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THE SEDIMENT DIGESTER

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INTRODUCTION

The material below was written in November, 2001. It represents an attempt to quantify processes observed by the author at the Porgera Gold Mine, Papua New Guinea in October, 2001.

THE PHENOMENON

Incompetent waste sediment from the Porgera Gold Mine, Papua New Guinea, is disposed into two waste dumps, the Anjolek Dump and the Anawe Dump. The sediment introduced into these dumps forms earthflows which move slowly downslope, and are eventually incorporated into the fluvial transport of rivers. Of interest here is the Anjolek Dump. Figure 1 shows a view of the site, including the Anjolek and Anawe Dumps and the Kaiya River. The sediment of the Anjolek Dump is incorporated fluvially into the Kaiya River and carried downstream. In the image part of the Kaiya River flows through the middle of the downstream end of the Anjolek earthflow. At the time of the author's site visit, however, the earthflow has pushed the Kaiya River up against the northern valley wall.

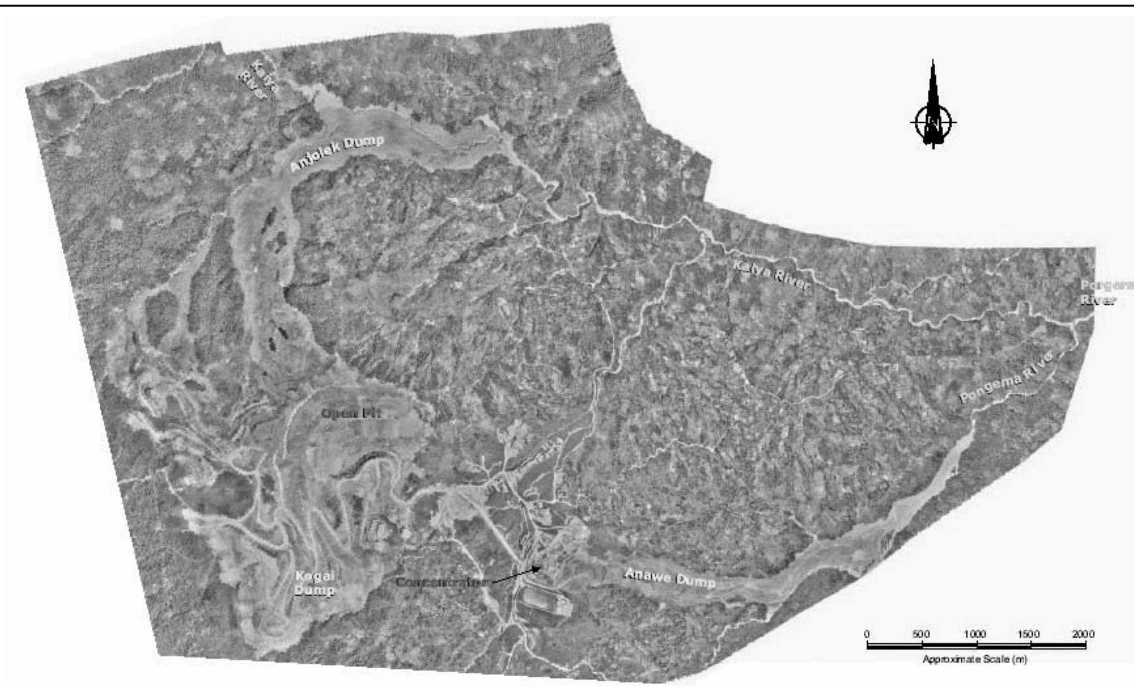

Figure 1. View of the Porgera Gold Mine, the earthflow of the Anjolek Dump, the earthflow of the Anawe Dump and the Kaiya River.

Figure 2-4 were taken by the author during a site visit in October, 2001. Figure 2 shows a view of the Anjolek earthflow looking downslope from the dumps.

Figure 3 shows a view of the same earthflow as it approaches the Kaiya River and pushes it up against the northern valley wall of the river. Figure 4 shows a view of the distal end of the same earthflow and the Kaiya River

Figure 1. View of the Anjolek earthflow looking downslope from the dump site. The Kaiya River is visible in the distance.

Figure 3. View of the Anjolek earthflow as it flows into and along the valley of the Kaiya River.

