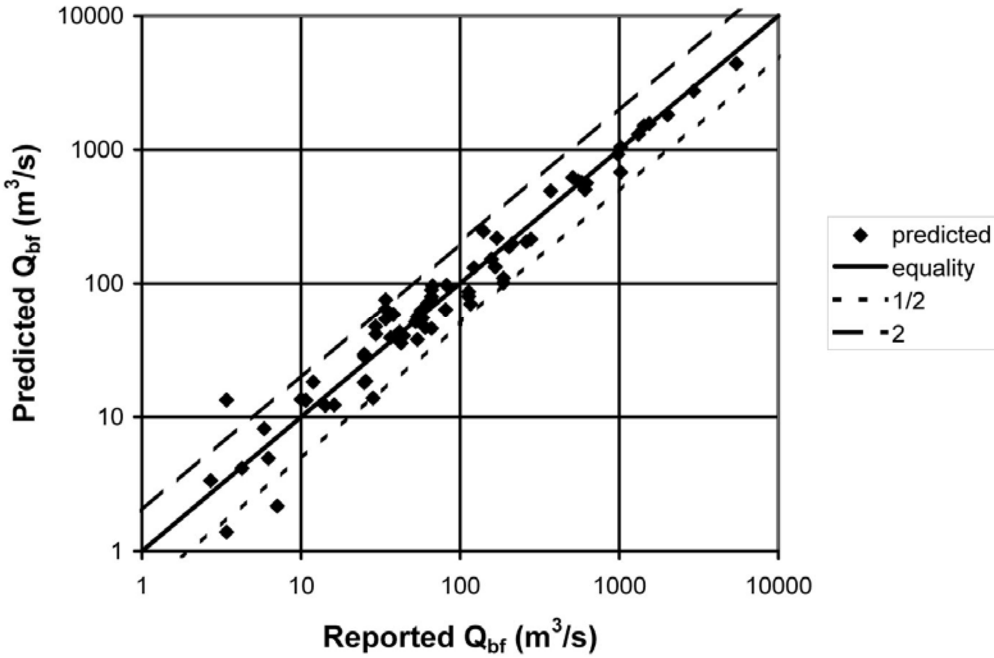

Figure 13 Predicted versus reported bankfull discharge for the baseline data set, discriminated according to subset.

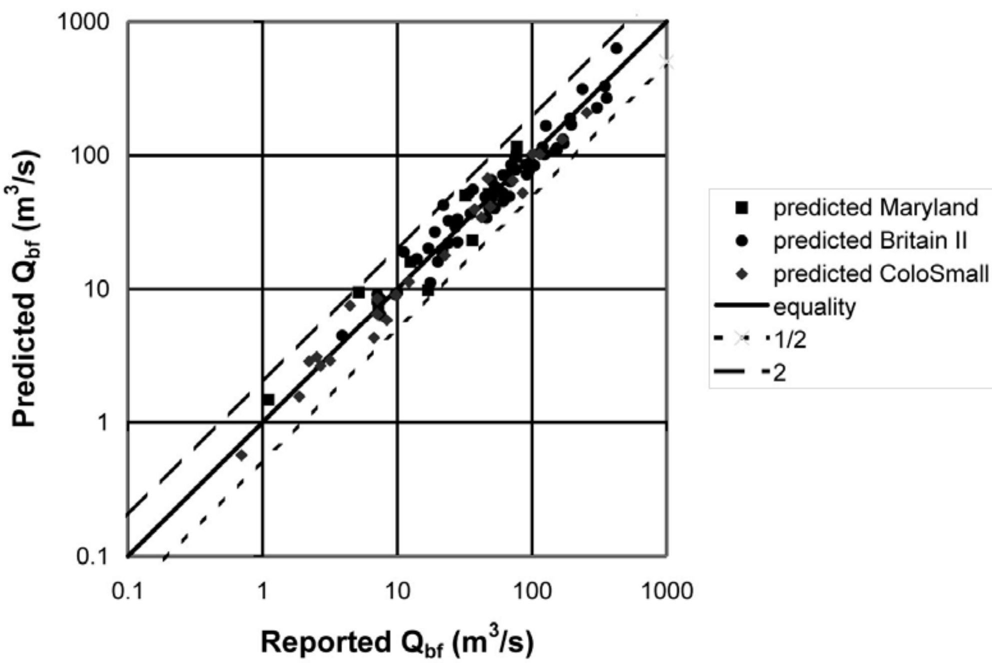

Figure 14 Predicted versus reported bankfull discharge for the Maryland and Britain II subsets.