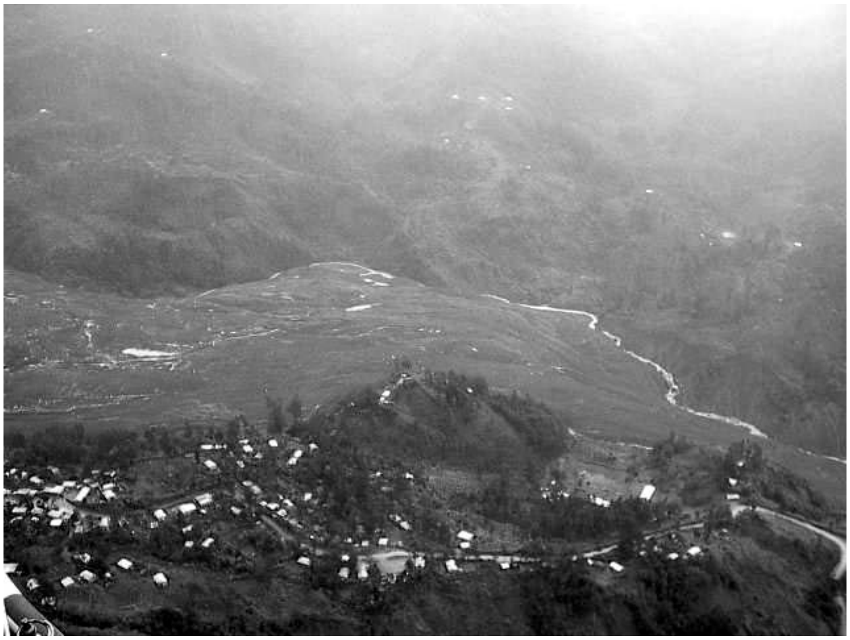

Figure 4. View of the distal end of the Anjolek earthflow and the Kaiva River.

OVERVIEW

The “sediment digester” is a model for describing how the Kaiya River, Papua New Guinea digests sediment from the Anjolek earthflow (or how the Pongema River digests sediment from the Anawe earthflow). These earthflows flow continuously from dump sites at the Porgera Gold Mine, Papua New Guinea. They constrict the river to one side of the river valley over a significant length. This results in a line source of sediment to the river rather than a point source. The case considered is illustrated below in Figures 5 (plan) and 6 (cross-section). In Figure 2 the flow in the river is directed out of the page, and the earthflow introduces sediment from the right side.

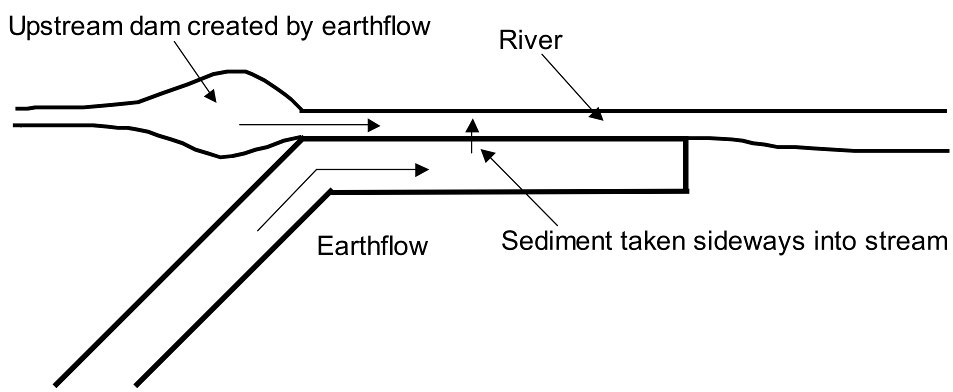

Figure 5. Schematic diagram illustrating how the Kaiya River “digests” the sediment from the Anawe earthflow.

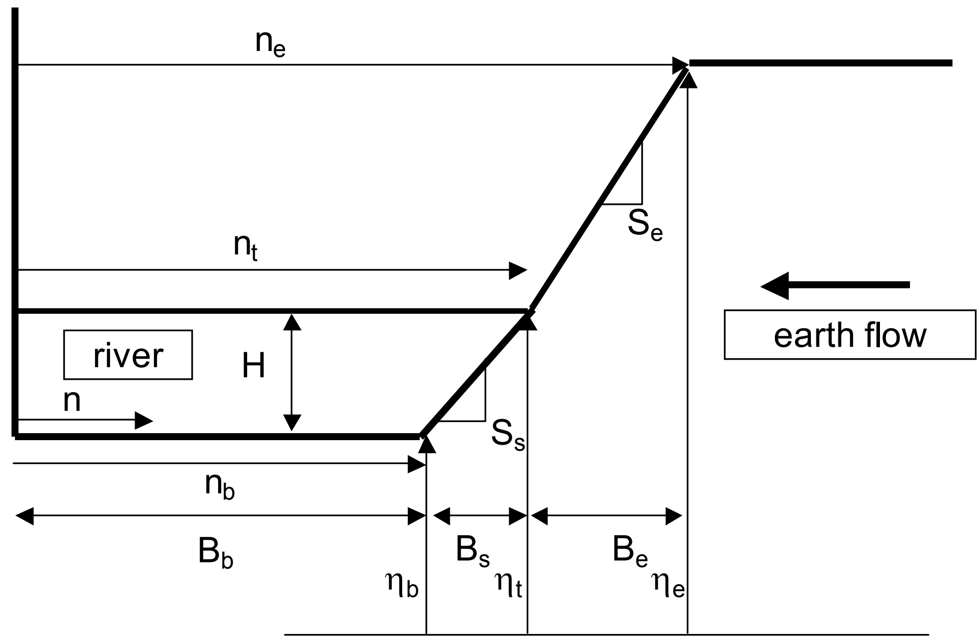

Figure 6. Schematic diagram illustrating the assumed cross-sectional geometry.

These figures imply an interesting interaction. Sideways sediment supply from the earthflow tends to narrow the channel, and in addition causes it to aggrade. The narrower channel responds by increasing its capacity for streamwise transport and lateral removal of earthflow material. The problem constitutes an interesting case of the interaction of stream width and channel bed elevation.

The same treatment be used to describe channel widening from an initial, overly narrow configuration to an equilibrium configuration with a finite, non-vanishing sediment transport rate in the channel and no further widening. In such a case the lateral input of sediment is assumed to be vanishing.

GEOMETRY

In Figure 6 n denotes a transverse coordinate directed from a fixed, vertical wall. The streamwise coordinate is directed into the page. The submerged side slope of the channel take the constant value S_s. The emergent side slope of the earthflow is S_e. Flow depth in the bed region of the channel is H. The width of the bed region of the channel, side region of the channel and earthflow above the channel are defined to be B_b, B_s and B_e respectively. Elevation of the channel bed is η_b, elevation of the water surface is η_t and elevation of the top of the earthflow is η_e. Transverse distance from the vertical wall to the channel bed–sidewall intersection is n_b; transverse distance to the water's edge is n_t and transverse distance to the top of the earthflow is n_e.

Many of these parameters are related geometrically. Some of these relations are

$$n_b = B_b \qquad n_t = B_b + B_s \qquad n_e = B_b + B_s + B_e$$
$$B_s = \frac{H}{S_s} \qquad B_e = \frac{\eta_e - \eta_t}{S_e} \qquad \eta_t = \eta_b + H \tag{1a,b,c,d,e,f}$$

In order to simplify the problem, the elevation η_e of the top of the earthflow is assumed to be constant herein.

CHANNEL HYDRAULICS

For simplicity normal flow (steady, streamwise uniform flow) is assumed here. Let τ_b denote the shear stress on the bed region of the channel and τ_{bs} denote the boundary shear stress on the bank, or side slope region of the channel (as well as the vertical wall). The condition for normal flow is

$$\tau_b B_b + \tau_{bs} H \left(1 + \sqrt{1 + \frac{1}{S_s^2}} \right) = \rho g (H B_b + \frac{1}{2} H B_s) S \tag{2}$$

where S denotes the streamwise bed slope (assumed to be the same for the bed and bank regions).

We now assume that

$$\tau_{bs} = \varphi \tau_b \tag{3}$$

where φ is a number typically between 2/3 and 4/5. This assumption allows for the possibility of a channel that does not erode its banks (because τ_{bs} is below the threshold of motion there) but can maintain an equilibrium sediment transport rate on the bed region (because τ_b exceeds the threshold of motion there.) This corresponds with the analysis of Parker (1978). Manipulating with (1d), it is found that

$$\tau_b \left(B_b + \varphi \frac{S_s + \sqrt{1 + S_s^2}}{S_s} H \right) = \rho g H S \left(B_b + \frac{1}{2} \frac{H}{S_s} \right) \tag{4}$$

In addition, it is assumed that the cross-sectionally averaged flow velocity U maintains the same value on the bed region as on the bank region, and that in addition a Manning-Strickler relation for bed resistance holds;

$$\tau_b = \rho C_f U^2$$

$$C_f^{-1/2} = \alpha_r \left(\frac{H}{k_s}\right)^{1/6} \qquad (5a,b,c)$$

$$k_s = n_k D$$

where C_f is a friction coefficient, k_s is roughness height, D denotes a characteristic sediment grain size (e.g. median or geometric mean), α_r is a dimensionless coefficient here taken to be equal to 8.1 and n_k is a dimensionless parameter that is typically between 2 and 4 depending on the standard deviation of the grain size distribution. Finally, water conservation requires that total water discharge Q_w is given as

$$Q_w = UH\left(B_b + \frac{1}{2}\frac{H}{S_s}\right) \qquad (6)$$

Solving for H between (4), (5) and (6) it is found that

$$H = \left(\frac{k_s^{1/3} Q_w^2}{\alpha_r^2 g B_b^2 S}\right)^{3/10} c_1 \qquad c_1 = \left[\frac{1 + \varphi \dfrac{S_s + \sqrt{1 + S_s^2}}{S_s}\dfrac{H}{B_b}}{\left(1 + \dfrac{1}{2S_s}\dfrac{H}{B_b}\right)^3}\right]^{3/10} \qquad (7a,b)$$

Note that the actual solution must be achieved iteratively since c_1 is a weak function of H. A Newton-Raphson iteration scheme is supplied toward the end of this document.