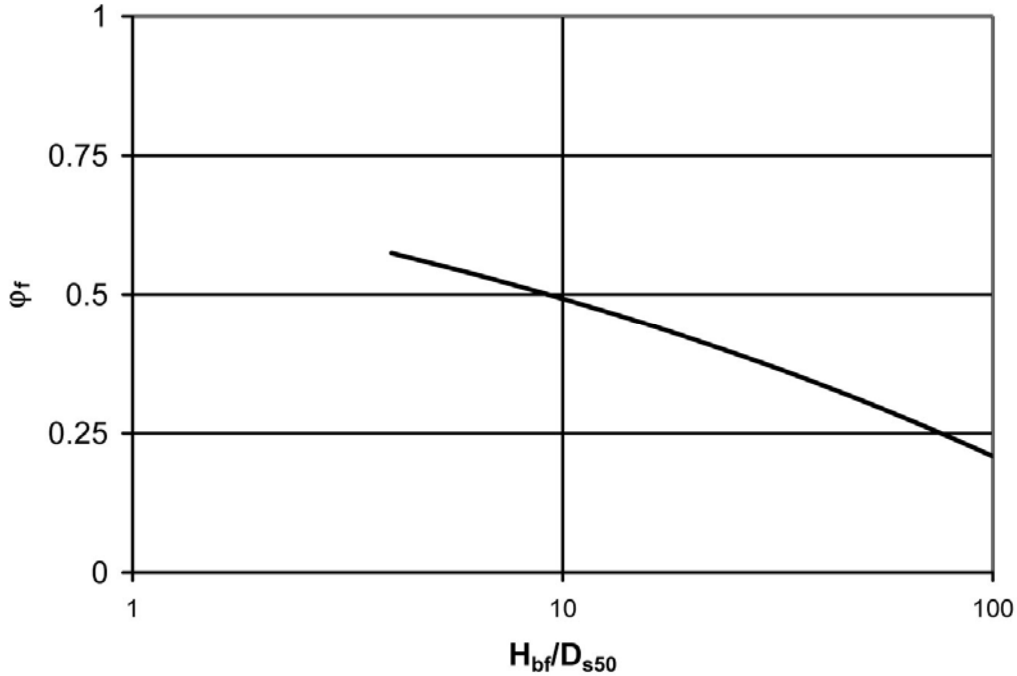

Figure 15 Estimated fraction of the resistance coefficient that is form drag versus the ratio H_{bf}/D_{s50}, based on the assumption that D_{s90}/D_{s50} is equal to 3.