The Shields number τ_b^* for the bed region is then given as

$$\tau_b^* = \frac{\tau_b}{\rho R g D} = \frac{HS}{RD}\left(\frac{1 + \dfrac{1}{2S_s}\dfrac{H}{B_b}}{1 + \varphi \dfrac{S_s + \sqrt{1 + S_s^2}}{S_s}\dfrac{H}{B_b}}\right) = \frac{1}{RD}\left(\frac{k_s^{1/3} Q_w^2}{\alpha_r^2 g B_b^2}\right)^{3/10} S^{7/10} c_2$$

$$c_2 = \left(\frac{1 + \dfrac{1}{2S_s}\dfrac{H}{B_b}}{1 + \varphi \dfrac{S_s + \sqrt{1 + S_s^2}}{S_s}\dfrac{H}{B_b}}\right) c_1 \qquad (8a,b)$$

In the above relations *R* denotes the submerged specific gravity of the sediment (usually close to 1.65). Note that c_2 is another parameter that can be approximated as unity for sufficiently small values of H/B_b. The Shields number τ_s^* on the bank or sidewall region is then

$$\tau_s^* = \frac{\tau_{bs}}{\rho R g D} = \varphi \tau_b^* \tag{9}$$

Equations (7) and (8) contain useful information about the sediment digestion process. They specifically indicate that as the width of the bed region is decreased, both the depth of flow and the Shields stress is increased.

STREAMWISE BEDLOAD TRANSPORT

Let q_{sbs} and q_{bss} denote the streamwise volume bedload transport rate per unit width on the bed and sidewall regions, respectively. It is assumed that both can be computed from a common bedload transport relation as long as the appropriate Shields stress is used. Here the following assumption is made:

$$q_{bsb} = \sqrt{RgD}D\alpha_s\left(\tau_b^*\right)^{1.5}\left(1-\frac{\tau_c^*}{\tau_b^*}\right)^{4.5}$$
$$q_{bss} = \sqrt{RgD}D\alpha_s\left(\varphi\tau_b^*\right)^{1.5}\left(1-\frac{\tau_c^*}{\varphi\tau_b^*}\right)^{4.5} \tag{10a,b}$$

In the above relations α_s = 11.2 and τ_c^* = 0.03. The relation itself is the Parker (1979) approximation to the Einstein (1950) bedload transport relation. Between (8), (9) and (10) it is seen that a decrease in width increases the Shields stress, and thus the bedload transport rate.

Assuming that q_{bsb} and q_{bss} have units of m^2/s, the total streamwise bedload transport rate G_{load} in Mt/a is given as

$$G_{load} = \left(B_b q_{bsb} + \sqrt{1+S_s^2}B_s q_{bss}\right)(R+1)\frac{31557600}{1x10^6} \tag{10c}$$

TRANSVERSE BEDLOAD TRANSPORT

In general, let q_{bs} denote the streamwise volume bedload transport rate per unit width and q_{bn} denote the corresponding volume bedload transport rate in the transverse direction. The following simple approximate relation for transverse bedload transport has been determined by a number of authors:

$$\frac{q_{bn}}{q_{bs}} = \frac{\tau_{bn}}{\tau_{bs}} - \alpha_n \sqrt{\frac{\tau_c^*}{\tau_s^*}} \frac{\partial \eta}{\partial n} \qquad (11)$$

In the above relation, τ_{bn} denotes the boundary shear stress in the transverse direction, τ^* denotes the Shields stress based on the streamwise shear stress on the sidewall region and α_n and b are constants. The relation says that a) in the absence of a side slope the bedload transport vector is parallel to that of the boundary shear stress, and b) in the presence of a side slope a component of bedload is invariably directed down the slope. In the Johannesson-Parker (1989) implementation of (11), α_n = 2.65.

For the purposes of the present analysis the channel is assumed to be straight, so that τ_{bn} can be approximated as vanishing. In addition, in the bed region of the channel the bed is assumed to be horizontal, so that $\partial\eta/\partial n$ = 0 there and thus q_{bn} can be assumed to be vanishing. In the sidewall region, however (11) takes the form

$$\begin{aligned} q_{bns} &= -q_{bss}\alpha_n \sqrt{\frac{\tau_c^*}{\varphi\tau_b^*}} S_s = \\ &-\sqrt{RgD}D\alpha_s\left(\varphi\tau_b^*\right)^{1.5}\left(1-\frac{\tau_c^*}{\varphi\tau_b^*}\right)^{4.5}\alpha_n\sqrt{\frac{\tau_c^*}{\varphi\tau_b^*}} S_s \end{aligned} \qquad (12a)$$

where τ_b^* is given by (8a). If q_{bns} is computed in m^2/s, then the corresponding transverse transport rate g_{sides} toward the channel region in Mt/m/a is given as

$$g_{sides} = -q_{bns}(R+1)\frac{31557600}{1x10^6} \qquad (12b)$$

The negative sign in (12b) insures that a transverse transport from sidewall region to bed region, which implies a negative value of q_{bns}, yields a positive value of g_{sides}.