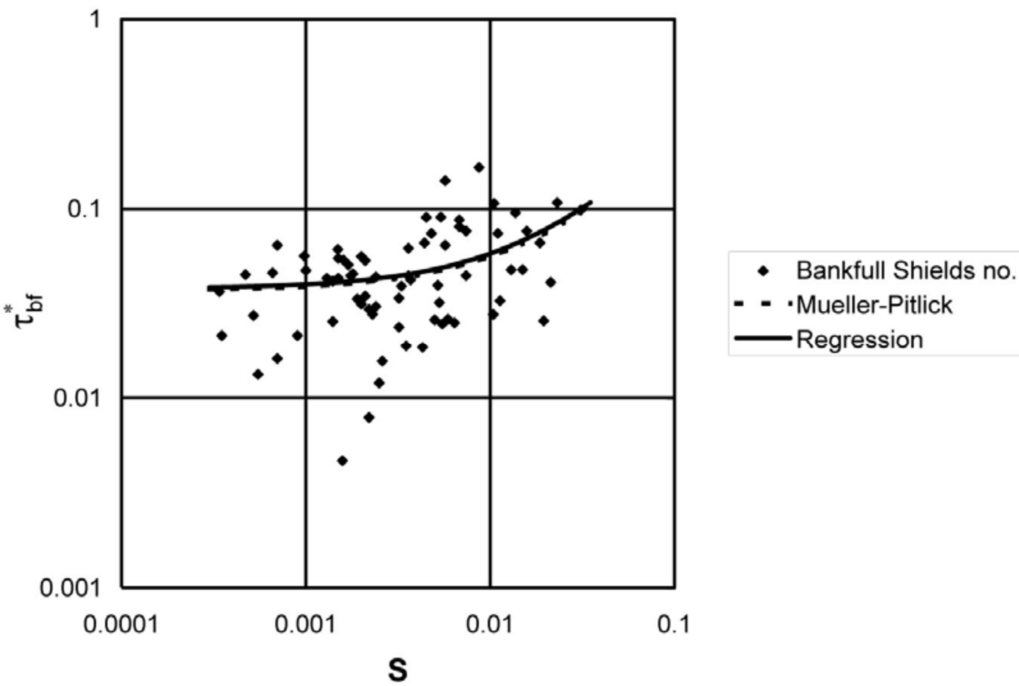

Figure 16 Plot of bankfull Shields number τ^*_{bf} versus bed slope S for the baseline data set. Also included is the relation (47) due to Mueller and Pitlick (2005a) and the linear regression relation (48) obtained from the baseline data set.

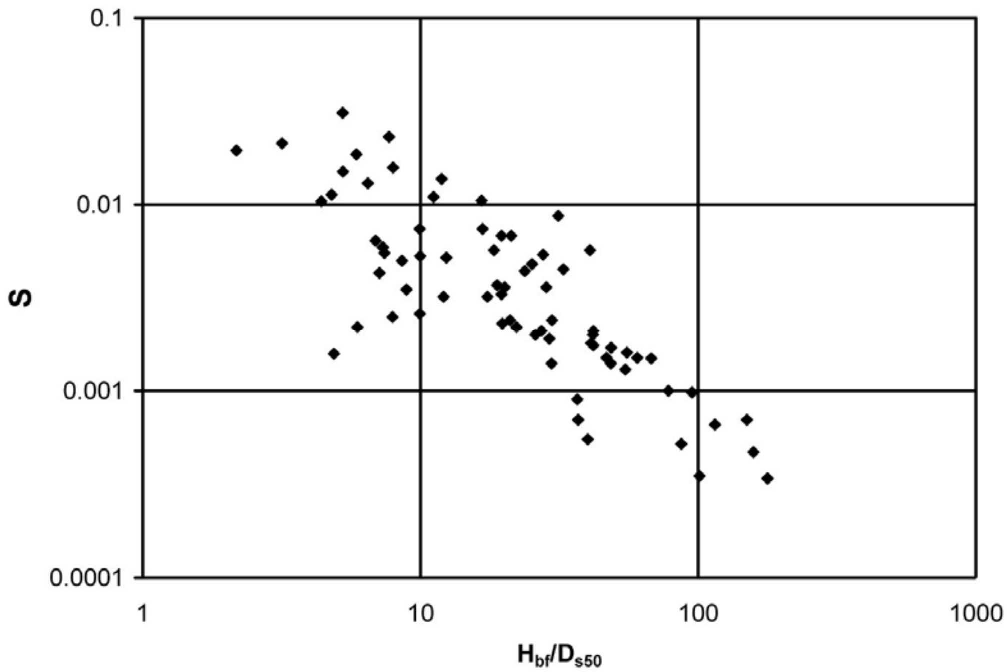

Figure 17 Plot of H_{bf}/D_{s50} versus S for the baseline data set.