SEDIMENT CONTINUITY IN THE BED REGION

The Exner equation for sediment continuity in the bed region was already worked out in class, but unfortunately that treatment contains some errors. (The errors are not important for most problems but are important for this one.) The problem is done correctly below.

In general the 2D version of the Exner equation of sediment continuity can be written as

$$(1-\lambda_p)\frac{\partial\eta}{\partial t} = -\frac{\partial q_{bs}}{\partial s} - \frac{\partial q_{bn}}{\partial n} \tag{13}$$

This equation may be integrated over the bed region to yield

$$(1-\lambda_p)\int_0^{n_b}\frac{\partial\eta}{\partial t}dn = -\int_0^{n_b}\frac{\partial q_{bs}}{\partial s}dn - \int_0^{n_b}\frac{\partial q_{bn}}{\partial n}dn \tag{14}$$

The first term in the above equation reduces with the aid of (1a) and Figure 6 to

$$\begin{aligned}(1-\lambda_p)\int_0^{n_b}\frac{\partial\eta}{\partial t}dn = (1-\lambda_b)\left(\frac{\partial}{\partial t}\int_0^{n_b}\eta dn - \eta\Big|_{n_b}\frac{\partial n_b}{\partial t}\right) = \\ (1-\lambda_p)\left(\frac{\partial B_b\eta_b}{\partial t} - \eta_b\frac{\partial B_b}{\partial t}\right) = (1-\lambda_p)B_b\frac{\partial\eta_b}{\partial t}\end{aligned} \tag{15}$$

According to Leibnitz rule and (1a),

$$\frac{\partial}{\partial s}\int_0^{n_b} q_{bs}dn = \int_0^{n_b}\frac{\partial q_{bs}}{\partial s}dn + q_{bs}\Big|_{n_b}\frac{\partial n_b}{\partial s} = \int_0^{n_b}\frac{\partial q_{bs}}{\partial s}dn + q_{bs}\Big|_{n_b}\frac{\partial B_b}{\partial s} \tag{16}$$

Thus the second term in (14) becomes

$$-\frac{\partial}{\partial s}\int_0^{n_b} q_{bs}dn = -\int_0^{n_b}\frac{\partial q_{bs}}{\partial s}dn + q_{bs}\Big|_{n_b}\frac{\partial B_b}{\partial s} \tag{17}$$

Approximating q_{bs} as everywhere equal to q_{bsb} on the bed region, the second term thus reduces to

$$-\frac{\partial}{\partial s}\int_0^{n_b} q_{bs}dn = -\frac{\partial B_b q_{bsb}}{\partial s} + q_{bsb}\frac{\partial B_b}{\partial s} = -B_b\frac{\partial q_{bsb}}{\partial s} \tag{18}$$

Finally, the third term in (14) reduces to

$$-\int_0^{n_b}\frac{\partial q_{bn}}{\partial n}dn = -q_{bn}\Big|_{n_b} + q_{bn}\Big|_0 \tag{19}$$

Here $q_{bn}\big|_o$ is assumed to vanish due to the inerodible wall at n = 0, and $q_{bn}\big|_{n_b}$ is specified in terms of (12);

$$q_{bn}\Big|_{n_b} \equiv -\hat{q}_{bns} = -\sqrt{RgD}D\alpha_s\left(\varphi\tau_b^*\right)^{1.5}\left(1-\frac{\tau_c^*}{\varphi\tau_b^*}\right)^{4.5}\frac{\alpha_n}{\left(\varphi\tau_b^*\right)^b}S_s \tag{20}$$

Here the hatted quantity is positive, insuring that a transverse sediment transport in the *negative* n direction (from the sidewall region to the bed region) gives a *positive* contribution to the bed sediment. Between (14), (15), (17), (19) and (20), then, the integral equation for sediment conservation in the bed region becomes

$$(1-\lambda_p)\frac{\partial \eta_b}{\partial t} = -\frac{\partial q_{bsb}}{\partial s} + \frac{\hat{q}_{bns}}{B_b} \tag{21}$$

SEDIMENT CONTINUITY IN THE SUBMERGED SIDEWALL REGION

Equation (13) is now integrated from n = n_b to n = n_t in accordance with the definitions of Figure 6.

$$(1-\lambda_p)\int_{n_b}^{n_t}\frac{\partial \eta}{\partial t}dn = -\int_{n_b}^{n_t}\frac{\partial q_{bs}}{\partial s}dn - \int_{n_b}^{n_t}\frac{\partial q_{bn}}{\partial n}dn \tag{22}$$

We now integrate these terms one by one. Between n_b and n_t bed elevation η obeys the relation

$$\eta = \eta_t - S_s(n_t - n) \tag{23}$$

so that

$$\frac{\partial \eta}{\partial t} = \frac{\partial \eta_t}{\partial t} - S_s\frac{\partial n_t}{\partial t} \tag{24}$$

Thus the following result is obtained;

$$(1-\lambda_p)\int_{n_b}^{n_t}\frac{\partial \eta}{\partial t}dn = (1-\lambda_p)(n_e - n_b)\left(\frac{\partial \eta_t}{\partial t} - S_s\frac{\partial n_t}{\partial t}\right) \tag{25}$$

Further reducing with (1b,d,f) it is found that

$$(1-\lambda_p)\int_{n_b}^{n_t}\frac{\partial \eta}{\partial t}dn = -(1-\lambda_p)H\left(\frac{\partial B_b}{\partial t} - \frac{1}{S_s}\frac{\partial \eta_b}{\partial t}\right) \tag{26}$$

Progressing to the next term of (22), according to Leibnitz' rule

$$\frac{\partial}{\partial s}\int_{n_b}^{n_t} q_{bs}dn = \int_{n_b}^{n_t}\frac{\partial q_{bs}}{\partial s}dn + q_{bs}\Big|_{n_t}\frac{\partial n_t}{\partial s} - q_{bs}\Big|_{n_b}\frac{\partial n_b}{\partial s} \tag{27a}$$

or thus

$$-\int_{n_b}^{n_t}\frac{\partial q_{bs}}{\partial s}dn = -\frac{\partial}{\partial s}\int_{n_b}^{n_t} q_{bs}dn + q_{bs}\Big|_{n_t}\frac{\partial n_t}{\partial s} - q_{bs}\Big|_{n_b}\frac{\partial n_b}{\partial s} \tag{27b}$$

It is assumed here that

$$q_{bs} = \begin{cases} q_{bsb}\ , 0 < n < n_b \\ q_{bss}\ , n_b < n < n_t \\ 0\ , n \geq n_t \end{cases} \tag{28}$$

where q_{bsb} and q_{bss} are given by (10a,b). Reducing (27) with (1d) and (28), then,

$$-\int_{n_b}^{n_t}\frac{\partial q_{bs}}{\partial s}dn = -\frac{1}{S_s}\frac{\partial H q_{bss}}{\partial s} - q_{bsb}\frac{\partial B_b}{\partial s} \tag{29}$$

Progressing to the last term,

$$-\int_{n_b}^{n_t}\frac{\partial q_{bn}}{\partial n}dn = q_{bn}\Big|_{n_b} - q_{bn}\Big|_{n_t} \tag{30}$$

The volume transverse rate of delivery of sediment per unit time per unit streamwise distance from the emerged sidewall region to the submerged sidewall region is here denoted as $\hat{q}_{bnt}$, where

$$\hat{q}_{bnt} = -q_{bn}\Big|_{n_t} \tag{31}$$

Note that the hatted quantity is again positive for the geometry of Figure 6. Thus (30) reduces with the aid of (31) and (20) to

$$-\int_{n_b}^{n_e}\frac{\partial q_{bn}}{\partial n}dn = -\hat{q}_{bns} + \hat{q}_{bnt} \tag{32}$$

The integral form (22) finally reduces with the aid of (26), (29) and (32) to

$$(1-\lambda_p)H\left(\frac{\partial B_b}{\partial t} - \frac{1}{S_s}\frac{\partial \eta_b}{\partial t}\right) = \frac{1}{S_s}\frac{\partial H q_{bss}}{\partial s} + q_{bsb}\frac{\partial B_b}{\partial s} + \hat{q}_{bns} - \hat{q}_{bnt} \tag{33}$$

SEDIMENT CONTINUITY IN THE EMERGENT EARTHFLOW SIDEWALL REGION

Equation (13) is now integrated from $n = n_t$ to $n = n_e$ in accordance with the definitions of Figure 6.

$$(1-\lambda_p)\int_{n_t}^{n_e}\frac{\partial\eta}{\partial t}dn = -\int_{n_t}^{n_e}\frac{\partial q_{bs}}{\partial s}dn - \int_{n_t}^{n_e}\frac{\partial q_{bn}}{\partial n}dn \tag{34}$$

We now integrate these terms one by one. Between n_t and n_e bed elevation η obeys the relation

$$\eta = \eta_e - S_e(n_e - n) \tag{35}$$

Differentiating the above expression with respect to time and recalling that η_e is here assumed to be a specified constant, it is found with the aid of (1c,d,e) that

$$\frac{\partial\eta}{\partial t} = -S_e\frac{\partial n_t}{\partial t} = -S_e\frac{\partial}{\partial t}\left(B_b + \frac{H}{S_s} + \frac{\eta_e - \eta_t}{S_e}\right) \tag{36}$$

Between (36) and (1e) the following result is obtained;

$$(1-\lambda_p)\int_{n_t}^{n_e}\frac{\partial\eta}{\partial t}dn = -(1-\lambda_p)(\eta_e - \eta_t)\left[\frac{\partial B_b}{\partial t} + \left(\frac{1}{S_s} - \frac{1}{S_e}\right)\frac{\partial H}{\partial t} - \frac{1}{S_e}\frac{\partial\eta_b}{\partial t}\right] \tag{37}$$

Since there is no streamwise fluvial transport in the emergent earthflow zone, q_{bs} can be assumed to vanish there, so that the second term in (34) can be dropped. The third term in (34) integrates to yield

$$-\int_{n_t}^{n_e}\frac{\partial q_{bn}}{\partial n}dn = q_{bn}\big|_{n_t} - q_{bn}\big|_{n_e} \tag{38}$$

The volume transverse rate of delivery of sediment per unit time per unit streamwise from the top of the earthflow is here denoted as $\hat{q}_{bne}$, where

$$\hat{q}_{bne} = -q_{bn}\big|_{n_e} \tag{39}$$

Note that the hatted quantity is again positive for the geometry of Figure 6. In the present analysis $\hat{q}_{bne}$ is assumed to be a given parameter determined by the movement of the earthflow, and independent of the stream itself. Thus (34) reduces with the aid of (31), (37 – 39) and (20) to

$$(1-\lambda_p)(\eta_e - \eta_t)\left[\frac{\partial B_b}{\partial t} + \left(\frac{1}{S_s} - \frac{1}{S_e}\right)\frac{\partial H}{\partial t} - \frac{1}{S_e}\frac{\partial\eta_b}{\partial t}\right] = \hat{q}_{bnt} - \hat{q}_{bne} \tag{40}$$

The parameter $\hat{q}_{bne}$ specified in m^2/s can be related to the input rate of earthflow material from the side g_{sidee} in Mt/m/a by the relation

$$g_{sidee} = \hat{q}_{bne}(R+1)\frac{31557600}{1x10^6} \tag{41}$$

REDUCTION TO RELATION FOR WIDTH VARIATION

The elimination of $\hat{q}_{bnt}$ from (33) and (40) results in the relation\

$$\begin{aligned}
&(1-\lambda_p)(\eta_e - \eta_b)\frac{\partial B_b}{\partial t} - (1-\lambda_p)\left[\left(\frac{1}{S_s} - \frac{1}{S_e}\right)H + \frac{\eta_e - \eta_b}{S_e}\right]\frac{\partial \eta_b}{\partial t} + \\
&(1-\lambda_p)(\eta_e - \eta_b - H)\left(\frac{1}{S_s} - \frac{1}{S_e}\right)\frac{\partial H}{\partial t} = \\
&\hat{q}_{bns} - \hat{q}_{bne} + \frac{1}{S_s}\frac{\partial Hq_{bss}}{\partial s} + q_{bsb}\frac{\partial B_b}{\partial s}
\end{aligned} \tag{42}$$

Substituting (21) into (42), the equation for the evolution of stream width is found to be

$$\begin{aligned}
&(1-\lambda_p)(\eta_e - \eta_b)\frac{\partial B_b}{\partial t} = \\
&\left[1 + \frac{\eta_e - \eta_b}{S_e B_b} + \left(\frac{1}{S_s} - \frac{1}{S_e}\right)\frac{H}{B_b} + \right]\hat{q}_{bns} - \hat{q}_{bne} - \\
&(1-\lambda_p)(\eta_e - \eta_b - H)\left(\frac{1}{S_s} - \frac{1}{S_e}\right)\frac{\partial H}{\partial t} - \\
&\left[\frac{\eta_e - \eta_b}{S_e} + \left(\frac{1}{S_s} - \frac{1}{S_e}\right)H\right]\frac{\partial q_{bsb}}{\partial s} + \\
&\frac{1}{S_s}\frac{\partial Hq_{bss}}{\partial s} + q_{bsb}\frac{\partial B_b}{\partial s}
\end{aligned} \tag{43}$$

The above equation can be further simplified by noting that according to (7),

$$\frac{\partial H}{\partial t} = -r\frac{\partial B_b}{\partial t} \tag{44a}$$

where

$$r = -\frac{\frac{3}{5} + \frac{3}{10}\frac{c_n - 1}{c_n} - \frac{9}{10}\frac{c_d - 1}{c_d}}{1 - c_1\left(\frac{3}{10}\frac{c_n - 1}{c_n} - \frac{9}{10}\frac{c_d - 1}{c_d}\right)\frac{H_o}{H}} c_1 \frac{H_o}{B_b} \tag{44b}$$

and

$$H_o = \left(\frac{k_s^{1/3} Q_w^2}{\alpha_r^2 g B_b^2 S}\right)^{3/10} \qquad c_1 = \left[\frac{c_d}{c_d^3}\right]^{3/10}$$
$$c_n = 1 + \varphi \frac{S_s + \sqrt{1 + S_s^2}}{S_s}\frac{H}{B_b} \tag{44c,d,e,f}$$
$$c_d = \left(1 + \frac{1}{2S_s}\frac{H}{B_b}\right)$$

Reducing (43) with (44a) the following result is obtained;

$$\begin{aligned}
&(1-\lambda_p)\left[(\eta_e - \eta_b) - r(\eta_e - \eta_b - H)\left(\frac{1}{S_s} - \frac{1}{S_e}\right)\right]\frac{\partial B_b}{\partial t} = \\
&\left[1 + \frac{\eta_e - \eta_b}{S_e B_b} + \left(\frac{1}{S_s} - \frac{1}{S_e}\right)\frac{H}{B_b} + \right]\hat{q}_{bns} - \hat{q}_{bne} - \\
&\left[\frac{\eta_e - \eta_b}{S_e} + \left(\frac{1}{S_s} - \frac{1}{S_e}\right)H\right]\frac{\partial q_{bsb}}{\partial s} + \\
&\frac{1}{S_s}\frac{\partial H q_{bss}}{\partial s} + q_{bsb}\frac{\partial B_b}{\partial s}
\end{aligned} \tag{45}$$

CASE OF VANISHING STREAMWISE VARIATION

Consider a case for which all parameters, including $\hat{q}_{bne}$ are constant in the streamwise direction. For this case (21) and (45) reduce to

$$(1-\lambda_p)\frac{d\eta_b}{dt} = \frac{\hat{q}_{bns}}{B_b} \tag{46}$$

$$\begin{aligned}
&(1-\lambda_p)\left[(\eta_e - \eta_b) - r(\eta_e - \eta_b - H)\left(\frac{1}{S_s} - \frac{1}{S_e}\right)\right]\frac{dB_b}{dt} = \\
&\left[1 + \frac{\eta_e - \eta_b}{S_e B_b} + \left(\frac{1}{S_s} - \frac{1}{S_e}\right)\frac{H}{B_b} + \right]\hat{q}_{bns} - \hat{q}_{bne}
\end{aligned} \tag{47}$$

It is this case that is considered in the spreadsheet program.

NEWTON-RAPHSON SCHEME FOR THE FLOW

Equations (7a,b) can be written in the form

$$f(H) = H - H_o c_1 = 0 \tag{48}$$

where

$$H_o = \left(\frac{k_s^{1/3} Q_w^2}{\alpha_r^2 g B_b^2 S} \right)^{3/10} \qquad c_1 = \left[\frac{c_d}{c_d^3} \right]^{3/10}$$
$$c_n = 1 + \varphi \frac{S_s + \sqrt{1 + S_s^2}}{S_s} \frac{H}{B_b} \tag{49a,b,c,d}$$
$$c_d = \left(1 + \frac{1}{2S_s} \frac{H}{B_b} \right)$$

Taking the derivative of (48) with respect to H,

$$f'(H) = 1 - H_o c_1'$$
$$c_1' = \frac{c_1}{H} \left[\frac{3}{10} \frac{c_n - 1}{c_n} - \frac{9}{10} \frac{c_d - 1}{c_d} \right] \tag{50a,b}$$

The Newton-Raphson scheme for depth is thus

$$H_{new} = H - \frac{f(H)}{f'(H)} \tag{51}$$

KINEMATIC WAVE IN WIDTH

The form of (45) suggests the possibility of channel width evolution in the form of an upstream-migrating kinematic wave. In particular, the equation can be rewritten as

$$\frac{\partial B_b}{\partial t} - c_B \frac{\partial B_b}{\partial s} = X \tag{52}$$

where

$$c_B = \frac{q_{bsb}}{(1-\lambda_p)\left[(\eta_e - \eta_b) - r(\eta_e - \eta_b - H)\left(\frac{1}{S_s} - \frac{1}{S_e}\right)\right]}$$

$$X = \frac{A}{B}$$

$$A = \left[1 + \frac{\eta_e - \eta_b}{S_e B_b} + \left(\frac{1}{S_s} - \frac{1}{S_e}\right)\frac{H}{B_b} + \right]\hat{q}_{bns} - \hat{q}_{bne} - \qquad (53a,b,c,d)$$

$$\left[\frac{\eta_e - \eta_b}{S_e} + \left(\frac{1}{S_s} - \frac{1}{S_e}\right)H\right]\frac{\partial q_{bsb}}{\partial s} + \frac{1}{S_s}\frac{\partial H q_{bss}}{\partial s}$$

$$B = (1-\lambda_p)\left[(\eta_e - \eta_b) - r(\eta_e - \eta_b - H)\left(\frac{1}{S_s} - \frac{1}{S_e}\right)\right]$$

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